

Solutions to Example Problems in Engineering Noise Control, 2nd Edn.

*A companion to
"Engineering Noise Control", 3rd Edn*

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All page, equation and table references are to the third edition of the textbook, Engineering Noise Control by DA Bies and CH Hansen

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Preface

This book provides detailed and instructive solutions to the book of problems on acoustics and noise control which is intended as a companion to the 3rd edition of the book "Engineering Noise Control" by David A. Bies and Colin H. Hansen. The problems and solutions cover chapters 1 to 10 and 12 in that text. Some of the problems and solutions are formulated to illustrate the physics underlying the acoustical concepts and others are based on actual practical problems. Many of the solutions extend the discussion in the text and illustrate the more difficult concepts by example, thus acting as a valuable source and understanding for the consultant and student alike.

Although most of the problems and solutions have been tested on students, it is highly likely that there exist errors of which I am unaware. I would dearly like to hear from any readers who may discover any errors, no matter how minor.

C.H.H., September, 2003
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Finally, the author would like to express his deep appreciation to his family, particularly his wife Susan and daughters Kristy and Laura for their patience and support during the many nights and weekends needed to assemble the problems and solutions in a form suitable for publication.

This book is dedicated
to Susan,
to Kristy
and to Laura.

Solutions to problems in Fundamentals

Unless otherwise stated an air temperature of 20°C corresponding to an air density of 1.206kg/m³ and a speed of sound of 343m/s has been assumed.

Problem 1.1

- undertake an assessment of the current environment where there appears to be a problem, including the preparation of noise level contours where required;
- establish the noise control objectives or criteria to be met;
- identify noise transmission paths and generation mechanisms;
- rank order noise sources contributing to any excessive levels;
- formulate a noise control program and implementation schedule;
- carry out the program; and
- verify the achievement of the objectives of the program.

See pages 8-10 in the third edition of the text for more details.

Problem 1.2

- (a) Speed of sound given by equation 1.4 in text. The only variables which are different for the two gases are γ and M . Thus:

$$\frac{c_{air}}{c_{He}} = \left(\frac{M_{He}}{\gamma_{He}} \times \frac{\gamma_{air}}{M_{air}} \right)^{1/2} = \left(\frac{4}{5/3} \right)^{1/2} \left(\frac{1.4}{29} \right)^{1/2} = 0.34$$

- (b) The wavelength of sound emitted from ones mouth is a function of the vocal cord properties which remain unchanged by the presence of helium. As $f\lambda = c$ and λ is fixed and c is faster in helium, the sound emanating from your mouth will be higher in pitch.

Problem 1.3

Using the relation for h given in the question and knowing that atmospheric pressure is 101.4kPa, we can write:

$$h = \frac{4240}{101.4 \times 10^3} \times \frac{95}{100} = 0.0397$$

The speed of sound in dry air at 30°C is given by:

$$c_{dry} = \sqrt{\gamma RT/M} = \sqrt{1.4 \times 8.314 \times 303.2 / 0.029} = 348.8 \text{ m/s}$$

The speed of sound in wet air is then:

$$c_{wet} = 348.8(1 + 0.0397 \times 0.16) = 351.0 \text{ m/s}$$

Problem 1.4

Gas volume flow rate = 250,000 m³ per day at STP

Gas Law also applies to a moving fluid, so:

$$P\dot{V} = \frac{\dot{m}}{M}RT$$

P and T are the static pressure and temperature respectively and

$$\dot{V} = \frac{250,000}{24 \times 3,600} = 2.894 \text{ m}^3/\text{s}$$

(a) The mass flow rate is given by:

$$\dot{m} = \frac{P\dot{V}M}{RT} = \frac{101,400 \times 0.029 \times 2.8935}{8.314 \times 288.2} = 3.550 \text{ kg/s}$$

(b) Density of gas in pipe is:

$$\rho = \frac{\dot{m}}{\dot{V}} = \frac{PM}{RT} = \frac{8 \times 10^6 \times 0.029}{8.314 \times 393.2} = 71.0 \text{ kg/m}^3$$

(c) Gas flow speed in discharge pipe is:

$$U = \frac{4}{\pi d^2} \dot{V} = \frac{4}{\pi d^2} \frac{\dot{m}}{\rho} = \frac{4}{\pi \times 0.01} \times \frac{3.551}{71.0} = 6.37 \text{ m/s}$$

Speed of sound (relative to fluid) is:

$$c = \left(\frac{\gamma P}{\rho} \right)^{1/2} = \sqrt{1.4 \times 8 \times 10^6 / 71.0} = 397 \text{ m/s}$$

(d) Speed of sound relative to the pipe is thus: $397.2 + 6.4 = 404 \text{ m/s}$.

Problem 1.5

The speed of sound is given by: $c = \sqrt{\gamma RT/M}$ and for any gas, (R/M) is fixed. We can find (R/M) by using the properties at 0°C and the expression:

$$PV = \frac{m}{M}RT \text{ or } \frac{R}{M} = \frac{PV}{mT}$$

As $(m/V) = 1.4 \text{ kg/m}^3$, $\frac{R}{M} = \frac{101400}{1.4 \times 273}$ and thus:

$$c = \left(\frac{1.35 \times 101400 \times 1273}{1.4 \times 273} \right)^{1/2} = 675 \text{ m/s}$$

Problem 1.6

The Universal gas law may be written as $PV = \frac{m}{M}RT$ or $P = \frac{m}{M} \frac{RT}{V}$.

Differentiating gives:

$$\frac{dP}{dV} = \frac{m}{M} \frac{R}{V} \frac{dT}{dV} - \frac{m}{M} \frac{RT}{V^2} \quad \text{or} \quad \frac{dp}{dV} = \frac{P}{T} \frac{dT}{dV} - \frac{P}{V}$$

which may be rewritten as:

$$\frac{dP}{P} + \frac{dV}{V} = \frac{dT}{T}$$

Another gas property associated with the adiabatic expansion and contraction of a sound wave is $PV^\gamma = \text{const}$, which leads to:

$$\frac{dV}{V} = -\frac{1}{\gamma} \frac{dP}{P}$$

Combining the above two equations gives:

$$\frac{dP}{P} \left(1 - \frac{1}{\gamma} \right) = \frac{dT}{T}$$

The acoustic pressure associated with a sound wave of intensity 95dB is calculated as follows:

Thus, $I = \frac{\bar{p}^2}{2\rho c} = I_{ref} 10^{L_I/10}$ and \bar{p} = pressure amplitude

$$\text{Thus, } \bar{p} = [2 \times 1.205 \times 343 \times 10^{-12} \times 10^{9.5}]^{1/2} = 1.62 \text{ Pa}$$

We can write the results of the preceding analysis as:

$$\frac{\bar{p}}{P} \left(1 - \frac{1}{\gamma} \right) = \frac{\bar{\tau}}{T}$$

Substituting in given values and rearranging the equation to give τ , we obtain:

$$\bar{\tau} = \frac{1.617(1 - 1/1.4)}{101400} \times 298 = 1.4 \times 10^{-3} \text{ }^\circ\text{C}$$

which is the amplitude of the temperature fluctuations.

Problem 1.7

Using equations 1.4a and b in the text, it can be shown that:

$$\rho c = \frac{\gamma P}{\sqrt{\gamma RT/M}} = \frac{1.4 \times 101400}{\sqrt{1.4 \times 8.314 \times 313.2/0.029}} = 400.4$$

Problem 1.8

$$f = c/4L = 343/(4 \times 4) = 21.4 \text{ Hz}$$

Problem 1.9

- (a) Assume that the tail pipe is effectively open at each end as it follows a muffler.

$$f = 250 = c/2L = c/2.4. \quad \text{Thus, } c = 600 \text{ m/s}$$

Thus:

$$c = \sqrt{\frac{\gamma R T}{M}} = \sqrt{\frac{1.4 \times 8.314 \times T}{0.035}}$$

$$T = \frac{600^2 \times 0.035}{1.4 \times 8.314} = 1083^\circ \text{K} = 810^\circ \text{C}$$

- (b)

$$\rho = \frac{\gamma P}{c^2} = \frac{1.4 \times 101.4 \times 10^3}{600^2} = 0.39 \text{ kg/m}^3$$

Assumption is that the gas in the pipe is at atmospheric pressure at sea level.

Problem 1.10

$$(a) \quad c = \sqrt{\frac{\gamma R T}{M}} = \sqrt{\frac{1.4 \times 8.314 \times 1873}{0.035}} = 789 \text{ m/s}$$

$$(b) \quad \rho = \frac{\gamma P}{c^2} = \frac{1.4 \times 101.4 \times 10^3}{789^2} = 0.23 \text{ kg/m}^3$$

$$(c) \quad \lambda = c/f = 789/40 = 19.7 \text{ m}$$

If we treat the furnace like a closed end tube, $f = Nc/2L$, so
 $L = c/2f = 789/80 = 9.9$ m

So we can expect one dimension of the furnace to be 9.9 m.

(d) Surface area, $S = \pi dL + 2\pi r^2 = \pi \times 4 \times 9.86 + 2\pi \times 4 = 149 \text{ m}^2$

$$\begin{aligned} L_w &= L_p - 10 \log_{10} \left[\frac{4(1 - \bar{\alpha})}{S\bar{\alpha}} \right] - 10 \log_{10} \left[\frac{\rho c}{400} \right] \\ &= 120 - 10 \log_{10} \left[\frac{4 \times 0.98}{149 \times 0.02} \right] - 10 \log_{10} \left[\frac{789 \times 0.23}{400} \right] = 122.3 \text{ dB} \\ W &= 10^{-12} \times 10^{122.3/10} = 1.7 \text{ W} \end{aligned}$$

- (e) Second burner has twice the sound power level, so increase in sound pressure level will be $10 \log_{10}(2)$ or an increase of 3.0 dB. So the new SPL = 125.3 dB.

Problem 1.11

- (a) The bulk modulus of the fluid is given by equation 1.2 in the text as:

$$D = -V \left(\frac{\partial V}{\partial p} \right)^{-1}$$

Consider a unit volume of fluid ($V = 1$), let the proportion (and volume V_g) of gas be x and the proportion (and volume V_L) of liquid is then $(1 - x)$. Assume a pressure increment of Δp , and a corresponding volume increment ΔV .

From the above equation, we can write, $\Delta V_L = -\Delta p(1 - x)/D$

Assuming adiabatic compression of gas,

$PV^\gamma = \text{const}$ or $P = \text{const} \times V^{-\gamma}$. Differentiating P with respect to V and rearranging gives:

$$\gamma \frac{dV_g}{V_g} + \frac{dP}{P} = 0$$

Substituting x for V_g and rearranging gives:

$$\Delta V_g = -\frac{x\Delta P}{\gamma P}$$

Thus, the total volume change for a change Δp in pressure = $\Delta V_L + \Delta V_g = \Delta V$. Thus:

$$\Delta V = \left(-\frac{1-x}{D} - \frac{x}{\gamma P} \right) \Delta P$$

Thus the effective bulk modulus of the fluid is given by:

$$D_{eff} = -\left(\frac{\Delta V}{\Delta P} \right)^{-1} = -\left(\frac{1-x}{D} - \frac{x}{\gamma P} \right)^{-1}$$

The effective density of the fluid is equal to the total mass for unit volume of fluid and is given by:

$$\rho_{eff} = (1-x)\rho_L + x\rho_g$$

The speed of sound is given by $c = \sqrt{D/\rho}$. Substituting in the effective values calculated for D and ρ above we obtain:

$$c = \sqrt{\frac{1}{\left(\frac{1-x}{D} + \frac{x}{\gamma P} \right) ((1-x)\rho_L + x\rho_g)}}$$

where x is the proportion of gas in the fluid.

(b) As x approaches 0, using the result of (a) above gives:

$$c = \sqrt{\frac{1}{(1/D)\rho_L}} = \sqrt{D/\rho_L} \quad \text{which is the speed of sound in the liquid.}$$

As x approaches 1, using the result of (a) above gives:

$$c = \sqrt{\frac{1}{(1/\gamma P)\rho_g}} = \sqrt{\frac{\gamma P}{\rho_g}} \quad \text{which is the speed of sound in the gas.}$$

Problem 1.12

Equation 1.40c in the text is the harmonic solution to the spherical wave equation. That is:

$$\varphi = \frac{A}{r} e^{j(\omega t - kr)}$$

For spherical waves, equation 1.6 may be written as $u = -\frac{\partial \varphi}{\partial r}$ and thus the particle velocity may be written as in equation 1.42. Using equations 1.7 and 1.40, the acoustic pressure may be written as in equation 1.41b. Using equations 1.41b and 1.42, we may write:

$$\frac{p}{u} = \frac{j\omega\rho}{1/r + jk}$$

Using $\omega = kc$ and multiplying the numerator and denominator of the above equation by r gives equation 1.43 in the text.

Multiplying the numerator and denominator of equation 1.43 by $(1 - jkr)$ gives:

$$\frac{p}{u} = \rho c \frac{jkr(1 - jkr)}{1 + k^2 r^2} = \frac{\rho c k r (j + kr)}{1 + k^2 r^2} = \frac{\rho c k r}{\sqrt{1 + k^2 r^2}} \times \frac{j + kr}{\sqrt{1 + k^2 r^2}}$$

Defining the phase $\beta = \tan^{-1}(1/kr)$ as the phase by which the pressure leads the particle velocity, the preceding equation may be written as:

$$\frac{p}{u} = \rho c \cos\beta (j\sin\beta + \cos\beta) = \rho c \cos\beta e^{j\beta}$$

which is the same as equation 1.72.

Harmonic intensity is defined as $I = 0.5 \text{Re}\{\bar{p}u^*\}$ where the bar denotes the complex amplitude. Using equation 1.72, we may write:

$$u = \frac{p}{\rho c \cos\beta} e^{-j\beta}$$

Thus, $0.5 \times \text{Re}\{\bar{p}u\} = 0.5 \frac{\bar{p}^2 \cos\beta}{\rho c \cos\beta} = \frac{\langle p^2 \rangle}{\rho c}$, which is the plane wave expression.

Problem 1.13

(a) At any location, $p = \frac{A}{r} e^{j\omega(t - r/c)} = \frac{A}{r} e^{j(\omega t - kr)}$. The velocity potential

is then $\varphi = \frac{1}{\rho} \int p dt = \frac{A}{j\rho r \omega} e^{j(\omega t - kr)}$ and the particle velocity is

$$u = -\frac{\partial \varphi}{\partial r} = \frac{Ak}{\rho r \omega} e^{j(\omega t - kr)} + \frac{A}{j\rho r^2 \omega} e^{j(\omega t - kr)}$$

(b) When $r = r_0$, $u = U$ and the particle velocity may be written as:

$$U = \frac{A}{\rho r_0 \omega} e^{j(\omega t - kr_0)} \left(k + \frac{1}{jr_0} \right)$$

But $U = U_0 e^{j\omega t}$, so $U_0 = \frac{A}{\rho r_0 \omega} \left(k + \frac{1}{jr_0} \right) e^{-jkr_0}$.

Thus, $A = \frac{\rho r_0 \omega U_0}{\left(k + \frac{1}{jr_0} \right)} e^{jkr_0}$

However, $e^{jkr_0} = \cos kr_0 + js \sin kr_0 \approx 1 + jkr_0$ for small kr_0

Thus, $A = \frac{\rho r_0 \omega U_0}{\left(k + \frac{1}{jr_0} \right)} (1 + jkr_0) = j\rho r_0^2 \omega U_0$

(c) $\omega = 200\pi$, $\rho = 1.206$, $U_0 = 2$, $r_0 = 0.05$

Power, $W = IS = \frac{(p^2/\rho c)4\pi r^2}{2\rho c} = \frac{|p|^2}{2\rho c} 4\pi r^2 = \frac{|A|^2}{2\rho c r^2} 4\pi r^2$ and

$|A|^2 = \omega^2 r_0^4 \rho^2 U_0^2$. Thus:

$$W = 2\pi \omega^2 r_0^4 \rho U_0^2 / c$$

$$= 2\pi(2\pi \times 100)^2 \times 0.05^4 \times 1.205 \times 2^2 / 344 = 0.22 W$$

Problem 1.14

Spherical wave solution to the wave equation is:

$$p = \frac{A}{r} e^{j(\omega t - kr)}$$

Using equation 1.7 in the text, the velocity potential is:

c

$$\varphi = \frac{A}{j\omega\rho r} e^{j(\omega t - kr)}$$

Using the one dimensional form of equation 1.6 in the text, the particle velocity is:

$$\begin{aligned} u &= -\frac{\partial\varphi}{\partial r} = \frac{jkA}{jr\omega\rho} e^{j(\omega t - kr)} + \frac{A}{jr^2\omega\rho} e^{j(\omega t - kr)} \\ &= \frac{A}{r\rho c} e^{j(\omega t - kr)} \left(1 - \frac{j}{kr} \right) = \frac{p}{\rho c} \left(1 - \frac{j}{kr} \right) \end{aligned}$$

Thus:

$$|u| = \frac{|p|}{\rho c} \left| 1 - \frac{j}{kr} \right|$$

Amplitude is twice $|p|/\rho c$ when $|1 - j/kr| = 2$ or $1 + (kr)^{-2} = 4$.

Thus $kr = 1/\sqrt{3} = 0.58$

Problem 1.15

(a) Plane wave: $p = \rho c u = 1.206 \times 343 \times 0.2$

$$L_p = 20 \log_{10} \frac{1.206 \times 343 \times 0.2}{2 \times 10^{-5}} = 132 \text{ dB}$$

$$(b) L_p = 20 \log_{10} \frac{988 \times 1486 \times 0.2}{10^{-6}} = 229 \text{ dB}$$

Non-linear effects are definitely important as the level in air is above 130dB

and the level in water is also very high.

Cavitation in water would occur if the sound pressure amplitude exceeds the mean pressure which would happen if the water were less than a certain depth.

The depth is calculated by calculating the weight of a column of water, 1 m^2 in cross section. Weight = $988 \times 9.81 = 9692.3 \text{ N}$, so the water pressure is 9692.3 Pa per meter depth. The acoustic pressure amplitude is $1.41 \times 988 \times 1486 \times 0.2 = 0.414 \text{ MPa}$. So the depth of water this corresponds to (allowing for atmospheric pressure) is:

$$h = (0.414 \times 10^6 - 101400)/9692.3 = 32.2 \text{ m deep.}$$

Problem 1.16

- (a) The instantaneous total pressure will become negative if the acoustic pressure amplitude exceeds the mean pressure. This corresponds to an r.m.s. acoustic pressure of $150/\sqrt{2} = 106.07 \text{ kPa}$. The sound pressure level is then:

$$L_p = 20 \log_{10} \left(\frac{p_{rms}}{10^{-6}} \right) = 20 \log_{10} \left(\frac{106.07 \times 10^3}{10^{-6}} \right) = 220.5 \text{ dB}$$

- (b) The particle velocity is given by $u = p/\rho c$. Thus the amplitude of the particle velocity is $u = 150 \times 10^3/(988 \times 1481) = 103 \text{ mm/s}$
- (c) Using the analysis of problem 1.14, we can show that for a spherical wave, the particle velocity amplitude is:

$$|u| = \frac{|p|}{\rho c} \left| 1 - \frac{j}{kr} \right| = \frac{150 \times 10^3}{988 \times 1481} \sqrt{1 + \left(\frac{1481}{2\pi \times 1000 \times 1} \right)^2}$$

$$= 105 \text{ mm/s}$$

Problem 1.17

- (a) The sound pressure level at 10m will be $20\log_{10}(10/1)$ less than at 1m, which translates to 90dB.
- (b) Spherical wave solution to the wave equation is:

$$p = \frac{A}{r} e^{j(\omega t - kr)}$$

Using equation 1.7 in the text, the velocity potential is:

$$\varphi = \frac{A}{j\omega\rho r} e^{j(\omega t - kr)}$$

Using the one dimensional form of equation 1.6 in the text, the particle velocity is:

$$\begin{aligned} u &= -\frac{\partial\varphi}{\partial r} = \frac{jkA}{jr\omega\rho} e^{j(\omega t - kr)} + \frac{A}{jr^2\omega\rho} e^{j(\omega t - kr)} \\ &= \frac{A}{r\rho c} e^{j(\omega t - kr)} \left(1 - \frac{j}{kr} \right) = \frac{p}{\rho c} \left(1 - \frac{j}{kr} \right) \end{aligned}$$

Thus:

$$|u| = \frac{|p|}{\rho c} \left| 1 - \frac{j}{kr} \right|$$

$$\rho c = 1.206 \times 343 = 413.7; k = 2\pi f/c = 2\pi \times 100/343 = 1.832$$

At 1m, $1/kr = 0.546$; at 10m, $1/kr = 0.0546$. The acoustic pressure amplitude at 1m is:

$$\begin{aligned} |p| &= \sqrt{2} \times p_{ref} \times 10^{L_p/20} = \sqrt{2} \times 2 \times 10^{-5} \times 10^{110/20} \\ &= 8.94 \text{ Pa} \end{aligned}$$

Thus at 1m, the particle velocity amplitude is:

$$|u| = \frac{8.94}{413.6} \sqrt{1 + 0.546^2} = 25 \text{ mm/s}$$

The acoustic pressure amplitude at 10m is:

$$\begin{aligned}
 |p| &= \sqrt{2} \times p_{ref} \times 10^{L_p/20} = \sqrt{2} \times 2 \times 10^{-5} \times 10^{90/20} \\
 &= 0.894 \text{ Pa}
 \end{aligned}$$

And at 10m, the particle velocity amplitude is:

$$|u| = \frac{0.894}{413.6} \sqrt{1 + 0.0546^2} = 2.2 \text{ mm/s}$$

Problem 1.18

(a) Spherical wave solution to the wave equation is:

$$p = \frac{A}{r} e^{j(\omega t - kr)}$$

Using equation 1.7 in the text, the velocity potential is:

$$\varphi = \frac{A}{j\omega\rho r} e^{j(\omega t - kr)}$$

Using the one dimensional form of equation 1.6 in the text, the particle velocity is:

$$\begin{aligned}
 u &= -\frac{\partial\varphi}{\partial r} = \frac{jkA}{jr\omega\rho} e^{j(\omega t - kr)} + \frac{A}{jr^2\omega\rho} e^{j(\omega t - kr)} \\
 &= \frac{A}{r\rho c} e^{j(\omega t - kr)} \left(1 - \frac{j}{kr} \right) = \frac{p}{\rho c} \left(1 - \frac{j}{kr} \right)
 \end{aligned}$$

(b) Specific acoustic impedance, $Z = p/u$. Thus:

$$Z = \rho c \left(1 - \frac{j}{kr} \right)^{-1}$$

(c) The modulus of the impedance of the spherical wave is half that of a plane wave (ρc) when

$$|Z| = \rho c k r \frac{\sqrt{1 + k^2 r^2}}{1 + k^2 r^2} = \frac{\rho c k r}{\sqrt{1 + k^2 r^2}} = \rho c / 2$$

Thus:

$$4k^2 r^2 = 1 + k^2 r^2 \text{ or } k^2 r^2 = 0.3333$$

Thus,
$$r = (\lambda/2\pi)\sqrt{0.3333} = 0.092\lambda$$

Problem 1.19

- (a) r.m.s. sound pressure

$$p_{rms} = \sqrt{I\rho c} = \sqrt{\frac{W\rho c}{4\pi r^2}} = \sqrt{\frac{1 \times 1.205 \times 343}{4\pi \times 0.3^2}} = 19.12 \text{ Pa}$$

(b) $SPL = 20 \log_{10} \frac{19.1}{2 \times 10^{-5}} = 119.6 \text{ dB}$

- (c) r.m.s. particle velocity (see equation 1.43 in text):

$$|u_r| = \frac{|p_r| \sqrt{1 + k^2 r^2}}{k r \rho c} = \frac{19.025 (1 + k^2 r^2)^{1/2}}{413.6 k r}$$

$$k r = \frac{2\pi \times 1000}{343} \times 0.3 = 5.50$$

$$\text{Thus, } u = \frac{19.125 [1 + 5.50^2]^{1/2}}{413.7 \times 5.50} = 0.047 \text{ m/s}$$

- (d) Phase between pressure and particle velocity given by equation 1.73 as:
 $\beta = \tan^{-1}[1/(kr)] = \tan^{-1}(1/5.50) = 10.3^\circ$, and the acoustic pressure leads the particle velocity.

- (e) As shown on p35 in the text, spherical wave intensity is $p_{rms}^2/(\rho c)$.
 Substituting in the value for p_{rms} calculated in (a) above gives:
 $I = 19.125^2/(1.205 \times 343) = 0.885 \text{ W/m}^2$

- (f) From equation 1.74 in the text, the amplitude of the reactive intensity is:

$$I_r = \frac{p_{rms}^2}{\rho c k r} = \frac{19.125^2}{1.205 \times 343 \times 5.50} = 0.161 \text{ W/m}^2$$

- (g) The sound intensity level is:

$$L_I = 10 \log_{10} 0.885 + 120 = 119.5 \text{ dB}$$

- (h) 1500Hz is a different frequency to 1000Hz so the mean square pressures add.

$$\text{Thus, } p_{rms} = \sqrt{2} \times 19.125 = 27.0 \text{ Pa.}$$

Problem 1.20

$$L_{oct} = 10 \log_{10} (10^{7.8} + 10^{7.3} + 10^8) = 80.8 \text{ dB}$$

Problem 1.21

- (a) For a spherical source:

$$p_{rms}^2 = \rho c I = \rho c W / S = \rho c W / 4\pi r^2.$$

As the sources are uncorrelated we may add p^2 for each. For source A, $r = 10$.

For source B, $r = 10\sqrt{2}$.

For source C, $r = 10$.

Thus the total p^2 at location D is:

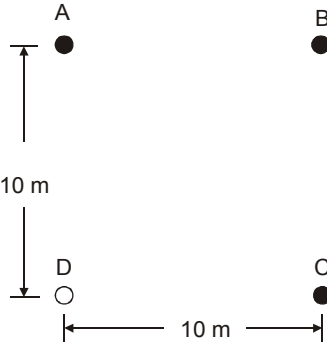
$$p_{rms}^2 = \frac{1.206 \times 343}{4\pi} \left(\frac{10}{100} + \frac{20}{200} + \frac{15}{100} \right) = 11.52 \text{ Pa}^2$$

Thus, the sound pressure level is:

$$L_p = 10 \log_{10} \frac{11.52}{(2 \times 10^{-5})^2} = 104.6 \text{ dB}$$

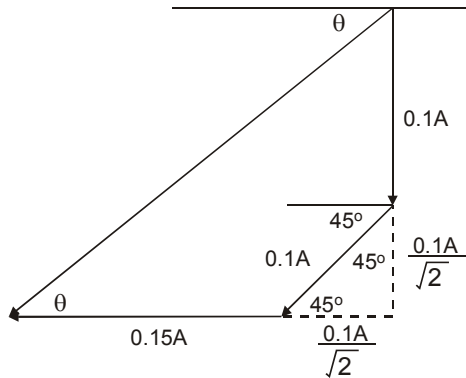
- (b) Intensity, $I \propto p^2$. Thus, $I \propto W/r^2 = AW/r^2$. The resultant intensity can thus be calculated using the figure on the next page.

$$\tan \theta = \frac{0.1A(1 + 1/\sqrt{2})}{0.1A(1.5 + 1/\sqrt{2})} = 0.7734$$



Thus the direction of the intensity vector is 37.7° below the horizontal.

- (c) Sound intensity is a measure of the net flow of energy in an acoustic disturbance. For a single frequency field the sound intensity is the product of the acoustic pressure with the in-phase component of the particle velocity. For a



broadband field it is the time average product of the acoustic pressure and particle velocity. Thus its measurement requires a knowledge of the acoustic pressure and particle velocity. Equations 1.6 and 1.7 in the text indicate that the particle velocity is proportional to the pressure gradient which can be approximated by subtracting the measurements from two microphones and dividing the result by the separation distance. In the far field of the source, the pressure and particle velocity are in phase and related by $p = \rho c u$. Thus, in the far field, only one microphone is necessary as the pressure gradient need not be calculated.

- (d) For the determination of sound power, sound intensity measurements can still give accurate results in the presence of reflecting surfaces or other nearby noisy equipment, whereas results obtained using sound pressure measurements are likely to be seriously in error.

Problem 1.22

- (a) Specific acoustic impedance is the ratio of acoustic pressure to particle velocity at any location in the medium containing the acoustic disturbance. It is a function of the type of disturbance as well as of the acoustic medium.
- (b) Characteristic impedance is a material or acoustic medium property, is equal to ρc and is the specific acoustic impedance of a plane wave in an infinitely extending medium.

- (c) Interference describes the interaction between two or more sound waves of the same frequency such that regions of reinforcement (increased sound pressure) and regions of cancellation (reduced sound pressure) are formed.
- (d) Phase speed is the speed at which a single frequency sound wave propagates and is proportional to the rate of change of phase experienced by a stationary observer as the sound wave propagates past.
- (e) Sound power is a measure of the total rate of energy emission by an acoustic source and has the units of watts.
- (f) Particle velocity describe the oscillatory motion of particles in an acoustic medium during propagation of an acoustic disturbance.

Problem 1.23

A flat spectrum level implies equal energy in each 1Hz wide frequency band. As the bandwidth of an octave band doubles from one band to the next, the energy level will increase by 3dB each time the octave band centre frequency is doubled.

Similarly, the level will increase by 1dB for one third octave bands, each time the band centre frequency is stepped up.

Problem 1.24

- (a) The noise levels due to the machine only at each of the three locations are:

$$10\log_{10}(10^{98/10} - 10^{95/10}) = 95.0 \text{ dB}$$

$$10\log_{10}(10^{102/10} - 10^{98/10}) = 99.8 \text{ dB}$$

$$10\log_{10}(10^{96/10} - 10^{94/10}) = 91.7 \text{ dB}$$

The L_{eq} at 500Hz is:

$$10\log_{10}(10^{95/10} + 10^{99.8/10} + 10^{91.7/10}) = 101.5 \text{ dB}$$

(b)

$$\text{dB reduction} = -10\log_{10}\frac{1}{3}\left[10^{-1.5} + 10^{-2} + 10^{-2.3}\right] = 18.1 \text{ dB}$$

Problem 1.25

Use equation 1.90 in text, 3rd edn. as the two signals will be coherent.

If the two speakers operate together, the phase difference between the two signals is zero as the speakers are identical and driven by the same amplifier. Thus $\cos\theta = 1$.

$$p_1^2 = p_2^2 = p_{ref}^2 10^{L_p/10} \text{ and } p_t^2 = 4 \times p_1^2$$

$$\text{Thus, } L_{p_t} = L_{p_1} + 10\log_{10}4 = 85 + 6 = 91 \text{ dB}$$

If a 45° phase shift were introduced, then $\cos\theta = 0.707$ and

$$p_t^2 = (2 + 2 \times 0.707)p_1^2 = 3.414p_1^2.$$

$$\text{Thus: } L_{p_t} = L_{p_1} + 10\log_{10}3.414 = 85 + 5.3 = 90.3 \text{ dB.}$$

Problem 1.26

Use equation 1.90 in text, as the signals to be added are coherent.

$$p_1 = p_2 = p_{ref} 10^{85/20}$$

$$p_t^2 = p_{ref}^2 (10^{8.5} + 10^{8.5} + 2 \times 10^{8.5} \times \cos 30)$$

$$L_{p_t} = 10\log_{10} \frac{p_t^2}{p_{ref}^2} = 90.7 \text{ dB}$$

For signals 180° out of phase, $\cos 180^\circ = -1$ and thus $p_t^2 = 0$ which means that $L_{p_t} = -\infty$. In practice the measured level would be greater than this due to electronic instrumentation noise.

Problem 1.27

- (a) Each signal has an amplitude of A , an r.m.s. value of $A/\sqrt{2}$ and the relative phase between them is φ radians.
- (b) As the signals are at the same frequency and are shifted in phase by a constant amount, they are coherent, so equation 1.90 in the text is used to add them together. Therefore the r.m.s. value of the combined signal is given by:

$$p_{rms} = \sqrt{A^2/2 + A^2/2 + A^2 \cos \varphi} = A\sqrt{(1 + \cos \varphi)}$$

With just a single source, $p'_{rms} = A/\sqrt{2}$. The difference in dB between the two is thus:

$$\Delta L = 10 \log_{10} \left(\frac{p_{rms}}{p'_{rms}} \right)^2 = 10 \log_{10} (2 + 2 \cos \varphi) = 4.8 \text{ dB}$$

Thus for a value of $\varphi = 60^\circ$, $\Delta L = 4.8 \text{ dB}$

Problem 1.28

- (a) As the waves are from the same source, one may be described by $p_1 = P_1 e^{j(\omega t - kx_1)}$ and the other by $p_2 = P_2 e^{j(\omega t - kx_2)}$, where x_1 and x_2 are the path lengths of the two waves. The phase difference is thus

$$\beta_2 - \beta_1 = k(x_1 - x_2) = \frac{2\pi f_c}{c}(x_1 - x_2), \text{ where } f_c \text{ is the centre frequency of the band of noise.}$$

- (b) If $(x_1 - x_2) = \lambda/2 = c/2f_c$, the phase difference is:

$$\beta_2 - \beta_1 = \frac{2\pi f_c}{c} \times \frac{c}{2f_c} = \pi$$

Substituting this result into equation 1.90 in the text gives:

$$\langle p_t^2 \rangle = \langle p_1^2 \rangle + \langle p_2^2 \rangle - 2\langle p_1 p_2 \rangle = \langle (p_1 - p_2)^2 \rangle, \quad p_1 > p_2$$

If $(x_1 - x_2) = \lambda = c/f_c$, the phase difference is:

$$\beta_2 - \beta_1 = \frac{2\pi f_c}{c} \times \frac{c}{f_c} = 2\pi$$

Substituting this result into equation 1.90 in the text gives:

$$\langle p_t^2 \rangle = \langle p_1^2 \rangle + \langle p_2^2 \rangle + 2\langle p_1 p_2 \rangle = \langle (p_1 + p_2)^2 \rangle$$

- (c) If all phases are present the total pressure is given by equation 1.90 averaged over all phases. Thus:

$$\begin{aligned} \langle p_t^2 \rangle &= \langle p_1^2 \rangle + \langle p_2^2 \rangle + 2\langle p_1 p_2 \rangle \frac{1}{2\pi} \int_0^{2\pi} \cos \alpha \, d\alpha \\ &= \langle p_1^2 \rangle + \langle p_2^2 \rangle + 2\langle p_1 p_2 \rangle \frac{1}{2\pi} [\sin \alpha]_0^{2\pi} \\ &= \langle p_1^2 \rangle + \langle p_2^2 \rangle \end{aligned}$$

which is the result for the incoherent case.

Problem 1.29

Following example 1.4 on p49 in the text, the level due to the "first" signal alone is given by $10 \log_{10}(10^{7.5} - 10^{6.9}) = 73.7 \text{ dB}$.

Problem 1.30

- (a) The phenomenon is the superposition of acoustic waves of the same frequency and fixed phase. Thus the total pressure, $\langle p_t^2 \rangle$, is given by:

$$\langle p_t^2 \rangle = \langle p_1^2 \rangle + \langle p_2^2 \rangle + 2\langle p_1 p_2 \rangle \cos(\beta_1 - \beta_2)$$

If the phase difference between the two waves is zero, then:

$$\langle p_t^2 \rangle = \langle p_1^2 \rangle + \langle p_2^2 \rangle + 2\langle p_1 p_2 \rangle$$

If $p_1 = p_2$, then $\langle p_t^2 \rangle = 4\langle p_1^2 \rangle$

When the two waves are 180° out of phase:

$$\langle p_t^2 \rangle = \langle p_1^2 \rangle + \langle p_2^2 \rangle - 2\langle p_1 p_2 \rangle$$

If $p_1 = p_2$, then $\langle p_t^2 \rangle = 0$

The noise level could be reduced substantially by using a control system which ensured that when one pump was turned on, the other was turned on at such a time that it was 180° out of phase with the first pump.

- (b) When the problem is noticed, the sound pressure level at the house is 60dB. This would occur when the two pumps are in phase. Thus:

$$20 \log_{10} \left(\frac{2p_1}{2 \times 10^{-5}} \right) = 60$$

If one pump only were operating, then the level should be:

$$20 \log_{10} \left(\frac{p_1}{2 \times 10^{-5}} \right) = 60 - 6 = 54 \text{ dB}$$

If the level which is measured with one pump operating is closer to 54dB than 57dB, then the theory of in-phase addition of sound waves would be verified. However if the level with one pump operating were closer to 57dB, then incoherent addition would be suggested and the problem would need further investigation.

Problem 1.31

Following example 1.4 in the text, the level due to the machine alone is equal to:

$$10 \log_{10} (10^{9.7} - 10^{9.4}) \approx 94 \text{ dB}$$

Thus the machine is in compliance with specifications.

Problem 1.32

Following example 1.4 in the text, the level due to the machine alone is equal to:

$$10 \log_{10}(10^{9.5} - 10^{9.1}) \approx 92.8 \text{ dB}$$

Problem 1.33

See pages 49 and 50 in text, "combining level reductions". The difference level with the barrier removed can be calculated by adding the barrier noise reduction to 60dB(A). The noise reduction is calculated using equation 1.97 and is:

$$\begin{aligned} NR &= 10 \log_{10}[10^{0/10} + 10^{-5/10}] \\ &\quad - 10 \log_{10}[10^{-8/10} + 10^{-13/10} + 10^{-13/10} + 10^{-8/10} + 2(10^{-18/10} + 10^{-12/10})] \\ &= 1.2 - (-2.4) = 3.6 \text{ dB} \end{aligned}$$

Thus the level with the barrier removed is $60 + 3.6 = 63.6 \text{ dB}$.

Problem 1.34

- (a) Level at the receiver due to both waves = 75dB.

Reflected signal has suffered a 5dB loss.

Let the signal due to the direct wave = $x \text{ dB}$. Then:

$$75 = 10 \log_{10}(10^{x/10} + 10^{(x-5)/10})$$

$$\text{or} \quad 75 = 10 \log_{10}(10^{x/10}) + 10 \log_{10}(1 + 10^{(-5/10)})$$

Thus, $x = 75 - 1.2 = 73.8 \text{ dB}$

- (b) Contributions to the total level from various paths are:

Path A: $73.8 - 4 - 7 = 62.8$

Path B: $73.8 - 5 - 5 = 63.8$

Path C: $73.8 - 4 = 69.8$

Sound pressure level at receiver =

$$10 \log_{10}(10^{6.28} + 10^{6.38} + 10^{6.98}) = 71.4 \text{ dB}$$

- (c) The answer can be found by calculating what the direct field contribution

is, using the level with the barrier in place (note incoherent addition with the barrier in place). Let the direct sound field = x dB. The total level with the barrier in place is 70dB. Thus:

$$70 = 10 \log_{10} (10^{(x-11)/10} + 10^{(x-10)/10} + 10^{(x-4)/10})$$

or

$$70 = 10 \log_{10} (10^{x/10}) + 10 \log_{10} (10^{-11/10} + 10^{-10/10} + 10^{-4/10})$$

Thus, $x = 70.0 + 2.4 = 72.4$ dB

Reduction due to destructive interference = $72.4 - 65 = 7.4$ dB

Problem 1.35

The positive going wave may be represented as $p_i = Ae^{j(\omega t - kx)}$ and the negative going wave as $(A/4)e^{j(\omega t + kx + \theta)}$. At $x=0$, the phase between the two waves is 0. Thus, $\theta = 0$. The total pressure field may then be written as:

$$p_{tot} = Ae^{j(\omega t - kx)} + (A/4)e^{j(\omega t + kx)} = Ae^{j\omega t} (e^{-jkx} + 0.25e^{jkx})$$

Combining equations 1.6 and 1.7 in the text gives for the particle velocity:

$$\begin{aligned} u_{tot} &= -\frac{1}{\rho} \frac{\partial}{\partial x} \int p \, dt = -\frac{A}{j\omega\rho} \frac{\partial}{\partial x} (e^{-jkx} + 0.25e^{jkx}) \\ &= \frac{kA}{\omega\rho} e^{j\omega t} (e^{-jkx} - 0.25e^{jkx}) \end{aligned}$$

The active acoustic intensity is then:

$$I = \frac{1}{2} \text{Re} \{ \bar{p} \bar{u}^* \} = \frac{1}{2} \text{Re} \left\{ A (e^{-jkx} + 0.25e^{jkx}) \frac{kA}{\omega\rho} (e^{jkx} - 0.25e^{-jkx}) \right\}$$

The preceding equation may be rearranged to give:

$$I = \frac{1}{2} \text{Re} \left\{ \frac{kA^2}{\omega\rho} (1.0 - 0.25^2 + 0.25e^{2jkx} - 0.25e^{-2jkx}) \right\} = 0.47 \frac{kA^2}{\omega\rho}$$

Alternatively, the intensity of each of the two waves could have been calculated separately and combined vectorially. That is, for the positive going

plane wave:

$$I_i = \frac{\langle p_i^2 \rangle}{\rho c} = \frac{A^2}{2\rho c}$$

and for the negative going plane wave:

$$I_i = \frac{\langle p_r^2 \rangle}{\rho c} = \frac{A^2}{4^2 \times 2\rho c}$$

The total intensity is then:

$$I_{tot} = I_i - I_r = \frac{A^2}{2\rho c} - \frac{A^2}{32\rho c} = 0.47 \frac{A^2}{\rho c} = 0.47 \frac{kA^2}{\omega\rho}$$

If the two waves had the same amplitude it is clear from the preceding equations that the active intensity would be zero.

Problem 1.36

- (a) Higher order mode cut-on frequency is: $f_{co} = 0.586c/d$, where d is the tube diameter (see p456 in text).

$$\text{Thus } f_{co} = 0.586 \times 343/0.05 = 4020\text{Hz}$$

Frequency range for plane waves = 0 to 4020Hz.

- (b) For plane waves, the acoustic power is:

$$W = IS = \frac{S\langle p^2 \rangle}{\rho c} = \rho c S \langle u^2 \rangle, \text{ where } S \text{ is the duct cross sectional area.}$$

$$\langle u^2 \rangle = \omega^2 \xi_0^2 / 2$$

$$= (2\pi \times 500 \times 0.0001)^2 / 2 = 0.04935 (\text{m/s})^2$$

$$\rho c = 1.206 \times 343 = 413.7, S = (\pi/4) \times 0.05^2$$

$$\text{Thus, } W = 413.7 \times 1.964 \times 10^{-3} \times 0.04935 = 0.04 \text{ watts}$$

- (c) As power is proportional to the square of the cone velocity, the cone velocity squared should be kept constant which means that the displacement squared of the speaker cone should vary inversely with frequency. That is, the displacement should vary inversely with the square root of the frequency.

Problem 1.37

(a) The acoustic pressure is given by:

$$p(x, t) = 5e^{j(500t - k_1x)} + 3e^{j(200t - k_2x)}$$

The particle velocity can be obtained using equation 1.7 and the one dimensional form of equation 1.6 as follows:

$$u(x, t) = -\frac{1}{j\omega\rho} \frac{\partial p(x, t)}{\partial x} = \frac{p(x, t)}{\rho c}$$

Using the above expression for acoustic pressure and $k_1 = 500/343$ $k_2 = 200/343$ and $x = 5$, the acoustic particle velocity can be written as:

$$\begin{aligned} u(x, t) &= \frac{1}{\rho c} \left(5e^{j(500t - k_1x)} + 3e^{j(200t - k_2x)} \right) \\ &= \frac{1}{343 \times 1.206} \left(5e^{j(500t - 2500/343)} + 3e^{j(200t - 1000/343)} \right) \\ &= \frac{1}{413.7} \left(5e^{j(500t - 7.29)} + 3e^{j(200t - 2.92)} \right) \end{aligned}$$

The displacement is given by $\zeta(x, t) = \int u \, dt$. Thus:

$$\begin{aligned} \zeta(x, t) &= \frac{1}{\rho c} \left(\frac{5}{500} e^{j(500t - k_1x - \pi/2)} + \frac{3}{200} e^{j(200t - k_2x - \pi/2)} \right) \\ &= \frac{1}{343 \times 1.206} \left(\frac{5}{500} e^{j(500t - 2500/343 - \pi/2)} \right. \\ &\quad \left. + \frac{3}{200} e^{j(200t - 1000/343 - \pi/2)} \right) \\ &= \frac{1}{413.7} \left(\frac{1}{100} e^{j(500t - 7.29 - \pi/2)} \right. \\ &\quad \left. + \frac{3}{200} e^{j(200t - 2.91 - \pi/2)} \right) \end{aligned}$$

- (b) r.m.s. values are:

$$u_{rms} = \frac{1}{\sqrt{2}} \times \frac{1}{413.6} (5^2 + 3^2)^{1/2} = 0.010 \text{ m/s}$$

$$\xi_{rms} = \frac{1}{\sqrt{2}} \times \frac{1}{413.6} [(1/100)^2 + (3/200)^2]^{1/2} = 3.1 \times 10^{-5} \text{ m}$$

- (c) **Active intensity.** As we have a plane wave propagating in only one direction, the sound intensity is given by equation 1.70 in the text. Also note that the mean square pressures for two different frequencies add. Thus:

$$I_a = \frac{p_{rms}^2}{\rho c} = 0.5 \frac{5^2 + 3^2}{413.7} = 0.041 \text{ W/m}^2$$

- (d) **Reactive intensity.** This is undefined as we have more than one frequency component in the wave.

- (e) The acoustic pressure for each wave is given by:

$$p_R(x, t) = 5e^{j(500t - k_1x)} + 3e^{j(200t - k_2x)}$$

$$p_L(x, t) = 4e^{j(500t + k_1x)} + 2e^{j(200t + k_2x)}$$

Total acoustic pressure, $p = p_R + p_L$. Thus:

$$p = 5e^{j(500t - k_1x)} + 3e^{j(200t - k_2x)} + 4e^{j(500t + k_1x)} + 2e^{j(200t + k_2x)}$$

The total acoustic particle velocity is then:

$$u(x, t) = -\frac{1}{j\omega\rho} \frac{\partial(p_R(x, t) - p_L(x, t))}{\partial x} = \frac{p_R(x, t) - p_L(x, t)}{\rho c}$$

Thus:

$$u(x, t) = \frac{1}{1.206 \times 343} \left(5e^{j(500t - k_1x)} + 3e^{j(200t - k_2x)} - 4e^{j(500t + k_1x)} - 2e^{j(200t + k_2x)} \right)$$

The active intensity is given by $I = 0.5 \text{Re}\{\bar{p} \bar{u}^*\}$ where the bar denotes the complex amplitude. The amplitude of the reactive component is not defined as there is more than one frequency present. The active intensity is calculated at each frequency and the results added together as follows:

$$I = \frac{0.5}{1.206 \times 343} \text{Re}\left\{\left(5e^{-jk_1x} + 4e^{jk_1x}\right)\left(5e^{jk_1x} - 4e^{-jk_1x}\right)\right\} \\ + \frac{0.5}{1.206 \times 343} \text{Re}\left\{\left(3e^{-jk_2x} + 2e^{jk_2x}\right)\left(3e^{jk_2x} - 2e^{-jk_2x}\right)\right\}$$

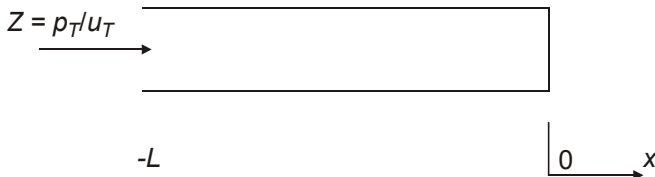
$$\text{Thus, } I = 1.20 \times 10^{-3} (9 + 5) = 0.017 \text{ W/m}^2$$

Problem 1.38

- (a) The acoustic pressure may be written as $p(x, t) = A e^{j(\omega t - kx)}$. Using equations 1.6 and 1.7 in the text, the acoustic particle velocity may be written as: $u(x, t) = \frac{kA}{\rho\omega} e^{j(\omega t - kx)} = \frac{A}{\rho c} e^{j(\omega t - kx)}$.
- (b) Particle velocity is the magnitude of the motion of the particles disturbed during the passage of an acoustic wave, whereas the speed of sound refers to the speed at which the disturbance propagates. Acoustic particle velocity is a function of the loudness of the noise, whereas the speed of sound is independent of loudness.
- (c) The specific acoustic impedance is the ratio of acoustic pressure to particle velocity. Using the preceding equations we obtain:

$$Z = \frac{p(x, t)}{u(x, t)} = \rho c \frac{A e^{j(\omega t - kx)}}{A e^{j(\omega t - kx)}} = \rho c$$

- (d)



To simplify the algebra, set the origin of the coordinate system at the rigid end of the tube as shown in the figure. As the tube is terminated non-anechoically, the pressure will include a contribution from the reflected wave. For a rigid termination, the phase shift on reflection is 0° and the amplitude of the reflected wave is equal to the amplitude of the incident wave. As the origin, $x = 0$ is at the point of reflection, the phase of the two waves must be the same when $x = 0$. Of course if the origin were elsewhere, this would not be true and the following expressions would have to include an additional term (equal to the distance from the origin to the point of reflection) in the exponent of the reflected wave. With the origin at the point of reflection, the total acoustic pressure and particle velocity at any point in the tube may be written as:

$$p_T = A(e^{j(\omega t - kx)} + e^{j(\omega t + kx)})$$

and

$$u_T = \frac{A}{\rho c}(e^{j(\omega t - kx)} - e^{j(\omega t + kx)})$$

The specific acoustic impedance is then:

$$\frac{Z}{\rho c} = \frac{(e^{-jkx} + e^{jkx})}{(e^{-jkx} - e^{jkx})} = \frac{\cos(kx)}{-j \sin(kx)} = j \cot(kx)$$

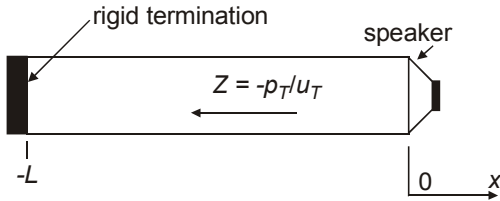
Problem 1.39

The coordinate system is as shown in the figure at right.

- (a) For a rigid termination, the phase shift on reflection is 0° and the amplitude of the reflected wave is equal to the amplitude of the incident wave. As the origin, $x = 0$ is at the loudspeaker location, the phase of the two waves must be the same when $x = -L$. Thus, the total acoustic pressure at any point in the tube may be written as:

$$p_T = A(e^{j(\omega t - kx)} + e^{j(\omega t + kx + 2kL)})$$

The velocity potential and acoustic particle velocity may be derived from



the above expression as:

$$\varphi_T = \frac{A}{j\rho\omega} \left(e^{j(\omega t - kx)} + e^{j(\omega t + kx + 2kL)} \right)$$

$$u_T = \frac{A}{\rho c} \left(e^{j(\omega t - kx)} - e^{j(\omega t + kx + 2kL)} \right)$$

(b) At $x = 0$, $u_T = U_0 e^{j\omega t}$,

$$\text{thus } U_0 = \frac{A}{\rho c} (1 - e^{j2kL}) \text{ and so } A = \frac{U_0 \rho c}{(1 - e^{j2kL})}$$

(c) Rewriting the expressions of (a) in terms of U_0 , we obtain:

$$p_T = \frac{U_0 \rho c}{(1 - e^{j2kL})} \left(e^{j(\omega t - kx)} + e^{j(\omega t + kx + 2kL)} \right)$$

and

$$u_T = \frac{U_0}{(1 - e^{j2kL})} \left(e^{j(\omega t - kx)} - e^{j(\omega t + kx + 2kL)} \right)$$

The real part of the acoustic intensity (where the bar denotes the complex amplitude which is time independent) is:

$$\begin{aligned} I &= \frac{1}{2} \text{Re} \left\{ \bar{p}_T u_T^* \right\} \\ &= \rho c U_0^2 \text{Re} \left\{ \frac{\left(e^{-jkx} + e^{j(kx + 2kL)} \right) \times \left(e^{jkx} - e^{-j(kx + 2kL)} \right)}{2(1 - e^{j2kL}) \times (1 - e^{-j2kL})} \right\} \\ &= \rho c U_0^2 \text{Re} \left\{ \frac{\left(e^{-jkx} + e^{j(kx + 2kL)} \right) \times \left(e^{jkx} - e^{-j(kx + 2kL)} \right)}{2(2 - e^{j2kL} - e^{-j2kL})} \right\} \\ &= \frac{\rho c U_0^2}{2(2 - 2\cos(2kL))} \text{Re} \left\{ 1 - 1 + e^{2jk(x+L)} - e^{-2jk(x+L)} \right\} = 0 \end{aligned}$$

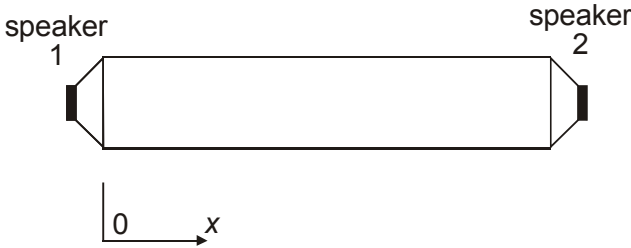
The amplitude of the imaginary part of the acoustic intensity can be derived in a similar way and from the last line in the above equation, it

is:

$$\begin{aligned}
 I &= \frac{1}{2} \text{Im} \{ \bar{p}_T \bar{u}_T^* \} \\
 &= \frac{\rho c U_0^2}{(2 - 2 \cos(2kL))} \sin[2k(x + L)]
 \end{aligned}$$

- (d) The acoustic intensity is a vector quantity and as the amplitudes of the two waves travelling in opposite directions are the same, their intensities will vectorially add to zero.

Problem 1.40



- (a) Incident wave and reflected wave pressures may be written as:

$$p_i = A e^{j(\omega t - kx)}$$

$$\text{and } p_r = B e^{j(\omega t + kx + \theta)}$$

The total pressure is thus:

$$p_T = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx + \theta)}$$

The maximum pressure amplitude occurs when the left and right going waves are in-phase which is at location x , such that the phase, $\theta = -2kx$, giving a pressure amplitude of $(A + B)$. The minimum pressure amplitude occurs when the left and right going waves are π radians out of phase, at the location x , such that the phase $\theta = -2kx + \pi$, with a corresponding amplitude of $A - B$. Thus, the ratio of maximum to minimum pressure is

$(A + B)/(A - B)$ and the standing wave ratio is $20\log_{10}[(A + B)/(A - B)]$

The location of the minimum closest to the end where $x = 0$ (left end) is 0.09m. Thus $\theta = -0.18k + \pi$. As θ is a constant:

$$\pi - 2kx_{\min} = -2kx_{\max}$$

So:

$$x_{\min} - x_{\max} = \frac{\pi}{2k} = \frac{\lambda}{4} = \frac{c}{4f}$$

and thus:

$$f = \frac{c}{4(x_{\min} - x_{\max})} = \frac{343}{4(0.09 - 0.03)} = 1430 \text{ Hz}$$

- (b) The total particle velocity can be calculated using equations 1.6 and 1.7 in the text as:

$$u_T = \frac{1}{\rho c} (p_i - p_r)$$

Thus:

$$u_T = \frac{1}{\rho c} (A e^{j(\omega t - kx)} - B e^{j(\omega t + kx + \theta)})$$

The complex particle velocity amplitude at $x = 0$ is then:

$$\bar{u}_T = \frac{1}{\rho c} (A - B e^{j\theta})$$

The phase angle θ is given by part (a) as:

$$\theta = -0.06k = -0.06 \left(\frac{\pi}{2(x_{\min} - x_{\max})} \right) = -\pi/2$$

Using the above 2 equations, the particle velocity amplitude can be written as:

$$|\bar{u}_T| = \frac{1}{413.7} (A^2 + B^2)^{1/2}$$

The standing wave ratio is given by:

$$\left| \frac{A+B}{A-B} \right| = 10^{(100-96.5)/20} = 1.496$$

Thus, $A = 5.03B$. However, the maximum pressure amplitude is $A + B$. So:

$$A + B = 2 \times 10^{-5} \times 1.414 \times 10^{100/20} = 2.828$$

From the preceding two equations, we have $A = 2.36$ and $B = 0.469$. Thus the velocity amplitude at $x = 0$ is:

$$|\bar{u}_T| = \frac{1}{413.7} (2.36^2 + 0.469^2)^{1/2} = 5.8 \text{ mm/s.}$$

Thus the volume velocity amplitude is $5.9 \times 10^{-6} \text{ m}^3/\text{s}$ (0.001 m^2 area) which is equivalent to an r.m.s volume velocity of $4.2 \times 10^{-6} \text{ m}^3/\text{s}$.

- (c) The mechanical impedance of the second loudspeaker is given by the cross-sectional area multiplied by the ratio of the pressure and particle velocity at the surface of the loudspeaker. Thus $Z_m = pS/u$. Using the relationships derived in parts (a) and (b), we have:

$$Z_m = \rho c S \frac{A + B e^{j(2kL + \theta)}}{A - B e^{j(2kL + \theta)}}$$

We previously found that $\theta = -\pi/2$ and $k = -\theta/0.06$. Also $L = 0.3$ and $2kL = (2\pi/0.12) \times 0.3 = 5\pi$. Thus Z_m may be written as:

$$\begin{aligned} Z_m &= 413.7 \times 0.001 \times \frac{5.032 + e^{j\pi/2}}{5.032 - e^{j\pi/2}} = 0.4136 \times \frac{5.032 + j}{5.032 - j} \\ &= 0.01571 \times (5.032 + j)^2 = 0.382 + 0.158j \end{aligned}$$

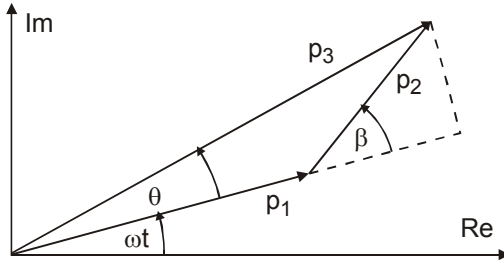
Problem 1.41

- (a) It is sufficient to show that adding two waves of the same frequency but shifted in phase will give a third wave of the same frequency but shifted

in phase. Assuming plane wave propagation, let the two waves to be added be described as:

$$p_1 = A_1 e^{j\omega t} \text{ and } p_2 = A_2 e^{j\omega t + \beta}$$

and the third wave as $p_3 = A_3 e^{j\omega t + \theta}$. To find A_3 and θ in terms of A_1 , A_2 and β , it is easiest to express p_1 , p_2 and p_3 as rotating vectors and use the cosine rule as shown in the figure.



From the cosine rule:

$$A_3^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\beta$$

which is the same as equation 1.90 in the text.

The phase angle θ is:

$$\theta = \tan^{-1} \left(\frac{A_2 \sin\beta}{A_1 + A_2 \cos\beta} \right)$$

- (b) It is sufficient to show that adding together two plane waves of slightly different frequency with the same amplitude result in a third wave. Let the two waves to be added be described as:

$$p_1 = A_1 \cos \omega t \text{ and } p_2 = A_2 \cos(\omega + \Delta\omega)t$$

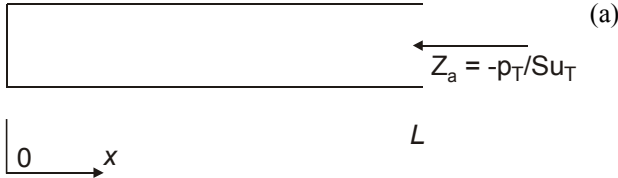
The sum of the two pressures written above may be expressed as:

$$\begin{aligned}
 p_1 + p_2 &= A(\cos\omega t + \cos(\omega + \Delta\omega)t) \\
 &= 2A \left(\cos\frac{t}{2}(\omega + \omega + \Delta\omega) \cos\frac{t}{2}(\omega - \omega - \Delta\omega) \right) \\
 &= 2A \cos\left(\frac{\Delta\omega}{2}t\right) \cos\left(\omega + \frac{\Delta\omega}{2}t\right)
 \end{aligned}$$

which is a sine wave of frequency $(\omega + \Delta\omega)$ modulated by a frequency $\Delta\omega/2$

- (c) If $\Delta\omega$ is small we obtain the familiar beating phenomenon (see page 46, fig 1.9 in text which shows a beating phenomenon where the two waves are slightly different in amplitude resulting in incomplete cancellation at the null points).

Problem 1.42



As the origin is at the left end of the tube, the incident wave will be travelling in the negative x direction. Assuming a phase shift between the incident and reflected waves of θ at $x=0$, the incident wave and reflected wave pressures may be written as:

$$p_i = A e^{j(\omega t + kx)} \quad \text{and} \quad p_r = B e^{j(\omega t - kx + \theta)}$$

The total pressure is thus:

$$p_T = A e^{j(\omega t + kx)} + B e^{j(\omega t - kx + \theta)}$$

The total particle velocity can be calculated using equations 1.6 and 1.7 in the text as:

$$u_T = \frac{1}{\rho c} (p_r - p_i)$$

Thus:

$$u_T = \frac{1}{\rho c} (B e^{j(\omega t - kx + \theta)} - A e^{j(\omega t + kx)})$$

At $x = 0$, $p_T = P_0 e^{j\omega t}$ and $u_T = U_0 e^{j\omega t}$. Thus:

$$P_0 = A + B e^{j\theta} \quad \text{and} \quad \rho c U_0 = B e^{j\theta} - A$$

Thus:

$$A = 0.5(P_0 - \rho c U_0) \quad \text{and} \quad B e^{j\theta} = 0.5(P_0 + \rho c U_0)$$

and the total acoustic pressure and particle velocity may be written as:

$$p_T = 0.5(P_0 - \rho c U_0) e^{j(\omega t + kx)} + 0.5(P_0 + \rho c U_0) e^{j(\omega t - kx)}$$

and

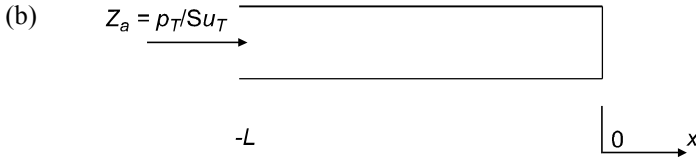
$$u_T = \frac{0.5}{\rho c} ((P_0 + \rho c U_0) e^{j(\omega t - kx)} - (P_0 - \rho c U_0) e^{j(\omega t + kx)})$$

The acoustic impedance looking towards the left in the negative x -direction is the negative ratio of the total acoustic pressure to the product of the duct cross-sectional area and the total acoustic particle velocity (see the preceding figure). Thus:

$$Z_a = -\frac{p_T}{S u_T} = -\frac{\rho c}{S} \frac{(P_0 - \rho c U_0) e^{jkx} + (P_0 + \rho c U_0) e^{-jkx}}{(P_0 + \rho c U_0) e^{-jkx} - (P_0 - \rho c U_0) e^{jkx}}$$

As $e^{jkx} = \cos(kx) + j\sin(kx)$ and $e^{-jkx} = \cos(kx) - j\sin(kx)$, the impedance may be written as:

$$Z_a = \frac{\rho c}{S} \frac{j\rho c U_0 \sin(kx) - P_0 \cos(kx)}{\rho c U_0 \cos(kx) - jP_0 \sin(kx)}$$



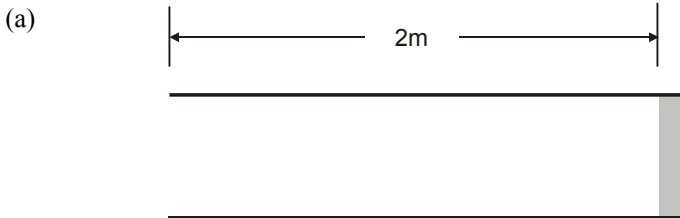
Using a similar analysis to that outlined above, the following expression is obtained for the acoustic impedance looking to the right in the positive x direction:

$$Z_a = \frac{\rho c}{S} \frac{P_0 \cos(kx) - j \rho c U_0 \sin(kx)}{\rho c U_0 \cos(kx) - j P_0 \sin(kx)}$$

The expressions in parts (a) and (b) for the impedance can be shown to be equal if evaluated at the open end of the tube ($x = L$ in part (a) and $x = -L$ in part (b)). In addition, the quantity $-P_0/U_0$ is equal to the termination impedance Z_0 in part (a), while in part (b), the quantity P_0/U_0 is equal to the termination impedance Z_0 . Making the appropriate substitutions, both expressions give the following for the impedance at the open end of the tube.

$$Z_a = \frac{\rho c}{S} \frac{Z_0 \cos(kL) + j \rho c \sin(kL)}{\rho c \cos(kL) + j Z_0 \sin(kL)}$$

Problem 1.43



Set the origin, $x = 0$ at the surface of the sample. Then the entrance of the tube is at $x = -2.0$. As the origin is at the surface of the sample, the incident wave will be travelling in the positive x direction. Assuming a phase shift between the incident and reflected waves of θ at $x = 0$, the incident wave and

reflected wave pressures may be written as:

$$p_i = A e^{j(\omega t - kx)} \quad \text{and} \quad p_r = B e^{j(\omega t + kx + \theta)}$$

The total pressure is thus:

$$p_T = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx + \theta)}$$

The total particle velocity can be calculated using equations 1.6 and 1.7 in the text as:

$$u_T = \frac{1}{\rho c} (A e^{j(\omega t - kx)} - B e^{j(\omega t + kx + \theta)})$$

The pressure reflection coefficient, R_p , is defined as p_r/p_i . Thus:

$$R_p = (B/A) e^{j\theta} = 0.5 + 0.5j$$

The reflection coefficient amplitude is B/A and the phase is θ . Thus, $B/A = 0.707$ and $\theta = 45^\circ = 0.7854$ radians.

Thus the specific acoustic impedance at any point in the tube may be written as:

$$\begin{aligned} Z &= \frac{p_T}{u_T} = \rho c \frac{A e^{-jkx} + B e^{jkx + j\theta}}{A e^{-jkx} - B e^{jkx + j\theta}} = \rho c \frac{A + B e^{j(2kx + \theta)}}{A - B e^{j(2kx + \theta)}} \\ &= \rho c \frac{A/B + \cos(2kx + \theta) + j\sin(2kx + \theta)}{A/B - \cos(2kx + \theta) - j\sin(2kx + \theta)} \end{aligned}$$

$k = \omega/c = 2\pi \times 100/343 = 1.832$ and $x = -2$. Thus, $2kx + \theta = -6.542$. $\cos(2kx + \theta) = 0.9667$ and $\sin(2kx + \theta) = -0.25587$ and $A/B = 1.414$.

Thus, $Z = 414 \times (2.3807 - j0.25587)/(0.4475 + j0.25587)$
 $= 1558(1 - j0.7237) = 1560 - j1130$

(b) The absorption coefficient is defined as $\alpha = 1 - |R_p|^2$, so $\alpha = 0.5$

Problem 1.44

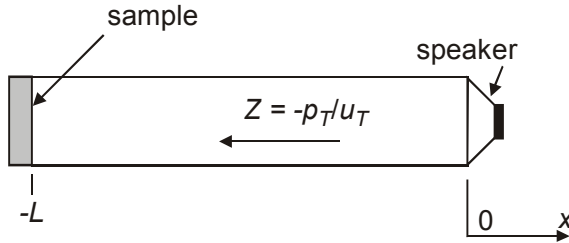
- (a) To simplify the algebra, assume that the tube is horizontal with the left end at $x = 0$ containing the sample of material whose absorption coefficient is to be determined, as shown in the figure. As the origin is at the left end of the tube, the incident wave will be travelling in the negative x direction. Assuming a phase shift between the incident and reflected waves of θ at $x = 0$, the incident wave and reflected wave pressures may be written as:

$$p_i = A e^{j(\omega t + kx)} \quad \text{and} \quad p_r = B e^{j(\omega t - kx + \theta)}$$

The total pressure is thus:

$$p_T = A e^{j(\omega t + kx)} + B e^{j(\omega t - kx + \theta)}$$

The maximum pressure will occur when $\theta = 2kx$, and the minimum will occur when $\theta = 2kx + \pi$. Thus:



$$p_{\max} = e^{jkx}(A + B) \quad \text{and} \quad p_{\min} = e^{jkx}(A - B)$$

and the ratio of maximum to minimum pressures is $(A + B)/(A - B)$

The standing wave ratio, L_0 , is defined as:

$$10^{L_0/20} = \frac{A + B}{A - B}$$

Thus the ratio (B/A) is:

$$\frac{B}{A} = \left[\frac{10^{L_0/20} - 1}{10^{L_0/20} + 1} \right]$$

The amplitude of the pressure reflection coefficient squared is $|R_p|^2 = (B/A)^2$ which can be written in terms of L_0 ($= 95 - 80$) as:

$$|R_p|^2 = \left[\frac{10^{L_0/20} - 1}{10^{L_0/20} + 1} \right]^2$$

The absorption coefficient is defined as $\alpha = 1 - |R_p|^2$, so:

$$\alpha = 1 - \left[\frac{10^{15/20} - 1}{10^{15/20} + 1} \right]^2 = 0.51$$

- (b) The total particle velocity can be calculated using the equation in part (a) for the acoustic pressure and equations 1.6 and 1.7 in the text as:

$$u_T = \frac{1}{\rho c} (p_r - p_i)$$

Thus:

$$u_T = \frac{1}{\rho c} (B e^{j(\omega t - kx + \theta)} - A e^{j(\omega t + kx)})$$

Thus the specific acoustic impedance at any point in the tube may be written as:

$$Z = -\frac{p_T}{u_T} = -\rho c \frac{A e^{jkx} + B e^{-jkx + j\theta}}{B e^{-jkx + j\theta} - A e^{jkx}} = -\rho c \frac{A + B e^{j(-2kx + \theta)}}{B e^{j(-2kx + \theta)} - A}$$

At $x = 0$, the specific acoustic impedance is the normal impedance, Z_s , of the surface of the sample. Thus:

$$\frac{Z_s}{\rho c} = -\frac{p_T}{\rho c u_T} = \frac{A + B e^{j\theta}}{A - B e^{j\theta}}$$

The above impedance equation may be expanded to give:

$$Z_s = \frac{A/B + \cos\theta + j\sin\theta}{A/B - \cos\theta - j\sin\theta} = \frac{(A/B)^2 - 1 + (2A/B)j\sin\theta}{(A/B)^2 + 1 - (2A/B)\cos\theta}$$

The modulus of the impedance is then:

$$|Z_s| = \frac{\sqrt{((A/B)^2 - 1)^2 + (2A/B)^2 \sin^2 \theta}}{(A/B)^2 + 1 - (2A/B) \cos \theta}$$

and the phase is given by:

$$\beta = \tan^{-1} \left(\frac{2(A/B) \sin \theta}{(A/B)^2 - 1} \right)$$

Using the previous analysis, $\frac{A}{B} = \frac{10^{L_0/20} + 1}{10^{L_0/20} - 1} = 1.433$.

At the pressure minimum, $\theta = 2kx - \pi$, where $k = 2\pi/\lambda$.

$x = 0.2\text{m}$ and $f = 250\text{Hz}$, thus $k = 2\pi f/c = 4.58$.

Thus, $\theta = 2 \times 4.58 \times 0.2 - \pi = -1.31$ radians

$\cos \theta = 0.258$ and $\sin \theta = -0.966$

Thus from the preceding equations:

$$\frac{|Z_s|}{\rho c} = \frac{\sqrt{(1.433^2 - 1)^2 + (2.866^2 \times 0.966^2)}}{1.433^2 + 1 - 2.866 \times 0.258} = 1.28$$

and the phase is:

$$\beta = \tan^{-1} \left(\frac{2 \times 1.433 \times (-0.966)}{1.433^2 - 1} \right) = -69.2^\circ$$

- (c) The statistical absorption coefficient is given by equation C.37 in the text as

$$\alpha_{st} = \left\{ \frac{8 \cos \beta}{\xi} \right\} \left\{ 1 - \left[\frac{\cos \beta}{\xi} \right] \log_e (1 + 2\xi \cos \beta + \xi^2) + \left[\frac{\cos(2\beta)}{\xi \sin \beta} \right] \tan^{-1} \left[\frac{\xi \sin \beta}{1 + \xi \cos \beta} \right] \right\}$$

Substituting in the modulus and phase of the impedance, we obtain''

$$\begin{aligned}
\alpha_{st} &= \left(\frac{8 \times \cos(-69.17)}{1.28} \right) \left\{ 1 - \left(\frac{\cos(-69.17)}{1.28} \right) \times \right. \\
&\quad \times \log_e(1 + 2 \times 1.28 \times \cos(-69.17) + 1.28^2) \\
&\quad \left. + \left(\frac{\cos(-138.4)}{1.28 \sin(-69.17)} \right) \tan^{-1} \left(\frac{1.28 \times \sin(-69.17)}{1 + 1.28 \times \cos(-69.17)} \right) \right\} \\
&= 2.22(1 - 0.278 \times 1.266 + 0.625 \times (-0.688)) \\
&= 2.22(1 - 0.352 - 0.430) = 0.485
\end{aligned}$$

Problem 1.45

(a)

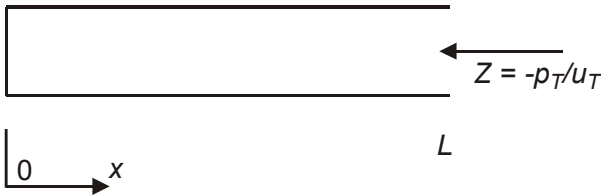
Assume a horizontal tube with the left end containing the termination impedance Z_0 at $x = 0$. As the origin is at the left end of the tube, the incident wave will be travelling in the negative x direction.

The power radiated by a source at the other end of the tube is related to the specific acoustic impedance, Z , it "sees" as follows:

$$W = \frac{S}{2} \operatorname{Re} \{ \bar{p}_T \bar{u}_T^* \} = \frac{S |\bar{u}_T|^2}{2} \operatorname{Re} \{ Z \}$$

where the bar represents the complex amplitude and S is the tube cross-sectional area.

Assuming a phase shift between the incident and reflected waves of θ at



$x = 0$, the incident wave and reflected wave pressures may be written as:

$$p_i = A e^{j(\omega t + kx)} \quad \text{and} \quad p_r = B e^{j(\omega t - kx + \theta)}$$

The total pressure is thus:

$$p_T = A e^{j(\omega t + kx)} + B e^{j(\omega t - kx + \theta)}$$

The total particle velocity can be calculated using equations 1.6 and 1.7 in the text as:

$$u_T = \frac{1}{\rho c} (p_r - p_i)$$

Thus:

$$u_T = \frac{1}{\rho c} (B e^{j(\omega t - kx + \theta)} - A e^{j(\omega t + kx)})$$

Thus the specific acoustic impedance at any point in the tube may be written as:

$$Z = -\frac{p_T}{u_T} = -\rho c \frac{A e^{jkx} + B e^{-jkx + j\theta}}{B e^{-jkx + j\theta} - A e^{jkx}} = -\rho c \frac{A + B e^{j(-2kx + \theta)}}{B e^{j(-2kx + \theta)} - A}$$

At $x = 0$, $Z = Z_0$. For convenience also set $p_T = P_0 e^{j\omega t}$ and $u_T = U_0 e^{j\omega t}$
Thus:

$$\frac{Z_0}{\rho c} = -\frac{P_0}{\rho c U_0} = -\frac{A + B e^{j\theta}}{B e^{j\theta} - A}$$

Thus:

$$A = 0.5(P_0 - \rho c U_0) \quad \text{and} \quad B e^{j\theta} = 0.5(P_0 + \rho c U_0)$$

and the total acoustic pressure and particle velocity may be written as:

$$p_T = 0.5(P_0 - \rho c U_0) e^{j(\omega t + kx)} + 0.5(P_0 + \rho c U_0) e^{j(\omega t - kx)}$$

and

$$u_T = \frac{0.5}{\rho c} ((P_0 + \rho c U_0) e^{j(\omega t - kx)} - (P_0 - \rho c U_0) e^{j(\omega t + kx)})$$

The specific acoustic impedance looking towards the left in the negative x direction may then be written as:

$$Z = -\rho c \frac{(P_0 - \rho c U_0) e^{jkx} + (P_0 + \rho c U_0) e^{-jkx}}{(P_0 + \rho c U_0) e^{-jkx} - (P_0 - \rho c U_0) e^{jkx}}$$

As $e^{jkx} = \cos(kx) + j\sin(kx)$ and $e^{-jkx} = \cos(kx) - j\sin(kx)$, the impedance may be written as:

$$Z = \rho c \frac{j\rho c U_0 \sin(kx) - P_0 \cos(kx)}{\rho c U_0 \cos(kx) - jP_0 \sin(kx)}$$

Dividing through by $\rho c U_0 \cos(kx)$, replacing x with L and replacing $-\frac{P_0}{U_0}$ with Z_0 gives:

$$Z = \rho c \left[\frac{Z_0 / \rho c + j \tan kL}{1 + j(Z_0 / \rho c) \tan kL} \right] = \rho c \left[\frac{(R_0 + jX_0) / \rho c + j \tan kL}{1 + j((R_0 + jX_0) / \rho c) \tan kL} \right]$$

Thus:

$$\text{Re}\{Z\} = \rho c \left[\frac{(R_0 / \rho c)(1 + \tan^2 kL)}{(1 - X_0 / \rho c)^2 \tan^2 kL + (R_0 / \rho c)^2 \tan^2 kL} \right]$$

and the power is then:

$$W = \frac{S \rho c U_L^2}{2} \left[\frac{(R_0 / \rho c)(1 + \tan^2 kL)}{(1 - X_0 / \rho c)^2 \tan^2 kL + (R_0 / \rho c)^2 \tan^2 kL} \right]$$

- (b) It can be seen from the equation derived in part (a) that when $R_0 = 0$, the power will be zero.
- (c) If all losses are zero, the impedance presented to the loudspeaker will be given by the previously derived expression for Z with $R_0 = 0$. In this case:

$$Z = \rho c \left[\frac{jX_0/\rho c + j \tan kL}{1 - (X_0/\rho c) \tan kL} \right] = j\rho c \left[\frac{X_0/\rho c + \tan kL}{1 - (X_0/\rho c) \tan kL} \right]$$

which is imaginary. Thus there will be no real power generated; only imaginary power which represents non-propagating energy stored in the near field.

- (d) As no real power is generated, the acoustic pressure and particle velocity must be 90° out of phase.
- (e) The pressure amplitude reflection coefficient is given by:

$$A = 0.5(P_0 - \rho c U_0) \quad \text{and} \quad B e^{j\theta} = 0.5(P_0 + \rho c U_0)$$

From part (a):

$$R_p = \frac{B e^{j\theta}}{A}$$

Thus:

$$R_p = \frac{B e^{j\theta}}{A} = \frac{(P_0 + \rho c U_0)}{(P_0 - \rho c U_0)}$$

Dividing numerator and denominator by U_0 and putting

$Z_0 = \rho c + jX_0 = -P_0/U_0$, we obtain:

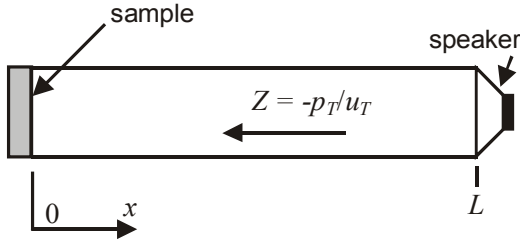
$$R_p = \frac{-\rho c - jX_0 + \rho c}{-\rho c - jX_0 - \rho c} = \frac{jX_0}{2\rho c + jX_0}$$

Thus:

$$|R_p| = X_0 [4\rho^2 c^2 + X_0^2]^{-1/2}$$

Problem 1.46

(a)



As given in the problem, the tube is assumed to be horizontal with the left end at $x = 0$ containing the sample of material whose impedance is to be determined, as shown in the figure. As the origin is at the left end of the tube, the incident wave will be travelling in the negative x direction. Assuming a phase shift between the incident and reflected waves of θ at $x = 0$, the incident wave and reflected wave pressures may be written as:

$$p_i = A e^{j(\omega t + kx)} \quad \text{and} \quad p_r = B e^{j(\omega t - kx + \theta)}$$

The total pressure is thus:

$$p_T = A e^{j(\omega t + kx)} + B e^{j(\omega t - kx + \theta)}$$

At the surface of the sample, the pressure amplitude reflection coefficient is thus $R = (B/A)e^{j\theta}$ and $B = (RA)e^{-j\theta}$. Thus the total pressure at any location, x , in the tube may be written as:

$$p_T = A \left(e^{j(\omega t + kx)} + R e^{j(\omega t - kx)} \right)$$

- (b) Returning to the first expression for the total pressure of part (a), the maximum pressure will occur when $\theta = 2kx$, and the minimum will occur when $\theta = 2kx + \pi$. Thus $p_{\max} = e^{jkx}(A + B)$, $p_{\min} = e^{jkx}(A - B)$ and the ratio of maximum to minimum pressures is $(A + B)/(A - B)$
- (c) The standing wave ratio (SWR) which is L_0 is defined as:

$$10^{SWR/20} = \frac{A + B}{A - B}$$

Thus the ratio (B/A) is:

$$\frac{B}{A} = \left[\frac{10^{L_0/20} - 1}{10^{L_0/20} + 1} \right]$$

From part (a), the amplitude of the pressure reflection coefficient squared is $|R_p|^2 = (B/A)^2$ which can be written in terms of L_0 as:

$$|R_p|^2 = \left[\frac{10^{L_0/20} - 1}{10^{L_0/20} + 1} \right]^2$$

- (d) The absorption coefficient is defined as $\alpha = 1 - |R_p|^2$, which on substituting the equation derived in part (c) for $|R_p|^2$, gives the required result.
- (e) The total particle velocity can be calculated using equations 1.6 and 1.7 in the text as:

$$u_T = \frac{1}{\rho c} (p_r - p_i)$$

Thus:

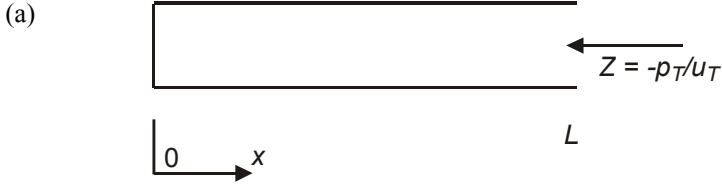
$$u_T = \frac{1}{\rho c} (B e^{j(\omega t - kx + \theta)} - A e^{j(\omega t + kx)})$$

Thus the specific acoustic impedance at any point in the tube may be written as:

$$Z = -\frac{p_T}{u_T} = -\rho c \frac{A e^{jkx} + B e^{-jkx + j\theta}}{B e^{-jkx + j\theta} - A e^{jkx}} = -\rho c \frac{A + B e^{j(-2kx + \theta)}}{B e^{j(-2kx + \theta)} - A}$$

At $x = 0$, the specific acoustic impedance is the normal impedance, Z_s , of the surface of the sample. Thus:

$$\frac{Z}{\rho c} = -\frac{p_T}{\rho c u_T} = \frac{A + B e^{j\theta}}{A - B e^{j\theta}} = \frac{1 + R_p}{1 - R_p}$$

Problem 1.47

Assume a horizontal tube with the left end containing the termination impedance Z_L at $x = 0$. As the origin is at the left end of the tube, the incident wave will be travelling in the negative x direction. Assuming a phase shift between the incident and reflected waves of θ at $x = 0$, the incident wave and reflected wave pressures may be written as:

$$p_i = A e^{j(\omega t + kx)} \quad \text{and} \quad p_r = B e^{j(\omega t - kx + \theta)}$$

The total pressure is thus:

$$p_T = A e^{j(\omega t + kx)} + B e^{j(\omega t - kx + \theta)}$$

The total particle velocity can be calculated using equations 1.6 and 1.7 in the text as:

$$u_T = \frac{1}{\rho c} (p_r - p_i)$$

Thus:

$$u_T = \frac{1}{\rho c} (B e^{j(\omega t - kx + \theta)} - A e^{j(\omega t + kx)})$$

Thus the specific acoustic impedance at any point in the tube may be written as:

$$\begin{aligned} Z &= -\frac{p_T}{u_T} = -\rho c \frac{A e^{j k x} + B e^{-j k x + j \theta}}{B e^{-j k x + j \theta} - A e^{j k x}} \\ &= -\rho c \frac{A + B e^{j(-2 k x + \theta)}}{B e^{j(-2 k x + \theta)} - A} \end{aligned}$$

At $x = 0$, $Z = Z_L$. For convenience also set $p_T = P_0 e^{j\omega t}$ and $u_T = U_0 e^{j\omega t}$. Thus:

$$\frac{Z_L}{\rho c} = -\frac{P_0}{\rho c U_0} = -\frac{A + B e^{j\theta}}{B e^{j\theta} - A}$$

Thus:

$$A = 0.5(P_0 - \rho c U_0) \quad \text{and} \quad B e^{j\theta} = 0.5(P_0 + \rho c U_0)$$

and the total acoustic pressure and particle velocity may be written as:

$$p_T = 0.5(P_0 - \rho c U_0) e^{j(\omega t + kx)} + 0.5(P_0 + \rho c U_0) e^{j(\omega t - kx)}$$

and:

$$u_T = \frac{0.5}{\rho c} \left((P_0 + \rho c U_0) e^{j(\omega t - kx)} - (P_0 - \rho c U_0) e^{j(\omega t + kx)} \right)$$

The specific acoustic impedance looking towards the left in the negative x direction may then be written as:

$$Z = -\rho c \frac{(P_0 - \rho c U_0) e^{jkx} + (P_0 + \rho c U_0) e^{-jkx}}{(P_0 + \rho c U_0) e^{-jkx} - (P_0 - \rho c U_0) e^{jkx}}$$

As $e^{jkx} = \cos(kx) + j\sin(kx)$ and $e^{-jkx} = \cos(kx) - j\sin(kx)$, the impedance may be written as:

$$Z = \rho c \frac{j\rho c U_0 \sin(kx) - P_0 \cos(kx)}{\rho c U_0 \cos(kx) - jP_0 \sin(kx)}$$

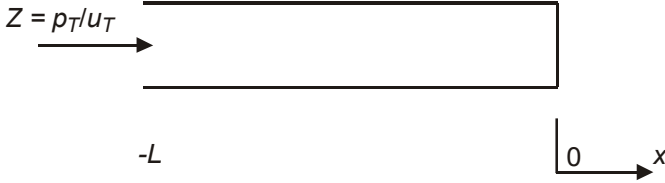
Dividing through by $\rho c U_0 \cos(kx)$, replacing x with L and replacing

$-\frac{P_0}{U_0}$ with Z_L gives:

The same result can be obtained by putting the open end of the tube to the left as shown in the figure above.

$$\left[\frac{Z_L / \rho c + j \tan kL}{1 + j(Z_L / \rho c) \tan kL} \right]$$

The same result can be obtained by putting the open end of the tube to the left as shown in the figure below.



The total pressure is thus:

$$p_T = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx + \theta)}$$

and the total particle velocity is thus:

$$u_T = \frac{1}{\rho c} \left(-B e^{j(\omega t + kx + \theta)} + A e^{j(\omega t - kx)} \right)$$

The specific acoustic impedance at any point in the tube is then:

$$Z = \frac{p_T}{u_T} = \rho c \frac{A e^{jkx} + B e^{-jkx + j\theta}}{-B e^{-jkx + j\theta} + A e^{jkx}} = \rho c \frac{A + B e^{j(-2kx + \theta)}}{A - B e^{j(-2kx + \theta)}}$$

The remaining part of the analysis is the same as before.

- (b) The reflection coefficient, R_p , is defined as the ratio of the complex amplitudes of the reflected to incident waves at $x = 0$. Thus, from part (a):

$$R_p = \frac{B e^{j\theta}}{A}$$

At $x = 0$:

$$\frac{Z_L}{\rho c} = \frac{A + B e^{j\theta}}{A - B e^{j\theta}} = \frac{1 + B e^{j\theta}/A}{1 - B e^{j\theta}/A} = \frac{1 + R_p}{1 - R_p}$$

Thus the pressure reflection coefficient may be written as:

$$R_p = \frac{Z_L/\rho c - 1}{Z_L/\rho c + 1}$$

which is equal to zero when $Z_L = \rho c$.

- (c) Assuming continuity of acoustic pressure and volume velocity (uS , where S is the tube cross sectional area) implies that the acoustic impedance is continuous across the hole and thus the specific impedance in the tube at $x = 0$ is related to the specific acoustic impedance, Z_H of the hole by:

$$Z_L = Z_H \frac{S_T}{S_H}$$

where S_H is the cross sectional area of the hole and S_T is the cross sectional area of the tube.

From equation 9.14 in the text we have $Z_H = j\rho c \tan k\ell$, where ℓ is the effective length of the hole. From part (a), we have at $x = 0$:

$$\frac{Z_L}{\rho c} = \frac{A + B e^{j\theta}}{A - B e^{j\theta}} = \frac{1 + B e^{j\theta}/A}{1 - B e^{j\theta}/A} = \frac{1 + R_p}{1 - R_p}$$

Thus the pressure reflection coefficient may be written as:

$$R_p = \frac{Z_L/\rho c - 1}{Z_L/\rho c + 1} = \frac{j(S_T/S_H)\tan k\ell - 1}{j(S_T/S_H)\tan k\ell + 1}$$

- (d) Using equation 9.8 in the text and the condition of continuity of acoustic pressure and acoustic volume velocity at the hole, the equality $Z_L/S_T = Z_H/S_H$ holds. Thus:

$$Z_L = j(S_T/S_H)\rho c \tan[k\ell(1 - M)] + R_a$$

- (e) For good absorption, $R_p = 0$, which as we showed in part (b), means that $Z_L = \rho c$. For this to be true, the first term in the equation derived in (d) must be large compared to the other terms. Thus, the product $k\ell M$ must be much less than 1. For a thin plate, the effective length of the hole is made up of two parts, one corresponding to the side of the hole in the tube and the other corresponding to the side looking into free space.

Using equations 9.16 and 9.19 in the text, assuming that the hole diameter is very small compared to the tube diameter, it can be shown that for small M , the effective length of equation 9.7 is $\ell = 0.73d_H$. Thus the condition that $k\ell/M \ll 1$ implies that $2\pi f/c \times 0.73d_H \ll M$, or $fd_H \ll 70M$.

- (f) The specific acoustic impedance looking into a tube was shown in part (a) to be:

$$Z = \rho c \left[\frac{Z_L/\rho c + j \tan kL}{1 + j(Z_L/\rho c) \tan kL} \right]$$

In the limit of small L and large λ (small k), $\tan(kL) = kL$. Also for a rigidly terminated tube, $Z_L = \infty$. Thus the preceding expression becomes:

$$Z = \rho c \left[\frac{\infty/\rho c + j kL}{1 + j(\infty/\rho c) kL} \right] = \frac{\rho c}{jkL} = \frac{-j\rho c}{kL}$$

The ratio of the pressure to the particle displacement is then:

$$\frac{p}{\xi} = \frac{j\omega p}{u} = \frac{\rho c \omega}{kL}$$

which indicates that the pressure is in phase with the particle displacement. Considering the analogy of a single degree of freedom spring-mass system the equation of motion is:

$$m\ddot{\xi} + K\xi = f \quad \text{or} \quad -\omega^2 m\xi + K\xi = f$$

where K is the spring stiffness and m is the mass. It can be seen from the above equation that for a stiffness only, the exciting force will be in phase with the displacement and for a mass only it will be 180° out of phase with the displacement. Thus it is clear from this analogy that the previous case of the pressure and particle displacement in-phase represents a stiffness.

For an open ended tube the impedance, $Z_L = 0$ and the impedance of the tube becomes:

$$Z = \rho c \left[\frac{0 + j kL}{1 + j0 kL} \right] = j\rho c kL$$

The ratio of the pressure to the particle displacement is then:

$$\frac{p}{\xi} = \frac{j\omega p}{u} = -\rho c \omega kL$$

which indicates that the acoustic pressure is 180° out of phase with the particle displacement. Thus by the preceding argument, this represents a lumped mass.

- (g) Using the equation given in the question and using the hint for maximum power we may substitute Z_0 for Z_L and Z to give kL at maximum power. Thus:

$$\frac{Z_0}{\rho c} = \left[\frac{Z_0/\rho c + j \tan kL}{1 + j(Z_0/\rho c) \tan kL} \right]$$

Rearranging gives:

$$\frac{Z_0}{\rho c} + j \left(\frac{Z_0}{\rho c} \right)^2 \tan kL = \frac{Z_0}{\rho c} + j \tan kL$$

For the left side to equal the right side, $\tan(kL) = 0$. Thus $kL = n\pi$, and the optimum length is given by $L = n\lambda/2$, where $n = 1, 2, 3, \dots$

- (h) The above result suggests that the frequency response will be characterised by a number of resonant peaks and so will be very poor and non-uniform. The frequency response could be made smoother by adding some absorptive porous acoustic material to the tube. This would have the effect of damping the resonances, thus reducing the difference between the peaks and troughs in the frequency response.
- (i) The resonances are less damped at low frequencies, thus resulting in bigger differences between the peaks and troughs in the frequency response.

Problem 1.48

Using the result derived in the answer to 1.45(a), the impedance seen by the loudspeaker may be written as:

$$Z = \rho c \left[\frac{Z_r/\rho c + j \tan kL}{1 + j(Z_r/\rho c) \tan kL} \right]$$

which on substituting $Z_r = R_r + jX_r$ may be rewritten as:

$$\frac{Z}{\rho c} = \frac{(R_r/\rho c + j(X_r/\rho c + \tan(kL))) \left(1 - (X_r/\rho c) \tan(kL) - j(R_r/\rho c) \tan(kL) \right)}{(1 - (X_r/\rho c) \tan(kL))^2 + ((R_r/\rho c) \tan(kL))^2}$$

The power output of the loudspeaker will vary with tube length because the power output of a source is dependent on impedance presented to it and the impedance it is presented is dependent on the tube length. The maximum power output will occur when the impedance presented to the loudspeaker is equal to the internal impedance of the loudspeaker which is infinite. Thus the maximum power output will occur when the denominator in the above equation is zero. Of course the real power output is only dependent on the resistive impedance while the imaginary power output is dependent on the reactive impedance. Thus the specific resistive impedance is:

$$\frac{R}{\rho c} = \frac{R_r/\rho c (1 + \tan^2(kL))}{(1 - (X_r/\rho c) \tan(kL))^2 + ((R_r/\rho c) \tan(kL))^2}$$

At 250Hz, $k = 2\pi f/c = 2\pi \times 250/343 = 4.58$

As $a = 0.075$, $ka = 4.58 \times 0.075 = 0.343$, $\pi a^2 = 0.0177$

Thus, $R_r/\rho c = 0.0177 \times 0.343^2/2 = 1.04 \times 10^{-3}$

and $X_r/\rho c = 0.0177 \times 0.343 \times 0.8 = 0.00486$

Substituting in the expression given for $R/\rho c$, we obtain:

$$\frac{R}{\rho c} = \frac{1.04 \times 10^{-3} (1 + \tan^2(kL))}{(1 - 0.00486 \tan(kL))^2 + (1.04 \times 10^{-3} \tan(kL))^2}$$

By trial and error it can be shown that the maximum value of the above expression occurs when $\tan(kL) = 3.636$, or $kL = 1.262$. Thus optimum L for maximum power out = $1.262/4.58 = 276\text{mm}$.

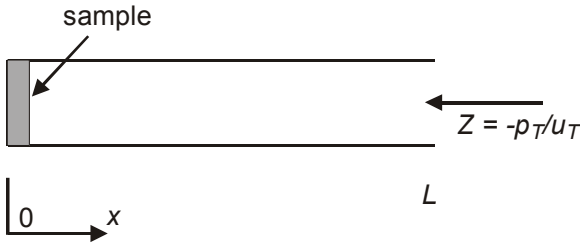
Problem 1.49

(a) A standing wave tube is used to determine the normal specific impedance

of a solid by placing the sample in one end of the tube which is then rigidly closed, and then generating a pure tone sound field in the tube with a loudspeaker placed at the other end of the tube. The ratio of maximum to minimum acoustic pressure in the standing wave generated in the tube is measured, as is the distance of the first minimum from the surface of the sample.

To simplify the algebra, a horizontal tube with the left end at $x = 0$ containing the solid whose impedance is to be determined will be assumed as shown in the figure. As the origin is at the left end of the tube, the incident wave will be travelling in the negative x direction. Assuming a phase shift between the incident and reflected waves of θ at $x = 0$, the incident wave and reflected wave pressures may be written as:

$$p_i = A e^{j(\omega t + kx)} \quad \text{and} \quad p_r = B e^{j(\omega t - kx + \theta)}$$



The total pressure is thus:

$$p_T = A e^{j(\omega t + kx)} + B e^{j(\omega t - kx + \theta)}$$

The total particle velocity can be calculated using equations 1.6 and 1.7 in the text as:

$$u_T = \frac{1}{\rho c} (p_r - p_i)$$

Thus:

$$u_T = \frac{1}{\rho c} (B e^{j(\omega t - kx + \theta)} - A e^{j(\omega t + kx)})$$

Thus the specific acoustic impedance at any point in the tube may be written as:

$$\begin{aligned}
 Z &= -\frac{p_T}{u_T} = -\rho c \frac{A e^{jkx} + B e^{-jkx + j\theta}}{B e^{-jkx + j\theta} - A e^{jkx}} \\
 &= -\rho c \frac{A + B e^{j(-2kx + \theta)}}{B e^{j(-2kx + \theta)} - A}
 \end{aligned}$$

At $x = 0$, the specific acoustic impedance is the normal impedance, Z_s , of the surface of the sample. Thus:

$$\frac{Z_s}{\rho c} = -\frac{p_T}{\rho c u_T} = \frac{A + B e^{j\theta}}{A - B e^{j\theta}}$$

To calculate the impedance, it is necessary to evaluate the constants A and B , and the phase angle, θ . Returning to the above expression for the total acoustic pressure, it can be seen that the maximum sound pressure in the tube will occur when $\theta = 2kx$, and the amplitude will be $(A + B)$. The minimum pressure amplitude occurs at the location, where $\theta = 2kx - \pi$, with a corresponding amplitude of $A - B$. Thus, the ratio of maximum to minimum pressure is $(A + B)/(A - B)$ and the standing wave ratio is $20 \log_{10}[(A + B)/(A - B)]$, which is the difference in dB between the maximum and minimum sound pressure levels in the tube. The phase angle θ is determined by the distance, x of the first minimum sound pressure level from the face of the sample by using $\theta = 2kx - \pi$. This is equivalent to the equation for θ , given on p623 the text, where $k = 2\pi/\lambda = 2\pi f/c$. The first minimum is used because the effect of any losses due to non rigid tube walls will be minimised. The minimum rather than the maximum is used because its location is much more sharply defined.

The equation for the impedance may be rewritten as:

$$\frac{Z_s}{\rho c} = \frac{(A/B) + e^{j\theta}}{(A/B) - e^{j\theta}}$$

The standing wave ratio (SWR) which can be measured is defined as:

$$10^{SWR/20} = \frac{A + B}{A - B}$$

Thus, $\frac{A}{B} = \frac{10^{SWR/20} + 1}{10^{SWR/20} - 1}$ and the impedance may be calculated.

This is done by expanding the above impedance equation to give:

$$\frac{Z_s}{\rho c} = \frac{A/B + \cos\theta + j\sin\theta}{A/B - \cos\theta - j\sin\theta} = \frac{(A/B)^2 - 1 + (2A/B)j\sin\theta}{(A/B)^2 + 1 - (2A/B)\cos\theta}$$

The modulus of the impedance is then:

$$\frac{|Z_s|}{\rho c} = \frac{\sqrt{((A/B)^2 - 1)^2 + (2A/B)^2 \sin^2\theta}}{(A/B)^2 + 1 - (2A/B)\cos\theta}$$

and the phase is given by:

$$\beta = \tan^{-1}\left(\frac{2(A/B)\sin\theta}{(A/B)^2 - 1}\right)$$

- (b) As can be seen in the above derivation, the amplitude of the pressure reflection coefficient is $|R_p| = \frac{B}{A} = \frac{10^{SWR/20} - 1}{10^{SWR/20} + 1}$. The sound power reflection coefficient is defined as $|R_p|^2$ and the absorption coefficient is given by $\alpha = 1 - |R_p|^2$.

(i) From part (a), $\frac{A}{B} = \frac{10^{SWR/20} + 1}{10^{SWR/20} - 1} = \frac{10^{0.21} + 1}{10^{0.21} - 1} = 4.216$.

$$\theta = 2kx - \pi, \text{ where } k = 2\pi/\lambda.$$

$$x = 0.4\lambda, \text{ thus } \theta = 4\pi \times 0.4 - \pi = 0.6\pi$$

$$\cos\theta = -0.309 \text{ and } \sin\theta = 0.951$$

Thus from the preceding equations:

$$\frac{|Z_s|}{\rho c} = \frac{\sqrt{(4.216^2 - 1)^2 + (8.432^2 \times 0.951^2)}}{4.216^2 + 1 + 8.432 \times 0.309} = 0.87$$

and the phase is:

$$\beta = \tan^{-1}\left(\frac{2 \times 4.216 \times 0.951}{4.216^2 - 1}\right) = 25.5^\circ$$

(ii) The normal incidence sound power reflection coefficient is given by:

$$|R_p|^2 = \left| \frac{10^{SWR/20} - 1}{10^{SWR/20} + 1} \right|^2 = 4.216^{-2} = 0.056$$

(iii) The absorption coefficient is given by $\alpha = 1 - |R_p|^2 = 0.94$

(iv) The intensity is given by:

$$\begin{aligned} I &= \frac{1}{2} \text{Re}\{p_T u_T^*\} = \frac{1}{2\rho c} \text{Re}\{(A + B e^{j\theta}) \times (B e^{-j\theta} - A)\} \\ &= \frac{1}{2\rho c} \text{Re}\{B^2 - A^2 - 2jAB \sin\theta\} = \frac{1}{2\rho c} (B^2 - A^2) \end{aligned}$$

The maximum sound pressure level is 70dB. Thus:

$$A + B = \sqrt{2} p_{ref} 10^{L_p/20} = 0.0894$$

Using $A = 4.216B$, we have $5.216B = 0.0894$. Thus $B = 0.0171$ and $A = 0.0723$. Thus the sound intensity is:

$$I = (2 \times 413.7)^{-1} \times (0.0171^2 - 0.0723^2) = -5.96 \times 10^{-6} \text{ W/m}^2.$$

The negative sign indicates that the net intensity is in the negative x -direction. The intensity will not vary along a lossless tube as the sound intensity is a vector quantity which in this case is the vector sum of the intensities in the left and right going waves which is independent of tube location for a lossless tube. This can be verified by using the expressions for p_T and u_T derived in part (a).

(v) Let successive minima be located at x_1 and x_2 . Then using the equation derived in part (a) for the total pressure, setting $\theta = 2kx$, and equating the pressures at x_1 and x_2 we obtain $(A + B)e^{jkx_1} = (A + B)e^{jkx_2}$. For this to be true, $e^{jk(x_2 - x_1)} = 1$. Thus $k(x_2 - x_1) = -\pi$, and $x_1 - x_2 = \lambda/2$, which implies that the minima are separated by half a wavelength. At 200Hz, this is equal to $343/(2 \times 200) = 0.86\text{m}$.

Problem 1.50

(a) Assuming no losses in the tube, the intensity of a single plane wave

propagating in any one direction is $\langle p^2 \rangle / \rho c$, where the amplitude of p as well as its r.m.s. value is independent of axial location. Thus the intensity of the positive going wave may be written as $\langle p_+^2 \rangle / \rho c$ and the negative going wave as $\langle p_-^2 \rangle / \rho c$. As intensity is a vector quantity, the intensities of the positive and negative going waves can be combined by adding the intensity of the positive going wave to the negative value of the negative going wave to give:

$$I = \frac{1}{\rho c} (\langle p_+^2 \rangle - \langle p_-^2 \rangle)$$

which is independent of location.

- (b) The intensity cannot be measured with a single microphone because it cannot distinguish between the pressures associated with the two wave components travelling in opposite directions.
- (c) Two identical microphones can be used because intensity is also the time averaged product of the acoustic pressure and particle velocity and from equations 1.6 and 1.7 in the text we can show that the acoustic particle velocity is related to the acoustic pressure gradient by:

$$u = -\frac{1}{\rho} \frac{\partial}{\partial x} \int p \, dt$$

where p_1 and p_2 are the pressures measured by microphones 1 and 2 respectively. The above equation may be rewritten as equation 3.21 in the text. Equations 3.21 and 3.22 may then be used to write equation 3.23 which is an expression for the intensity as a function of the measurements made by microphones 1 and 2. Equation 3.23 needs a slight modification as in this case (replacement of \mathbf{n} with 1) the intensity is in the positive x -direction down the tube rather than in an arbitrary direction \mathbf{n} .

Problem 1.51

(a)

$$R_p = \frac{Z_s - \rho c}{Z_s + \rho c}$$

and

$$\alpha = 1 - |R_p|^2 = 1 - \frac{|Z_s - \rho c|^2}{|Z_s + \rho c|^2} = \frac{|Z_s + \rho c|^2 - |Z_s - \rho c|^2}{|Z_s + \rho c|^2}$$

Rearranging gives:

$$\begin{aligned} \alpha &= \frac{[\operatorname{Re}\{Z_s\} + \rho c]^2 + [\operatorname{Im}\{Z_s\}]^2 - [\operatorname{Re}\{Z_s\} - \rho c]^2 - [\operatorname{Im}\{Z_s\}]^2}{|Z_s + \rho c|^2} \\ &= \frac{[\operatorname{Re}\{Z_s\}]^2 + 2\rho c \operatorname{Re}\{Z_s\} + (\rho c)^2 - [\operatorname{Re}\{Z_s\}]^2 + 2\rho c \operatorname{Re}\{Z_s\} - (\rho c)^2}{|Z_s + \rho c|^2} \\ &= \frac{4\rho c \operatorname{Re}\{Z_s\}}{|Z_s + \rho c|^2} \end{aligned}$$

- (b) From the above expression it can be seen that the maximum value of α is 1 which would occur when $Z_s = \rho c$.

Problem 1.52

- (a) The sound power reflection coefficient is simply the square of the modulus of the pressure amplitude reflection coefficient. Referring to equation 5.129 in the text, the required result can be obtained by allowing $\theta = 0$ (normal incidence) and substituting $\rho_2 c_2$ for Z_m and $\rho_1 c_1$ for ρc . Note that if $\theta = 0$, then $\sin\theta = 0$ and from equation 5.130, $\cos\psi = 1$.
- (b) Again, referring to equation 5.129 in the text, the required result can be obtained by substituting $\rho_2 c_2$ for Z_m and $\rho_1 c_1$ for ρc . The angle ψ is defined in equation 5.130, with the substitutions, $k_1 = k$ and $k_2 = k_m$.

Problem 1.53

- (a) As a result of the property of zero pressure at the pressure release boundary, we have at the boundary ($y = 0$), $p_R = -p_I$. Thus using equations 5.118 and 5.119 in the text and setting $y = 0$, we obtain $A_R = -A_I$. As the wave is propagating along the y -axis, $\theta = 0$, $k_y = k$ and $k_x = 0$. Thus the total pressure (adding the time dependent term, $e^{j\omega t}$) is given by:

$$\begin{aligned} p_T &= p_I + p_R = A_I e^{j\omega t} \left(e^{-j(k_x x - k_y y)} - e^{-j(k_x x + k_y y)} \right) \\ &= A_I e^{j\omega t} e^{-jk_x x} \left(e^{jk_y y} - e^{-jk_y y} \right) \\ &= 2j A_I \sin ky e^{j\omega t} \end{aligned}$$

- (b) For normally incident sound (and with $Z_m = \rho_w c_w$), equation 5.129 in the text can be written as:

$$R_p = \frac{\rho_w c_w - \rho c}{\rho_w c_w + \rho c} = \frac{\frac{\rho_w c_w}{\rho c} - 1}{\frac{\rho_w c_w}{\rho c} + 1}$$

From the preceding equation it can be seen that if $\rho_w c_w \gg \rho c$, then $R_p = 1$ and the reflected wave amplitude will be equal to the incident wave amplitude. For an air/water interface, $\rho_w c_w = 1026 \times 1500 = 1.54 \times 10^6$ which is much greater than $\rho c = 413.7$ thus satisfying the required condition.

- (c) Using equations 1.6 and 1.7 in the text and the result of part (a), the acoustic particle velocity amplitude is $2A_I/\rho c$. Using equations 1.6, 1.7 and 5.118, the acoustic particle velocity amplitude (normally incident wave) is $A_I/\rho c$. Thus the total particle velocity amplitude is twice the incident wave particle velocity.

Problem 1.54

- (a) Referring to the analysis on pages 210 – 212 in the text, for normal

incidence, $\theta = 0$. Thus, from equation 5.130, $\psi = 0$, and equation 5.129 becomes (with $Z_m = \rho_2 c_2 = 830$):

$$R_p = \frac{830 - \rho c}{830 + \rho c} = \frac{830 - 413.6}{830 + 413.6} = 0.335$$

The r.m.s. amplitude of the incident wave is given by:

$$P_i = 2 \times 10^{-5} \times 10^{60/20} = 0.02 \text{ Pa}$$

Thus the amplitude of the reflected wave is:

$$P_r = 0.335 P_i = 0.0067 \text{ Pa}$$

(b) When all the energy is reflected, $|R_p| = 1$, Thus:

$$\rho_2 c_2 \cos \theta - \rho c \cos \psi = \rho_2 c_2 \cos \theta + \rho c \cos \psi$$

Using Snell's Law, $\sin \psi = \frac{c_2 \sin \theta}{c}$. Thus:

$$\cos \psi = \left(1 - \frac{c_2^2}{c^2} \sin^2 \theta \right)^{1/2}$$

Substituting this result into the previous equation gives:

$$\begin{aligned} \rho_2 c_2 \cos \theta - \rho c \left(1 - \frac{c_2^2}{c^2} \sin^2 \theta \right)^{1/2} \\ = \rho_2 c_2 \cos \theta + \rho c \left(1 - \frac{c_2^2}{c^2} \sin^2 \theta \right)^{1/2} \end{aligned}$$

This is true only if $(c_2/c) \sin \theta = 1$, or $\theta = \sin^{-1}(c/c_2)$ which is the angle above which all incident energy will be reflected.

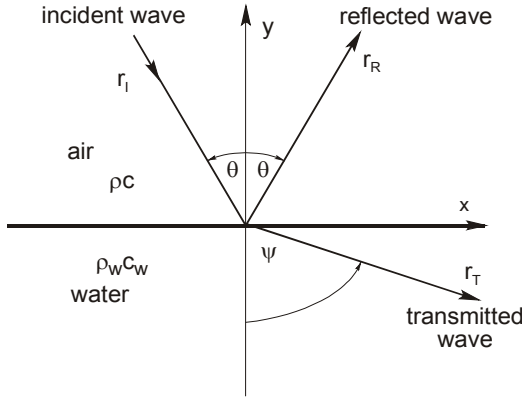
Problem 1.55

(a) Referring to the analysis on pages 210 – 212 in the text and substituting

$\rho_w c_w$ for Z_m in equation 5.129, the required result is obtained.

- (b) The transmission coefficient, τ , is an energy related quantity and is defined by:

$$\tau = \frac{|p_T|^2}{|p_I|^2} \frac{\rho c}{\rho_w c_w} = \frac{|A_T|^2}{|A_I|^2} \frac{\rho c}{\rho_w c_w}$$



Using equation 5.121 and 5.127 in the text and substituting $Z_1 = \rho c$ and $Z_2 = \rho_w c_w$, we obtain:

$$\frac{p_T}{p_I} = \frac{A_T}{A_I} = \frac{2 \cos \theta / (\rho c)}{\frac{\cos \theta}{\rho c} + \frac{\cos \psi}{\rho_w c_w}} = \frac{2 \rho_w c_w \cos \theta}{\rho_w c_w \cos \theta + \rho c \cos \psi}$$

Thus:

$$\tau = \frac{4 \rho c \rho_w c_w \cos^2 \theta}{(\rho_w c_w \cos \theta + \rho c \cos \psi)^2} = (1 - R_p^2) \cos \theta / \cos \psi$$

- (c) For air, $\rho c = 413.6$ and for sea water
 $\rho_w c_w = 1.026 \times 10^3 \times 1500 = 1539 \times 10^3$.
 For $\theta = 10^\circ$, $\cos \theta = 0.9848$ and $\sin \theta = 0.1736$.
 Using Snell's Law,

$\sin\psi = \frac{c_w \sin\theta}{c} = 1500 \times 0.1736 / 343 = 0.7594$. Thus $\psi = 49.4^\circ$ and $\cos\psi = 0.6506$.

Substituting these values into the result of part (a) gives:

$$R_p = \frac{1.539 \times 10^6 \times 0.9848 - 413.6 \times 0.6506}{1.539 \times 10^6 \times 0.9848 + 413.6 \times 0.6506} = 0.99964$$

$$\tau = (1 - R_p^2) \cos\theta / \cos\psi = 0.0011$$

(d) When all the energy is reflected, $|R_p| = 1$, Thus

$$\rho_w c_w \cos\theta - \rho c \cos\psi = \rho_w c_w \cos\theta + \rho c \cos\psi$$

Using Snell's Law, $\sin\psi = \frac{c_w \sin\theta}{c}$. Thus,

$$\cos\psi = \left(1 - \frac{c_w^2}{c^2} \sin^2\theta \right)^{1/2}$$

Substituting this result into the previous equation gives

$$\begin{aligned} & \rho_w c_w \cos\theta - \rho c \left(1 - \frac{c_w^2}{c^2} \sin^2\theta \right)^{1/2} \\ &= \rho_w c_w \cos\theta + \rho c \left(1 - \frac{c_w^2}{c^2} \sin^2\theta \right)^{1/2} \end{aligned}$$

This is true only if $(c_w/c)\sin\theta = 1$, or $\theta = 13^\circ$, which is the angle above which all incident energy will be reflected.

(e) If the sound source were a point source, the amount of sound power entering the water would be independent of altitude (except for atmospheric losses) as the same amount of power is contained in any specified included angle. However, the power would be distributed over an ever increasing area of ocean surface. A distributed sound source

such as a helicopter would exhibit similar behaviour.

- (f) Velocity reflection coefficient.

Using equations 1.6, 1.7, 5.115, 5.116, 5.118 and 5.119 in the text, the acoustic velocity amplitude of the incident wave at $y = 0$ is:

$$\bar{u}_I = \frac{A_I}{\rho c} (\sin\theta - \cos\theta) e^{-jk_{1x}x}$$

and the velocity amplitude of the reflected wave is:

$$\bar{u}_R = \frac{A_R}{\rho c} (\sin\theta + \cos\theta) e^{-jk_{1x}x}$$

The velocity amplitude reflection coefficient is the ratio of reflected to incident velocity amplitudes. Thus:

$$\frac{\bar{u}_R}{\bar{u}_I} = \frac{A_R}{A_I} \frac{(\sin\theta + \cos\theta)}{(\sin\theta - \cos\theta)}$$

Problem 1.56

- (a) Assuming that the bubble is a spherical source, the specific acoustic impedance is given by equation 1.43 in the text as:

$$\frac{p}{u} = \rho c \frac{jkr}{1 + jkr}$$

At the bubble surface, $kr \ll 1$, so:

$$\frac{p}{u} \approx \rho c jkr$$

Acceleration is defined as $\dot{u} = j\omega u$, so:

$$p = \rho c \frac{\dot{u}kr}{\omega} = \rho \dot{u}r$$

- (b) Adiabatic compression $PV^\gamma = \text{const}$ or $P = \text{const} \times V^{-\gamma}$
Differentiating P with respect to V and rearranging gives:

$$\gamma \frac{dV}{V} + \frac{dP}{P} = 0$$

$dP \approx p$, where p is the acoustic pressure. Thus:

$$\frac{p}{P} = -\gamma \frac{dV}{V}$$

Substituting the result for p from part (a) gives:

$$\frac{\rho \dot{u} r}{P} = -\gamma \frac{dV}{V}$$

or

$$\dot{u} = -\frac{\gamma P}{\rho r} \frac{dV}{V}$$

- (c) Resonance frequency is the frequency at which the bubble prefers to vibrate given the physical parameters.

$$\dot{u} = ju\omega$$

$$dV = \frac{4\pi r^2 u}{j\omega} ; \quad V = \frac{4}{3}\pi r^3$$

Substituting for dV and V in the expression of part (b), we obtain:

$$ju\omega = -\frac{\gamma P 4\pi r^2 u}{\rho r j\omega (4/3)\pi r^3} = -\frac{3\gamma P u}{\rho r^2 j\omega}$$

Rearranging gives:

$$\omega^2 = \frac{3\gamma P}{\rho r^2} = \frac{3c^2}{r^2}$$

Thus:

$$f_{res} = \frac{\omega}{2\pi} = \frac{c\sqrt{3}}{2\pi r}$$

- (d) At resonance, the bubble screen should act like a Helmholtz resonator (see Ch. 9) and remove considerable energy from the sound field.

Solutions to problems relating to the Human Ear

Problem 2.1

- (a) Weight of a column of water equivalent to the weight of a column of atmosphere of cross sectional area 1m^2 is equal to 101.4kN , which is equal to $101400/9.81\text{ kg}$. Density of water = 1000kg/m^3 ,
 thus volume of water = $101.4/9.81\text{ m}^3$ and
 thus height of water column = $\text{volume}/(1\times 1) = 10.34\text{m}$.

Minimum audible sound = $0\text{dB} = 20 \times 10^{-6}\text{ Pa r.m.s} = 28.2 \times 10^{-6}\text{ Pa peak}$. This corresponds to a variation in water height of

$$\pm 28 \times 10^{-9}/9.81 = \pm 2.87 \times 10^{-9}\text{ m}$$

- (b) 120dB represents an increase in pressure by a factor of 10^6 over 0dB , so variation in column height would be $\pm 2.9\text{mm}$.
- (c) Figure 2.9 indicates an increase of 60dB for 31.5Hz sound which represents a factor of $10^{60/20} = 1000$.
- (d) See p56, text. Overall mechanical advantage = $15:1 = 23.5\text{dB}$ sound pressure level. Linkage mechanical advantage = $3:1 = 9.5\text{dB}$.

Problem 2.2

- (a) A scaling of physical dimensions by a factor of 10 would mean that the mouse's range of hearing is 200Hz to 180kHz .
- (b) Figure 3.4 and equations 3.14 and 3.16 indicate that the sensitivity is proportional to d^4 , so the mouse's ear should be 4 orders of magnitude (or 40dB) less sensitive.

- (c) The mouse's transduction mechanism must be 4 orders of magnitude more sensitive than the human mechanism.
- (d) Yes, because the differences would be necessary to make the mouse transduction mechanism more sensitive.
- (e) The differences could take the form of a larger mechanical advantage in the mouse's middle ear as well as differences in the relative physical dimensions of the inner ear.

Referring to possible inner ear differences, a critical component of the inner ear in regard to sensitivity is the hinge mechanism of the tectorial membrane. The sensing of sound by the inner and outer hair cells is by a shearing action imposed on the hair cell stereocelia caused by relative movement between the tectorial membrane and the rods of Corti. To increase the sensitivity, it would be necessary to increase this relative movement which could be achieved by lengthening the rods of Corti.

Referring to the middle ear differences, a much smaller oval window and a different arrangement of the bone linkage could account for a large sensitivity increase.

Problem 2.3

- (a) Sound introduced to the ear using ear muffs will be fairly reverberant, having frontal as well as lateral components; thus, we would expect the MAP to be lower than the MAF. If the earmuffs distort the pinna sufficiently, the MAP may approach the MAF in magnitude.
- (b) As the sound field within the ear muffs is similar to a diffuse field, one would expect the MDF to be similar to the MAP and less than the MAF.

Problem 2.4

The first symptoms of noise induced hearing loss are difficulties with understanding conversation in a noisy environment, in focussing on the speaker and in localising noise sources. These symptoms are caused by the breaking off of stereocilia on the outer hair cells leaving only the inner hair cell stereocilia functional. The loss usually occurs first in the 4kHz range and then extends to higher and lower frequencies. A conventional hearing aid which amplifies all frequencies by the same amount will not be much help as it will only amplify the "noise" experienced and not help with the symptoms mentioned above. A frequency selective hearing aid will do a little better by amplifying the signal at the frequencies most affected but it is difficult to see how even this type of hearing aid will ameliorate the above mentioned symptoms of early noise induced hearing loss.

Problem 2.5

Not necessarily. As stated in the text, repeated exposures to noise which results in a temporary threshold shift will result in permanent damage. On the other hand, there is evidence that damage is accumulative, so even one exposure will contribute to the eventual permanent damage.

Problem 2.6

Pitch: Determined by location on the basilar membrane which responds most to the noise. Also the hair cells are tuned to maximum output at frequencies corresponding to the resonance frequencies of the parts of the basilar membrane to which they are attached. However, at low frequencies and loud noise, some neurons will fire for hair cells all along the basilar membrane and a second mechanism by which neurons fire in locked phase with the acoustic signal (at acoustic signal maxima or once per cycle) dominates the pitch determination. As the frequency increases, the localisation of the basilar membrane excitation (and associated resonant hair cells) becomes increasingly important as the method for determining pitch until at 5000Hz the locked phase phenomenon of neuron firing ceases altogether and the neurons fire randomly.

Loudness: Determined by the rate of neuron firing, which is controlled by the motion of the hair cell stereocilia which in turn is controlled by the basilar membrane. There is also a hair cell feedback mechanism whereby the voltage generated by the hair cells causes them to deform, thus increasing the movement of the tectorial membrane. This feedback mechanism effectively increases the dynamic range of the hearing mechanism and also results in improved pitch resolution.

Problem 2.7

- (a) Two signals may have exactly the same spectral content and thus sound the same, but they may have entirely different phase relationships between the particular spectral components making up the signal. Thus the time histories as seen on an oscilloscope could look quite different. Both amplitude and relative phase of the spectral components of a signal are necessary to reconstruct a signal uniquely. However, the ear discards phase information.
- (b) In 0.05 seconds sound will travel a distance of 17.15m. This corresponds to a wavelength at 20Hz. Thus for a 20Hz tone, the peak sound pressures will occur at intervals of 0.05 seconds. Below this frequency there will be greater intervals between the peak sound pressures and the sound will be heard as a sequence of auditory events.
- (c) The dimensions of an auditorium must be such that the sound arriving at any location after being reflected from a wall, ceiling or floor must not arrive longer than 0.05 seconds after the direct sound. This means the difference between direct and reflected paths should be less than 17m.

Problem 2.8

- (a) A low frequency warning device would be more effective mainly because it is not as easily masked by the 500Hz noise as higher frequencies would be but also because it would diffract more effectively around obstacles to create a more uniform coverage.
- (b) See fig 2.10(a) in text and read off values as accurately as possible.

Octave band centre frequency	63	125	250	500	1k	2k	4k	8k
SPL forward	40	44	49	54	56	55	45	33
SPL side	39	43	47	51	53	49	39	27
SPL rear	38	42	44	48	49	44	27	15
Sones forward	0	0.6	1.7	2.8	3.8	4.3	2.8	1.6
Sones side	0.0 5	0.5 5	1.4	2.4	3.2	3	2	1.1
Sones rear	0.0 2	0.5	1.1	2	2.5	2.2	0.9	0.3
Phons forward	0	33	48	55	59	61	55	46.8
Phons side	0	33	45	53	57	55.8	50	41.4
Phons rear	0	30	41	50	53	51.4	39	22.6

Overall Levels:

$$\text{Sones Forward} = 4.3 + 0.3[0.0+0.6+1.7+2.8+3.8+2.8+1.6] = 8.3$$

$$\text{Sones Side} = 3.2 + 0.3[0.05+0.55+1.4+2.4+3.0+2.0+1.1] = 6.3$$

$$\text{Sones Rear} = 2.5 + 0.3[0.02+0.5+1.1+2.0+2.2+0.9+0.3] = 4.7$$

Using equation 2.1,

$$\text{Phons} = 40 + (10\log_{10}S)/(\log_{10}2).$$

Thus, Phons forward = 70.5

Phons side = 66.6

Phons rear = 62.3

- (c) Yes, a person with no hearing loss would be able to hear, as the levels in the bands important for speech recognition are well above the hearing threshold level or MAF.
- (d) If the distance increases to 10m from 2m, the sound pressure level in all bands will decrease by $20\log_{10}(10/2) = 14\text{dB}$ (assuming free field conditions). The MAF from figure 2.9 and the new sound pressure levels are in the table below.

Octave band centre frequency (Hz)	63	125	250	500	1k	2k	4k	8k
MAF	39	22	15	9	4	0	0	12
New SPL forward	26	30	35	40	42	41	31	19
New SPL side	25	29	33	37	39	35	25	13
New SPL rear	24	28	30	34	35	30	13	1

Difficulty in hearing sound in the 63Hz and 8kHz bands would be encountered but as these bands are not important for speech, it is expected that there would be no difficulty in understanding speech for any head orientation.

- (e) Speech recognition is just possible for $S = 8$ Sones. A speaker speaking twice as loudly will increase the rear level from 4 to 8 sones, making speech recognition just possible again, so the situation will be improved.

Problem 2.9

See figure 2.9(a). Masking tone is 800Hz at 60dB. From the figure, the threshold of detection of an 630Hz tone would be increased by 20dB. Figure 2.5 indicates that the MAF at 630Hz is approximately 5dB, so the sound level would need to be 25dB in order to be heard.

Solutions to problems relating to instrumentation and measurement

Problem 3.1

- (a) **Pressure response:** microphone response when subjected to a uniform pressure field which is accomplished in practice electrostatically.

Free-field response: microphone response when subjected to a sound wave coming from a specified direction in an otherwise free field (free of reflected sound).

Random incidence response: microphone response averaged over all possible angles of sound wave incidence.

Microphones are designed so that the roll-off in pressure response at high frequencies is just compensated for by the increase in free field pressure due to diffraction of a normally incident wave (normal incidence microphone) or by the increase in pressure due incident waves averaged over all possible directions of incidence (random incidence microphone).

- (b) Random incidence microphones are used to measure noise in reverberant test chambers, reverberation times in auditoria and industrial noise when the direction of origin is uncertain or there are a number of sources located in various directions from the observer.

Free field microphones are used in anechoic test chambers and in industrial noise measurement cases where the origin of the noise is from a single direction or a narrow direction angle.

Problem 3.2

- (a) Using equation 3.16, the sound level represented by the noise floor on the instrument is:

$$L_p = 20\log_{10}E - S + 94 = -110 + 26 + 94 = 10\text{ dB}$$

If the sound level meter actually reads 13dB, then the contribution due to the actual noise is:

$$L_p = 10\log_{10}(10^{1.3} - 10^{1.0}) = 10\text{ dB}$$

- (b) The ear can hear 0dB of frontally incident sound at 2kHz and it can probably discern signals just above its noise floor. So the equivalent sensitivity of the sound level meter would be approximately 10dB, implying that the ear is 10dB more sensitive.

Problem 3.3

- (a) Sensitivity = -25dB re 1V per Pa. Using equation 3.15, we have: $-25 = 20\log_{10}[E/p]$ which gives a sensitivity of $10^{(-25/20)}$ volts/Pa which = 56.2 mV/Pa.
- (b) The pressure response of the microphone would have to be 4.5dB less at 10kHz than at 250Hz for the overall response to remain flat; that is, -29.5dB re 1V per Pa.
- (c) The microphone overall sensitivity for a 0° incident field at 10kHz is $-29.5 + 5 = -24.5\text{ dB}$.
- (d) The microphone overall sensitivity for a reverberant field at 10kHz is $-29.5 + 1.5 = -28.0\text{ dB}$.
- (e) At 250Hz, it can be seen from figure 3.3 in the text that the correction for a 180° angle of incidence would be 0dB. Thus the overall microphone sensitivity at 250Hz would be equal to -24.5dB, the pressure sensitivity. At 10kHz, the overall sensitivity is: $-29.5 - 0.2 = -29.7\text{ dB}$.

Problem 3.4

The MAF is the minimum audible field to frontally incident sound and would correspond to the free field calibration of a microphone. Similarly the MDF would correspond to the diffuse field calibration of a microphone. The pressure calibration of a microphone would correspond to the MAP which is the minimum pressure audible at the tympanic membrane of the ear.

Problem 3.5

The random incidence microphone is used in cases where one is not sure of the direction from which the sound is coming or if it is coming from a number of directions simultaneously.

To minimise measurement error, the microphone would be held vertically, thus resulting in most sound being incident at angles close to 90° . The difference between the microphone diffuse field response and the 90° response is much smaller than the error resulting from pointing a free field microphone in the wrong direction (see figure 3.3 in text). In both cases measured sound pressure levels will be less than actual levels.

Problem 3.6

This would be similar to connecting the microphone to ground through a low impedance and this would seriously reduce the microphone sensitivity, resulting in large measurement errors.

Problem 3.7

The A-weighted sound pressure level is related to loudness perception of low level environmental noise as well as to hearing damage (although not to loudness perception of loud industrial noise), and regulations are written in terms of this quantity due to its ease of measurement by unskilled people. It is a reasonably valid measure of noise exposure as there is a direct correspondence between A-weighted sound level and hearing loss suffered by noise exposed people.

The advantages of the A-weighted level for characterising equipment and workplaces are its direct relationship to noise exposure and its single number simplicity. However it does not provide the frequency content information necessary for effective noise control measures to be specified and this is the main disadvantage.

Problem 3.8

- (a) The A-weighted levels are calculated by adding the A-weighting corrections (most are negative) to each octave band level and then logarithmically adding the results as described on page 48 of the text. The overall linear level is calculated by adding the values given in the problem together logarithmically as described on page 48 of the text. The answers are: A-weighted = 89.4dB(A) and Linear = 98.5dB.
- (b) The main source of error is a result of the assumption that all frequencies in each octave band can be weighted by a single quantity (the weighting corresponding to the band centre frequency) when in fact the A-weighting is a smoothly varying function of frequency. The maximum possible error can be estimated by comparing the results using the A-weighting corresponding to the band centre frequency with results obtained by using the A-weighting corresponding to the upper and lower band limit frequencies.

Problem 3.9

Plot out the values given in Table 3.1 on graph paper using a logarithmic frequency scale. Join all of the points by a straight line which is as good as a smooth curve for the present purposes. Then plot the upper and lower octave band frequency limits from table 1.2 and read off from your graph the A-weighting corrections at these frequencies. The difference between these values and the octave band centre frequency A-weighted values represent the largest errors which could occur if all of the energy just happened to be at frequencies at the edge of each octave band. To find the overall possible variation calculate the dB(A) levels for each extreme and compare them to the overall level calculated using band centre frequency corrections as illustrated in the table on the next page.

Combining the band levels together as described on p 38 in the text gives:

Upper limit = 84.7dB(A)

Lower limit = 83.3dB(A)

Centre value = 84.1dB(A) (corresponding to A-weighted corrections at band centre frequency).

The A-weighted levels peak at high frequencies so the noise would sound a little "hissy".

Octave band centre freq. (Hz)	63	125	250	500	1000	2000	4000	8000
Upper <i>f</i> limit	88	176	353	707	1,414	2,825	5,650	11,300
Lower <i>f</i> limit	44	88	176	353	707	1,414	2,825	5,650
Upper dB(A) adj.	-21	-12.2	-5.8	-1.2	1.2	1.3	1.1	0.0
Centre dB(A) adj.	-26.2	-16.1	-8.6	-3.2	0.0	1.2	1.0	-1.1
Lower dB(A) adj.	-35	-21	-12.2	-5.8	-1.2	1.1	0.0	-3.5
SPL	76	71	68	70	73	76	79	80
Upper dB(A)	55	58.8	62.2	68.8	74.2	77.3	80.1	80.0
Centre dB(A)	49.8	54.9	59.4	66.8	73	77.2	80	78.9
Lower dB(A)	41	50	55.8	64.2	71.8	77.1	80.0	76.5

Problem 3.10

First remove the background noise contribution from **each** octave band measurement as described in example 1.4, p.49 in the text. Then arithmetically add (some numbers are negative) the A-weighted corrections to each octave band level as was done for problem 3.9. Then add the A-weighted octave band levels together logarithmically as described on p.48 in the text. The final answer is 89.0dB(A).

Problem 3.11

- (a) A-weighted level = 86.8dB(A)
- (b) It is not a good way to calculate overall weighted levels because errors arise from the inherent assumption that the A-weighting is uniform across any particular octave band.

Problem 3.12

To begin, assume an arbitrary level of 50dB in the 63Hz octave band, calculate the A-weighted levels in all bands, the overall A-weighted level and thus the amount to add to each band level to reach an overall level of 105dB(A). The calculations are summarised in the following table, where it is noted that a constant spectrum level is reflected in octave band levels increasing at the rate of 3dB per octave (reflecting a doubling of bandwidth per octave).

Octave band centre frequency (Hz)	63	125	250	500	1000	2000	4000	8000	total
Un-weighted level	50	53	56	59	62	65	68	71	
A-weighted level	23.8	36.9	47.4	55.8	62	66.2	69	69.9	73.8
Adjustment needed	31.2	31.2	31.2	31.2	31.2	31.2	31.2	31.2	
Unweighted level for 105dB(A)	81.2	84.2	87.2	90.2	93.2	96.2	99.2	102.2	105.0

Problem 3.13

Although the signal levels in the experiments to determine equal loudness contours varied substantially, the A-weighted scale corresponds approximately to the loudness contour of 60dB. As industrial noise is usually much louder than this and equal loudness contours for high sound levels do not have the same shape as those at 60dB, it is unlikely that the A-weighted scale will indicate correct loudness levels for most industrial noise.

Problem 3.14

- (a) If the observer is too close to the microphone when noise measurements are being taken, then reflections from the observer can affect the noise levels being measured. In tonal sound fields, the reflections could result in an increase in the measured sound level of up to 5dB (and also decreases) and in broadband sound fields, the increase measured could be up to 2dB. The reasons for the above numbers not being 6 and 3dB respectively is because the observer will absorb and scatter some of the noise while reflecting it.
- (b) The fast response (0.1s time constant) of the sound level meter approximates the way the ear hears but the slow response (1.0s time constant) is useful for determining L_{Aeq} levels (average of the upper swings of the needle on an analog meter) and L_{90} levels (average of lower meter swings). However most modern instrumentation allows direct digital readout of these quantities. In addition, most legislation is written in terms of measurements taken using the "slow" response as some researchers suppose that this is more representative of the hearing damage caused by the noise.
- (c) The "frontal/diffuse" control is used for selecting the microphone characteristic most suitable for the measurement being undertaken. Note that the microphone remains unchanged - the electronics in the sound level meter effectively change its characteristics to compensate for the different diffraction effects of the microphone grid for each of the two types of field.

Problem 3.15

$p = (t^2 + 8t + 4) \times 10^{-2}$, thus

$$p^2 = (t^4 + 16t^3 + 72t^2 + 64t + 16) \times 10^{-4}$$

$$\begin{aligned} \text{Average } p^2 &= \int_0^8 (t^4 + 16t^3 + 72t^2 + 64t + 16) \times 10^{-4} dt \\ &= \frac{1}{8} \left(\frac{8^5}{5} + 4 \times 8^4 + 24 \times 8^3 + 32 \times 8^2 + 16 \times 8 \right) \times 10^{-4} = 0.4675 \end{aligned}$$

$$L_{eq} = 10 \log_{10} \frac{p^2}{p_{ref}^2} = 10 \log_{10} \frac{0.4675}{4 \times 10^{-10}} = 90.7 \text{ dB}$$

A-weighting at 250Hz is -8.6dB, so $L_{Aeq} = 82.1 \text{ dB(A)}$.

Problem 3.16

L_{Aeq} is generally used to describe noise as it is an A-weighted energy average which seems to be related to loudness perception of low level environmental noise as well as to hearing damage (although not to loudness perception of loud industrial noise), and regulations are written in terms of this quantity due to its ease of measurement by unskilled people.

$$\begin{aligned} L_{Aeq} &= 10 \log_{10} \left\{ \frac{1}{1/4 + 2 + 2 + 1/12 + 4} \times \right. \\ &\quad \left. \times (0.25 \times 10^8 + 2 \times 10^7 + 2 \times 10^9 + (1/12) \times 10^{9.9} + 4 \times 10^{7.5}) \right\} \\ &= 85.3 \text{ dB(A)}. \end{aligned}$$

Problem 3.17

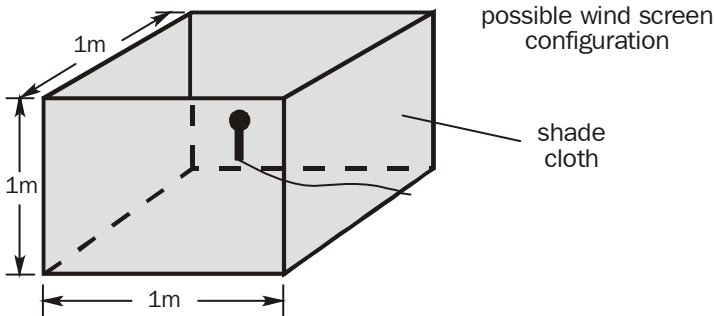
A sound level meter on site is preferable, as a tape recorder is not sufficiently accurate for legal disputes (see table 3.3 in the text).

Problem 3.18

Sources of measurement error:

Microphone vibration;
 SLM vibration;
 background noise;
 overloading input amplifier when taking octave or 1/3 octave band measurements with an old SLM;
 too cold or too hot;
 moisture or dust on microphone diaphragm;
 reflections from nearby surfaces; and
 wind noise.

Can minimise effects of wind noise by placing a foam wind shield on the microphone AND placing the mic in an enclosure made using shade cloth as



shown in the figure below.

Problem 3.19

(a) See text, p114 – 120.

- (b) (i) At low frequencies, errors arise because the phase difference between the two microphone signals (due to the spacing being small compared to a wavelength) is not sufficiently large compared to the phase accuracy of the microphones.
- (ii) At high frequencies, errors arise because the microphone spacing becomes significant compared to a wavelength

causing the finite difference approximation for the pressure gradient to be inaccurate.

- (iii) In very reactive sound fields, the phase between the acoustic pressure and particle velocity is close to 90° , so any phase errors translate to a large error in the intensity as it is proportional to the cosine of the phase angle.
- (iv) In the presence of external noise sources, sound power measurements could exhibit significant errors if the sound pressure level of the external noise is sufficiently high (usually about 10dB or more above the level from the noise source being measured). This is because the power measurement relies on averaging normal intensity measurements and the result of the external source will be to create a situation where small differences between large numbers will dominate the result.

Problem 3.20

- (a) See part (a) in previous question.
- (b) See part (b) in previous question.
- (c) **Applications** include:
 - sound power measurement;
 - localisation and identification of noise sources;
 - sound transmission loss measurement;
 - determining the importance of flanking sound transmission paths in noise control applications;

Problem 3.21

- (a) Sound pressure associated with 95dB sound level is

$$p_{rms} = p_{ref} 10^{L_p/20} = 2 \times 10^{-5} \times 10^{95/20} = 1.12 \text{ Pa}$$

Force on microphone diaphragm is

$$F_{rms} = 1.12 \times \pi \times (0.012)^2 / 4 = 1.27 \times 10^{-4} \text{ N}$$

- (b) r.m.s. velocity of the diaphragm is the same as the air particle velocity. From equation 9.35 in the text, the ratio of the sound pressure to particle velocity in a cavity of dimensions much smaller than a wavelength is

$$\frac{p}{uA} = -\frac{j\rho c^2}{V\omega}$$

The sound pressure measured by the monitoring microphone is 65dB which corresponds to an r.m.s. pressure of

$$p_{rms} = 2 \times 10^{-5} \times 10^{65/20} = 3.557 \times 10^{-2}$$

Thus the particle velocity is

$$\begin{aligned} u_{rms} &= \frac{p\omega V}{\rho c^2 S} ; \quad (S \text{ is the area of the microphone diaphragm}) \\ &= \frac{3.557 \times 10^{-2} \times 2\pi \times 500 \times 0.01 \times 0.02^2 \times (\pi/4)}{1.206 \times 343^2 \times 0.012^2 \times (\pi/4)} \\ &= 2.19 \times 10^{-5} \text{ m/s} \end{aligned}$$

- (c) The volume displacement in the cavity corresponding to a sound pressure level of 65dB can be calculated using the same equation as used in part (b). Thus the volume displacement is

$$\begin{aligned} \text{Vol. displ. ampl} &= \frac{\sqrt{2} u_{rms} S}{\omega} \\ &= \frac{\sqrt{2} \times 2.188 \times 10^{-5} \times \pi \times 0.012^2}{4 \times 2 \times \pi \times 500} \\ &= 1.11 \times 10^{-12} \text{ m}^3 \end{aligned}$$

- (d) Mechanical input impedance, $Z_m = F/u$. Thus

$$Z_m = \frac{1.27 \times 10^{-4}}{2.188 \times 10^{-5}} = 5.8 \text{ N-s/m}$$

- (e) The volume displacement of the monitoring microphone will be 30dB (95 - 65) below the displacement of the test microphone. This represents a percentage difference of $100/10^{1.5} = 3\%$ which will not affect the sound pressure sensed by the test microphone significantly.
- (f) Upper test frequency is limited by the onset of resonant cavity modes. As the largest dimension is the radius, cross modes will occur before axial modes. This is explained in Chapter 7 in the text.

4

Solutions to problems relating to criteria

Problem 4.1

- (a) Using equation 4.42 in the text, the allowable exposure time using European criteria is

$$T_a = 8 \times 2^{-(99 - 90)/3} = 1 \text{ hour}$$

- (b) The allowable exposure using USA criteria is

$$T_a = 8 \times 2^{-(99 - 90)/5} = 2.3 \text{ hours}$$

Problem 4.2

- (a) A-weighted SPL is given by:

$$L_{pA} = 10 \log_{10}(10^{(9.5 - 0.86)} + 10^{(9.7 - 0.32)} + 10^{9.9}) = 100 \text{ dB(A)}$$

- (b) Allowed daily exposure time in Australia is:

$$T_a = 8 \times 2^{-(100 - 90)/3} = 0.8 \text{ hours}$$

Allowed daily exposure time in USA is: $T_a = 8 \times 2^{-(100 - 90)/5} = 2 \text{ hours}$

Problem 4.3

Fan noise = 91dB(A)

Saw idling noise = 88dB(A)

Saw cutting noise = 93dB(A)

Let required fan noise for $L_{Aeq,8h} = 90\text{dB(A)}$ be $x \text{ dB(A)}$.

(a) European criteria

Using equation 4.3 or 4.39 with $L_B = 90$ and $L = 3$:

$$90 = 10 \log_{10} \left(\frac{1}{8} \left\{ 6.4 (10^{88/10} + 10^{x/10}) + 1.6 (10^{93/10} + 10^{x/10}) \right\} \right)$$

Solving for x gives:

$$10^{x/10} = 9.618 \times 10^7$$

Thus, $x = 79.8 \text{ dB(A)}$

The required fan noise reduction is then $91 - 79.8 = 11.2 \text{ dB(A)}$

(b) USA criteria

Using equation 4.39 with the integral replaced with a sum and with with $L_B = 90$ and $L = 5$, we obtain:

$$90 = 16.667 \log_{10} \left(\frac{1}{8} \left\{ 6.4 \left(10^{0.3 \times [10 \log_{10} (10^{8.8} + 10^{x/10}) - 90]/5} \right) \right. \right. \\ \left. \left. + 1.6 \left(10^{0.3 \times [10 \log_{10} (10^{9.3} + 10^{x/10}) - 90]/5} \right) \right\} \right) + 90$$

In solving for x , we must remember the proviso that combined fan and saw noise levels of less than 90 dB(A) at any time do not contribute to the noise exposure results in a value of x as close to 90 dB(A) as possible. Assuming a precision of 0.1 dB(A) , the allowed fan noise plus saw idle noise is 89.9 dB(A) . This is because if the fan noise plus saw idle noise is greater than 90 dB(A) , the overall L'_{Aeq} is greater than 90 dB(A) . Thus the maximum allowed fan noise is

$x = 10 \log_{10} (10^{8.99} - 10^{8.8}) = 85.4 \text{ dB(A)}$. Thus the required fan noise reduction is $91 - 85.4 = 5.6 \text{ dB(A)}$.

Problem 4.4

(a) $L_{Aeq,8h} = 10 \log_{10} (1/8) [2 \times 10^{95/10} + 6 \times 10^{70/10}] = 89.0 \text{ dB(A)}$

(b) $E_{A,8} = 32 \times 10^{(89 - 100)/10} = 2.54 \text{ Pa}^2 \cdot \text{h}$

(c) We may assume that the 70 dB(A) does not contribute significantly, so we

need to find the allowed exposure to 95 dB(A). This is given by
 $T_a = 8 \times 2^{-(95-90)/3} = 2.52$ hours

(d) SPL due to machine only is: $10 \log_{10} [10^{9.5} - 10^{9.1}] = 92.8$ dB(A)

Problem 4.5

$$HDI = 10 \log_{10} \int_0^t 10^{L_p/20} dt$$

In this case,

$$59.5 = 10 \log_{10} (10^{110/20} \times T) = 55 + 10 \log_{10} T$$

where T is the number of years to cross the hearing loss criterion.

Thus, $T \approx 3$ years, and he will be 23 years old before he joins the old folks (assuming that he is in the 20% more sensitive part of the population).

Problem 4.6

(a) Using equation 4.3 in the text:

$$L_{Aeq,8h} = 10 \log_{10} \left\{ \frac{1}{8} (4.5 \times 10^{10.5} + 1.5 \times 10^{9.5}) \right\} = 102.6 \text{ dB(A)}$$

(European criteria)

Using equation 4.41 in the text with the integral replaced with a summation sign:

$$\begin{aligned} L'_{Aeq,8h} &= 16.667 \\ &\times \log_{10} \left\{ (1/8) (10^{0.3 \times (105-90)/5} \times 4.5 + 10^{0.3 \times (95-90)/5} \times 1.5) \right\} + 90 \\ &= 101.4 \text{ dB(A)} \quad (\text{USA criteria}) \end{aligned}$$

(b) European criteria

Using equation 4.42 in the text,

$$T_a = 6 \times 2^{-(102.6-90)/3} = 0.33 \text{ hours}$$

USA criteria

$$T_a = 6 \times 2^{-(101.43 - 90)/5} = 1.23 \text{ hours}$$

Problem 4.7

$$\begin{aligned} HDI &= 10 \log_{10} \sum_i T_i \times 10^{L_{pi}/20} \\ &= 10 \log_{10} (5 \times 10^{8.5/2} + 3 \times 10^{9/2} + 6 \times 10^{9.5/2} + 1 \times 10^{10/2} + 10 \times 10^{8/2}) \\ &= 58.6 \end{aligned}$$

From figure 4.4(b) in the text, there is a 22% risk of developing a 22dB handicap.

Problem 4.8(a) USA criteria

Using equation 4.41 with $L_B = 90$ and $L = 5$, we obtain:

$$\begin{aligned} L'_{Aeq,8h} &= 16.667 \\ &\times \log_{10} \left(\frac{1}{8} \{ 2 \times 10^{0.3 \times (91 - 90)/5} + 2 \times 10^{0.3 \times (96 - 90)/5} \} \right) + 90 \\ &= 88.9 \text{ dB} \end{aligned}$$

$$\text{Daily noise dose} = 2^{(L'_{Aeq} - 90)/5} = 0.86$$

No reduction in exposure time is necessary.

(b) European criteria

Using equation 4.3 or 4.39 with $L_B = 90$ and $L = 3$,

$$\begin{aligned} L_{Aeq} &= 10 \log_{10} \left(\frac{1}{8} \{ 2.4 \times 10^{85/10} + 1.6 \times 10^{88/10} \right. \\ &\quad \left. + 2 \times 10^{91/10} + 2 \times 10^{96/10} \} \right) \\ &= 91.8 \text{ dB(A)} \end{aligned}$$

$$\text{Daily noise dose} = 2^{(L_{Aeq} - 90)/3} = 1.53$$

Allowable exposure time:

$$T_a = \frac{8}{2^{(91.8 - 90)/3}} = 5.2 \text{ hours}$$

Thus a reduction of 2.8 hours is required.

Problem 4.9

$$\text{Number of impacts per day} = 80 \times 60 \times 8 \times 0.6 = 23,040$$

$$\text{B-duration} = 100 \text{ msec}$$

$$\text{B-duration} \times \text{number of impacts} = BN = 2.3 \times 10^6$$

$$\text{Peak SPL} = 125\text{dB}$$

Allowable level for $BN = 2.3 \times 10^6$ is obtained from fig 4.5 in the text.

European criteria

$$L_a = 112.5\text{dB}, \text{ and noise dose} = 2^{(125 - 112.5)/3} = 18$$

U.S.A. criteria

$$L_a = 121\text{dB}, \text{ and noise dose} = 2^{(125 - 120.5)/5} = 1.9$$

Allowable BN for 125dB peak - see fig 4.5 in text.

$$\text{European criteria} \quad BN = 1.40 \times 10^5, \text{ and noise dose} = 23.04/1.4 = 16.4$$

$$\text{U.S.A. criteria} \quad BN = 1.29 \times 10^6, \text{ and noise dose} = 2.304/1.29 = 1.8$$

The small differences in results obtained using the two methods (allowable level vs allowable BN) are due to difficulties in reading the figure any more accurately.

The operator is overexposed according to both criteria.

Required work day decrease

European criteria

Assuming that the press accounts entirely for the exposure of the employee,

then from figure 4.6, the allowed BN product is 1.4×10^5 . In terms of hours, this is equal to: $1.4 \times 10^5 / (80 \times 60 \times 100) = 0.29$ hours of press operation. Accounting for the background noise, the exposure will be controlled by this for a minimum of 7.71 hours ($8 - 0.29$). This corresponds to a noise dose of $(7.71/8) \times 2^{(85-90)/3} = 0.3$. Thus the allowable time of press operation is $0.7 \times 0.29 = 0.20$ hours. [Iterating again does not affect the result significantly].

Thus required workday decrease = $4.8 - 0.2 = 4.6$ hours of press operation.

USA criteria

Background noise of 85dB(A) does not contribute

From figure 4.6, allowable BN product = 1.2×10^6 . In terms of operating hours, this is equal to: $1.29 \times 10^6 / (80 \times 60 \times 100) = 2.69$ hours of press operation. The background noise when the press is not operating does not contribute to the exposure according to USA criteria.

Thus required work day decrease = $4.8 - 2.7 = 2.1$ hours of press operation.

Problem 4.10

Number of impacts per day = 40,000

B-duration = 60 msec

B-duration \times number of impacts = $BN = 2.4 \times 10^6$

Peak SPL = 135dB

Allowable level for $BN = 2.4 \times 10^6$ is obtained from fig 4.6 in the text.

European criteria

$L_a = 112$ dB, and noise dose = $2^{(135-112)/3} = 200$.

U.S.A. criteria

$L_a = 121$ dB, and noise dose = $2^{(135-121)/5} = 7$.

Allowable BN for 135dB peak - see fig 4.6 in text.

European criteria

$BN = 1.2 \times 10^4$

and noise dose = $240/1.2 = 200$

U.S.A. criteria

$$BN = 3.4 \times 10^5$$

$$\text{and noise dose} = 240/34 = 7$$

Allowable number of impactsUSA criteria

The background noise of 87dB(A) contributes nothing to the daily noise dose because it is less than 90dB(A). Thus the allowed number of impacts = $3.4 \times 10^5 / 60 = 5700$.

Thus required decrease = $40,000 - 5,700 = 34,300$.

European criteria

Assuming that the press accounts entirely for the exposure of the employee, then from figure 4.6, the allowed BN product is 1.2×10^4 . The allowable number of impacts is then $1.2 \times 10^4 / 60 = 200$. However this represents such a small part of the 8-hour day, that the 87dB(A) background can be considered to dominate the exposure for almost 8 hours, resulting in a noise dose of 0.5 due to this alone. Thus the allowable dose due to the impact noise is 0.5, which corresponds to 100 impacts.

Problem 4.11

Use figure 4.7 in the text.

Expected level - "very loud voice to shout"

Required level - "shout".

Problem 4.12

See figure 4.7 in text. The answers are:

No. No. No. Too Loudly.

Problem 4.13

The one third octave band levels would have to be first combined into octave band levels by logarithmically adding together three third octave bands for

each octave band result. The three bands to add would be one with a centre frequency the same as the octave band and one band above and one below that one. For example, if the 200Hz, 250Hz and 315Hz one third octave band levels were 60dB, 65dB, and 63dB respectively, then the 250Hz octave band level would be:

$$L_{p250} = 10 \log_{10} (10^{60/10} + 10^{65/10} + 10^{63/10}) = 67.9 \text{ dB}$$

Problem 4.14

A-weighted overall sound levels are inadequate for noise level specification and control because they give no indication of the frequency content of the noise which is necessary for assessing annoyance and determining the type of noise control approach which may be feasible. It is preferable to have data as octave band levels for control purposes, although often for specification purposes, NR, NC or RC numbers are adequate as they take into account the spectral content of the noise. However overall dB(A) numbers are adequate for the purposes of assessing hearing damage risk and for comparing noise (either occupational or environmental) with permitted levels according to local regulations.

Problem 4.15

(a) A-weighted levels are calculated and tabulated below

Frequency (Hz)	63	125	250	500	1k	2k	4k	8k
L_p (dB re 20μPa)	100	101	97	91	90	88	86	81
A-weighting	-26.2	-16.1	-8.6	-3.2	0.0	1.2	1.0	-1.1
A-weighted level	73.8	84.9	88.4	87.8	90	89.2	87	79.9

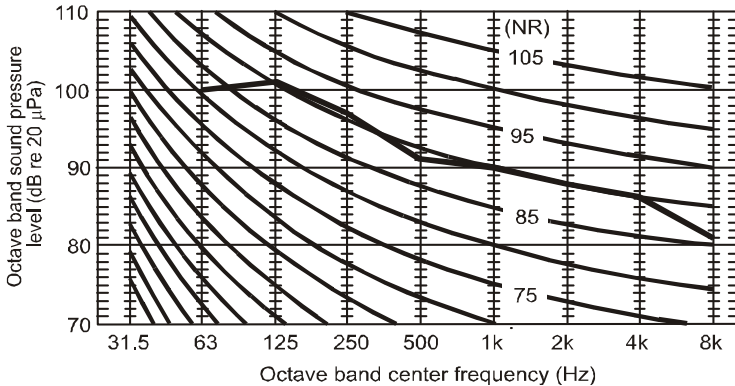
The overall A-weighted level is:

$$\begin{aligned}
 L_p &= 10 \log_{10} (10^{7.38} + 10^{8.49} + 10^{8.84} \\
 &\quad + 10^{8.78} + 10^9 + 10^{8.92} + 10^{8.7} + 10^{7.99}) \\
 &= 96.1 \text{ dB(A)}
 \end{aligned}$$

(b) Using equation 4.42 in the text, the allowable number of hours is:

$$T_a = 8 \times 2^{-(96.1 - 90)/3} \approx 1.95 \text{ hours}$$

(c) *NR* level of noise - plot levels on *NR* curves as shown below, where it may be seen that *NR* = 91.



(d) Loudness level

Frequency (Hz)	63	125	250	500	1k	2k	4k	8k
L_p (dB re 20μPa)	100	101	97	91	90	88	86	81
Sones	28.5	38	35.3	28.5	33	35.3	38	33

The overall level in sones is calculated using equation 2.33. Thus:

$$L = 38 + 0.3[28.5 + 35.3 + 28.5 + 33 + 35.3 + 38 + 33] = 107.5 \text{ sones}$$

From equation 2.32:

$$P = 40 + 10 \frac{\log_{10} S}{\log_{10} 2} = 107.5 \text{ phons}$$

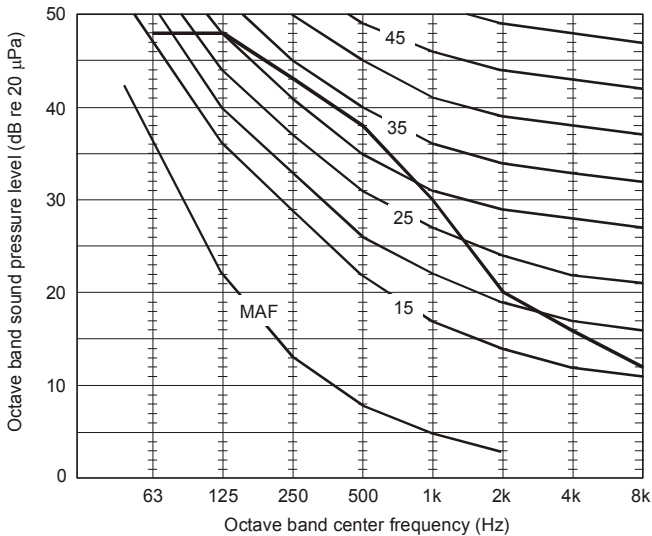
Note that the above equation is inaccurate for levels above 100 phons.

- (e) Following example 1.4, the contribution of the machine to the overall level is x dB(A) where x is defined as:

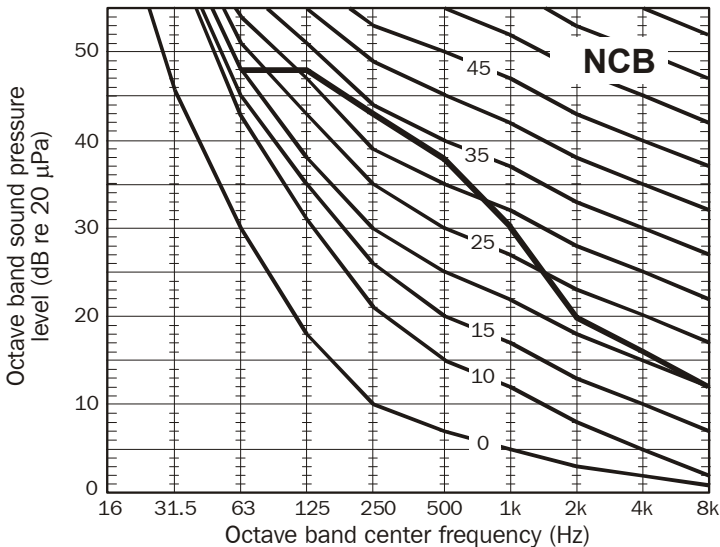
$$x = 10 \log_{10} (10^{96.1/10} - 10^{(96.1 - 1.5)/10}) = 90.8 \text{ dB(A)}$$

Problem 4.16

- (a) The levels are first plotted on NC and NCB curves



The result is $NC = 33$ and $NCB = (38 + 30 + 20 + 16)/4 = 26$

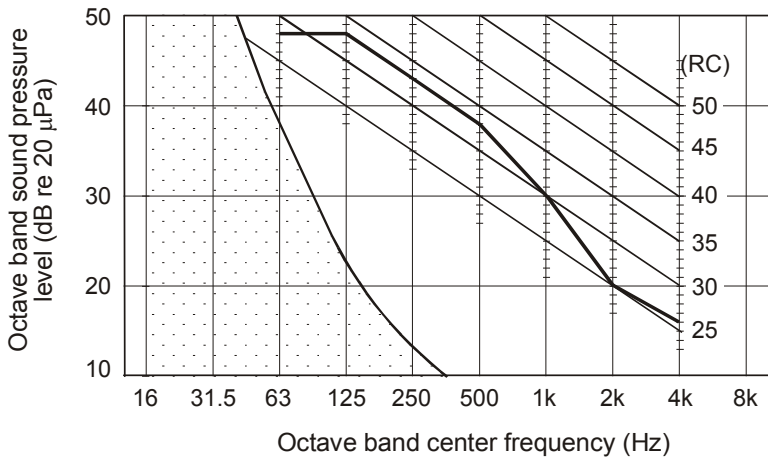


The system is sufficiently quiet for churches holding less than 250 people (see table 4.8 in the third edition of the textbook and table 4.2 in the first edition or look up AS2107-1987), however for larger churches, the level should be about 5dB lower.

- (b) This NCB level is exceeded by more than 3dB in 125Hz, 250Hz and 500Hz bands so it will sound rumble. Note that RC criteria would result in a neutral classification (not rumble or hissy).

Best fit between 125Hz and 500Hz is $NCB = 33$. No levels in the octave bands between 1000Hz and 8000Hz are above $NCB = 33$ so sound is not hissy.

This conclusion can be checked by plotting on RC curves. As can be seen from the following RC plot, the noise will be neutral (not rumble or hissy -see 3rd. Edn. text, page 159) as no levels in bands below 500 Hz exceed the RC level by more than 5 dB. Note $RC = (38+30+20)/3 = 29$. Although this conclusion is different to that drawn using NCB curves, it can be seen that the RC classification is close to rumble.



- (c) Optimum spectrum levels of added masking noise would be equal to the RC-33 levels (as this corresponds to the highest existing spectrum levels) with the existing levels logarithmically subtracted from it. For example, in the 63Hz band, the RC-33 value is 53dB and the desired added level is:

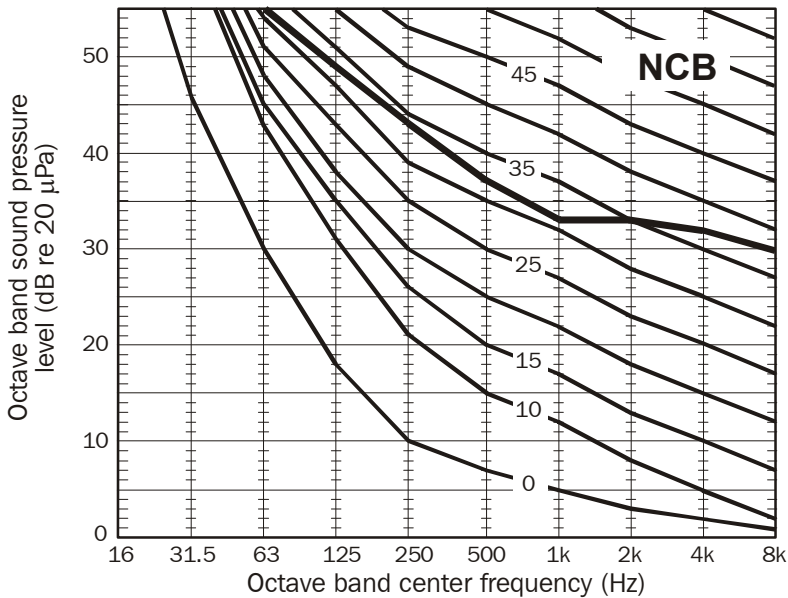
$$L_A = 10 \log_{10} (10^{53/10} - 10^{48/10}) = 51.3 \text{ dB}$$

Desired spectrum levels of masking noise are listed in the table below.

Octave band centre frequency (Hz)	63	125	250	500	1000	2000	4000	8000
Sound pressure level (dB)	48	48	43	38	30	20	16	12
RC-30 values	50	48	43	35	33	28	23	
Desired added levels	51.3	0	0	0	30	27	22	

Problem 4.17

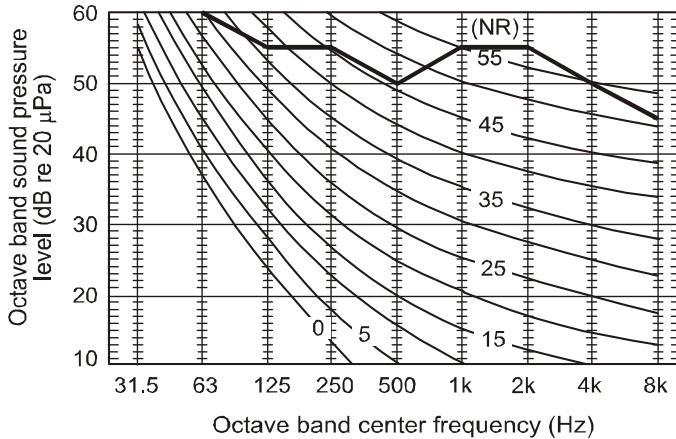
- (a) The NCB value for the noise is $(37 + 33 + 33 + 32)/4 = 34$
- (b) The line of best fit for the NCB curve between 125 Hz and 500 Hz is 32 or 33 NCB. Three high frequency bands exceed this curve so the noise sounds hissy. No bands below 500 Hz exceed 34 NCB so noise is not rumbling. See following figure.

**Problem 4.18**

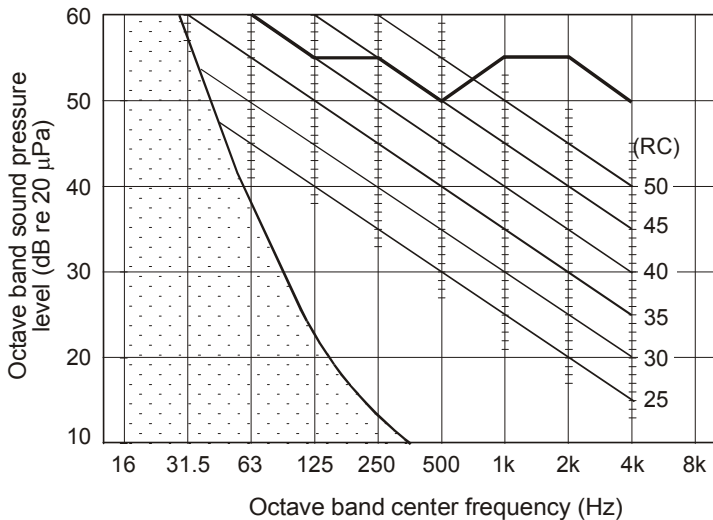
- (a) A-weighted level is:

$$\begin{aligned}
 L_A &= 10 \log_{10} \left(10^{(60 - 26.2)/10} + 10^{(55 - 16.1)/10} + 10^{(55 - 8.6)/10} \right. \\
 &\quad \left. + 10^{(50 - 3.2)/10} + 10^{(55 - 0)/10} + 10^{(55 + 1.2)/10} \right. \\
 &\quad \left. + 10^{(50 + 1.0)/10} + 10^{(45 - 1.1)/10} \right) \\
 &= 59.9 \text{ dB(A)}
 \end{aligned}$$

NR level from following figure (NR curves) = 58.



- (b) The levels are plotted on RC curves below, where it can be seen that the spectrum would sound hissy.



- (c) The noise level is 59.9dB(A). The dB(A) adjustments to the base level of 40dB(A), the resulting allowable noise levels and the expected public reactions are:

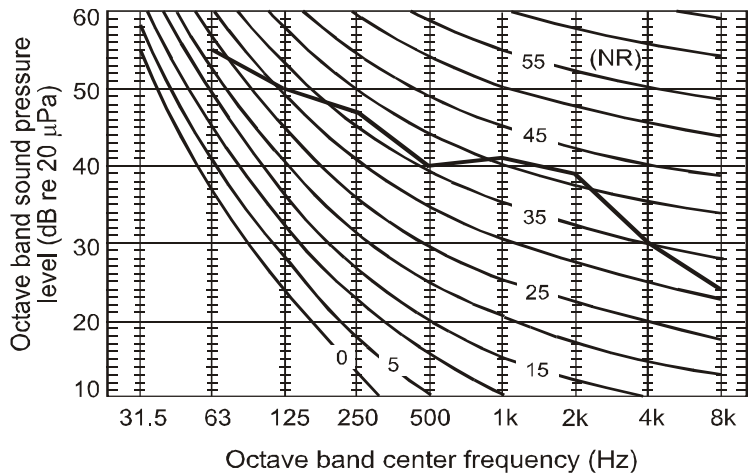
day: +20 -2 = 58dB(A) (marginal public reaction)
evening: +20 -5 -2 = 53dB(A) (little public reaction)
night: +20 -10 -2 = 48dB(A) (medium public reaction)

See table 4.11 in the text for public reaction estimates.

- (d) The noise reductions between inside and outside and the resulting inside levels are in the table below.

Octave band centre frequency (Hz)	63	125	250	500	1000	2000	4000	8000
Exterior sound pressure levels	60	55	55	50	55	55	50	45
Expected noise reduction (dB)	5	5	8	10	14	16	20	21
Interior sound pressure levels	55	50	47	40	41	39	30	24

The spectrum in the last line of the table is plotted in the figure below and represents an *NR* value of 42.



From Table 4.12 in the text, daytime base level = $NR\ 30$ and nighttime base level = $NR\ 25$.

Daytime adjustments = $+5 +10$, which give an allowable $NR = 45$
 nighttime adjustment = $+10$, which gives an allowable $NR = 35$

Thus we would expect complaints during the evening and night but not during the day if the windows are closed.

When the windows are open, 5dB is added to levels in all octave bands and the NR value of the interior noise becomes $NR\ 47$. This would result in a few complaints during the day and an increase in the number of nighttime complaints.

- (e) Factory could be built provided it only operated during the day. If the noise occurred only 25% of the time:

$$L_{Aeq} = 10 \log_{10}(0.25 \times 10^{59.9/10}) = 53.9 \text{ dB(A)}.$$

From the results of (c) above, it can be seen that the factory could now operate in the evening as well, but not at night. However from the results of (d) above, and table 4.11 in text, the NR criteria would indicate that it should still only operate during the day.

Problem 4.19

From table 4.10 in the text, the acceptable level for nighttime operation is $L_{Aeq} = 40 - 10 + 15 = 45 \text{ dB(A)}$. From table 4.11 in the text, the expected community response would be widespread complaints.

Solutions to problems relating to sound sources and outdoor sound propagation

Problem 5.1

- (a) Intensity, $I = W/S = 10^{-2} / (4\pi \times 0.5^2)$ Watts/m² = 3.18 mWatts/m²
- (b) $p_{rms} = \sqrt{I\rho c} = \sqrt{3.18 \times 1.206 \times 343 \times 10^{-3}} = 1.15$ Pa and thus the pressure amplitude = $p_{rms}\sqrt{2} = 1.62$ Pa.
- (c) For outwardly travelling spherical waves in free space, equation 1.40c may be written as:

$$\varphi = \frac{A}{r} e^{j(\omega t - kr)}$$

Using equations 1.6 and 1.7, the acoustic pressure and particle velocity may be written respectively as:

$$p = j\omega\rho \frac{A}{r} e^{j(\omega t - kr)}$$

and

$$\begin{aligned} u &= \frac{A}{r^2} e^{j(\omega t - kr)} + \frac{jkA}{r} e^{j(\omega t - kr)} \\ &= \frac{A}{r} e^{j(\omega t - kr)} \left(\frac{1}{r} + jk \right) \\ &= \frac{p}{j\omega\rho} \left(\frac{1}{r} + jk \right) = \frac{p}{\rho c} \left(1 - \frac{j}{kr} \right) \end{aligned}$$

Thus: $|u| = \frac{|p|}{\rho c} \left| 1 - \frac{j}{kr} \right|$

or,

$$|u| = \frac{|p|}{\rho c} \sqrt{1 + \left(\frac{c}{2\pi fr} \right)^2}$$

$$= \frac{1.62}{1.206 \times 343} \sqrt{1 + \left(\frac{343}{2 \times \pi \times 400 \times 0.5} \right)^2} = 4.06 \text{ mm/s}$$

(d) $L_p = 20 \log_{10} p_{rms} + 94 = 95.2 \text{ dB}$

(e) $L_w = 10 \log_{10} W + 120 = 100 \text{ dB}$

(f) Source strength, Q , may be calculated using equation 5.12 in the text. Thus:

$$Q = \sqrt{\frac{4\pi W}{k^2 \rho c}} = \sqrt{\frac{Wc}{\pi f^2 \rho}}$$

$$= \sqrt{\frac{0.01 \times 343}{\pi \times 400^2 \times 1.206}} = 2.38 \times 10^{-3} \text{ m}^3/\text{s}$$

Problem 5.2

(a) From equation 5.13(b) in the text, it can be seen that for a simple source the r.m.s. pressure² is inversely proportional to the distance² from the source. Thus the sound pressure level at 10m would be $110 - 20 \log_{10}(10/1) = 90 \text{ dB}$.

(b) We can use the result of 5.1(c) above. The acoustic pressure amplitude at 1m is $\bar{p} = 2 \times 10^{-5} \times \sqrt{2} \times 10^{110/20} = 8.945 \text{ Pa}$ and the amplitude at 10m is

$$\bar{p} = 2 \times 10^{-5} \times \sqrt{2} \times 10^{90/20} = 0.8945 \text{ Pa}.$$

At 1m, $kr = 2\pi f/c = 2\pi \times 100/343 = 1.832$. At 10m, $kr = 18.32$. The

result of 5.1(c) may be written as:

$$\bar{u} = \frac{\bar{p}}{\rho c} \sqrt{1 + \frac{1}{(kr)^2}}$$

Thus at 1m, $\bar{u} = \frac{8.945}{1.206 \times 343} \sqrt{1 + \frac{1}{1.832^2}} = 24.6 \text{ mm/s}$ and the phase relative to the acoustic pressure is:

$$\beta = -\tan^{-1}\left(\frac{1}{kr}\right) = -\tan^{-1}\frac{1}{1.832} = -28.6^\circ$$

At 10m, $\bar{u} = \frac{0.8945}{1.206 \times 343} \sqrt{1 + \frac{1}{18.32^2}} = 2.2 \text{ mm/s}$ and the phase relative to the acoustic pressure is:

$$\beta = -\tan^{-1}\left(\frac{1}{kr}\right) = -\tan^{-1}\frac{1}{18.32} = -3.1^\circ$$

Problem 5.3

Source volume velocity = $4\pi r^2 u_{rms} = 4\pi \times 0.01^2 \times 0.5 = 6.28 \times 10^{-4}$

At 100Hz, $k = 2\pi \times 100/343 = 1.832 \text{ m}^{-1}$ and at 800Hz, $k = 14.655 \text{ m}^{-1}$.

Using equation 5.12 in the text, the acoustic power radiated at 100Hz is:

$$W = \frac{Q^2 k^2 \rho c}{4\pi} = \frac{6.28^2 \times 10^{-8} \times 1.832^2 \times 1.206 \times 343}{4\pi} = 43.6 \mu \text{ Watts}$$

and at 800Hz, the acoustic power radiated is:

$$W = \frac{Q^2 k^2 \rho c}{4\pi} = \frac{6.28^2 \times 10^{-8} \times 14.655^2 \times 1.206 \times 343}{4\pi} = 2.79 \text{ milli Watts}$$

The corresponding sound power levels are calculated using:

$$L_w = 10 \log_{10} W + 120 \text{ dB}$$

Thus at 100Hz, $L_w = 76.4 \text{ dB}$ and at 800Hz, $L_w = 94.5 \text{ dB}$.

Problem 5.4

- (a) The amplitude of the pressure fluctuations can be calculated using equation 5.13(a) in the text. The amplitude of the volume velocity, \bar{Q} , is given by:

$$\bar{Q} = 4\pi r^2 \bar{u} = 4\pi \times 0.01^2 \times 0.1 = 1.257 \times 10^{-4} \text{ m}^3/\text{s}$$

At 50Hz, $k = 2\pi f/c = 2\pi \times 50/343 = 0.916$, and the amplitude of the pressure fluctuations is then:

$$\bar{p} = \frac{\bar{Q}k\rho c}{4\pi r} = \frac{1.257 \times 10^{-4} \times 0.916 \times 1.206 \times 343}{4\pi \times 10} = 3.79 \times 10^{-4} \text{ Pa}$$

- (b) Using the equation from problem 5.2(c), the phase of the pressure minus the phase of the particle velocity is given by:

$$\beta = -\tan^{-1}\left(\frac{1}{kr}\right)$$

At $r = 0.5\text{m}$, the above expression gives $\beta = 65.4^\circ$ and at $r = 10\text{m}$, $\beta = 6.2^\circ$.

This indicates that close to the source the acoustic pressure field is dominated by near field effects, whereas at 10m from the source, the near field effects will be small and the field may be approximated as a propagating plane wave.

Problem 5.5

The radiation impedance per unit area is equivalent to the specific acoustic impedance, Z which is simply p/u .

For outwardly travelling spherical waves in free space, equation 1.40c may be written as:

$$\varphi = \frac{A}{r} e^{j(\omega t - kr)}$$

Using equations 1.6 and 1.7, the acoustic pressure and particle velocity may be written respectively as:

$$p = j\omega\rho \frac{A}{r} e^{j(\omega t - kr)}$$

and

$$\begin{aligned} u &= \frac{A}{r^2} e^{j(\omega t - kr)} + \frac{jkA}{r} e^{j(\omega t - kr)} \\ &= \frac{A}{r} e^{j(\omega t - kr)} \left(\frac{1}{r} + jk \right) \\ &= \frac{p}{j\omega\rho} \left(\frac{1}{r} + jk \right) = \frac{p}{\rho c} \left(1 - \frac{j}{kr} \right) \end{aligned}$$

Thus, setting $r = a$,

$$\begin{aligned} \frac{p}{u} &= \rho c \left(1 - \frac{j}{ka} \right)^{-1} = \rho c \left(\frac{1 + \frac{j}{ka}}{1 + \frac{1}{(ka)^2}} \right) \\ &= \rho c \left(\frac{(ka)^2 + jka}{(ka)^2 + 1} \right) = \rho c \left(\frac{\omega^2 a^2 + j\omega a c}{c^2 + \omega^2 a^2} \right) \end{aligned}$$

Problem 5.6

Follow the analysis in the text described by equations 5.1 to 5.12. In equation 5.12, replace the mean square volume velocity \mathcal{Q}^2 with the product of half the velocity amplitude and the surface area of the pulsating sphere; that is,

$$\mathcal{Q}^2 = \frac{|U|^2}{2} (4\pi a^2)^2. \text{ The required result is then obtained.}$$

Problem 5.7

The wave equation is:

$$\left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial \psi^2} \right] \varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = 0$$

- (a) The solution given by equation 5.25 in the text is:

$$\varphi = 2f'(ct - r)(h/r)\cos\theta$$

Substituting the solution into the various terms in the wave equation gives:

$$r^2 \frac{\partial \varphi}{\partial r} = -2f''(ct - r)(hr)\cos\theta - 2f'(ct - r)h\cos\theta$$

$$\begin{aligned} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \varphi}{\partial r} \right) &= -2f'''(ct - r)(h/r)\cos\theta - 2f''(ct - r)(h/r^2)\cos\theta \\ &\quad + 2f''(ct - r)(h/r^2)\cos\theta \end{aligned}$$

$$\sin\theta \frac{\partial \varphi}{\partial \theta} = -2f'(ct - r)(h/r)\sin^2\theta$$

$$\frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial \varphi}{\partial \theta} \right) = -2f'(ct - r)(h/r^3)2\cos\theta$$

$$\frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial \psi^2} = 0$$

$$-\frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = -2f'''(ct - r)(h/r)\cos\theta$$

Adding all the above terms together gives:

$$-4f'(ct - r)(h/r^3)\cos\theta$$

which must equal zero to satisfy the wave equation. This is true provided that r is very large compared to h .

- (b) Equations 5.32 and 5.33 in the text may be written as:

$$p = \rho \frac{\partial \varphi}{\partial t} = \frac{A \cos\theta}{kr} \left[1 - \frac{j}{(kr)} \right] e^{j(\omega t - kr)}$$

$$u_r = -\nabla \varphi = \frac{A \cos\theta}{kr\rho c} \left[1 - \frac{2}{(kr)^2} - \frac{2j}{kr} \right] e^{j(\omega t - kr)}$$

Using the first of the above equations and omitting the integration constant:

$$\varphi = \frac{1}{\rho} \int_0^T p \, dt = -\frac{jA \cos \theta}{\rho c k^2 r} \left(1 - \frac{j}{kr} \right) e^{j(\omega t - kr)}$$

Checking the expression for u , by evaluating $-\nabla \varphi$, we obtain:

$$\begin{aligned} -\frac{\partial \varphi}{\partial r} &= -\frac{jA \cos \theta}{\rho c k^2 r^2} \left(1 - \frac{j}{kr} \right) e^{j(\omega t - kr)} \\ &\quad + \frac{jA \cos \theta}{\rho c k^2 r} \left(\frac{j}{kr^2} \right) e^{j(\omega t - kr)} \\ &\quad + \frac{jA \cos \theta}{\rho c k^2 r} \left(1 - \frac{j}{kr} \right) (-jk) e^{j(\omega t - kr)} \end{aligned}$$

To verify that the expression obtained above for φ is a solution to the wave equation we substitute it into the wave equation and calculate the result term by term as follows:

$$\begin{aligned} r^2 \frac{\partial \varphi}{\partial r} &= \frac{A r \cos \theta}{\rho c k} \left(1 - \frac{2j}{kr} - \frac{2}{(kr)^2} \right) e^{j(\omega t - kr)} \\ \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \varphi}{\partial r} \right) &= -\frac{A \cos \theta}{r^2 \rho c k} \left(1 - \frac{2j}{kr} - \frac{2}{(kr)^2} \right) e^{j(\omega t - kr)} \\ &\quad - \frac{A r \cos \theta}{r^2 \rho c k} \left(\frac{2j}{kr^2} + \frac{4}{k^2 r^3} \right) e^{j(\omega t - kr)} \\ &\quad - \frac{A r \cos \theta}{r^2 \rho c k} \left(1 - \frac{2j}{kr} - \frac{2}{(kr)^2} \right) (-jk) e^{j(\omega t - kr)} \\ &= -\frac{A \cos \theta}{r \rho c k} \left(-\frac{1}{r} + \frac{2}{k^2 r^3} - jk + \frac{2j}{kr^2} \right) e^{j(\omega t - kr)} \end{aligned}$$

$$\sin\theta \frac{\partial\varphi}{\partial\theta} = \frac{jA \sin^2\theta}{\rho c k^2 r} \left(1 - \frac{j}{kr} \right) e^{j(\omega t - kr)}$$

$$\frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial\varphi}{\partial\theta} \right) = \frac{2jA \cos\theta}{\rho c k^2 r^3} \left(1 - \frac{j}{kr} \right) e^{j(\omega t - kr)}$$

$$\frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial\psi^2} = 0$$

$$-\frac{1}{c^2} \frac{\partial^2\varphi}{\partial t^2} = \frac{\omega^2}{c^2} \varphi = -\frac{j k^2 A \cos\theta}{\rho c k^2 r} \left(1 - \frac{j}{kr} \right) e^{j(\omega t - kr)}$$

Adding all the above terms together gives:

$$\frac{jA \cos\theta}{\rho c r} \left(-\frac{j}{kr} + \frac{2j}{(kr)^3} + 1 - \frac{2}{(kr)^2} + \frac{2}{(kr)^2} - \frac{2j}{(kr)^3} - 1 + \frac{j}{kr} \right) e^{j(\omega t - kr)}$$

which is equal to zero. Thus the solutions given by equations 5.32 and 5.33 satisfy the spherical wave equation exactly.

Problem 5.8

Equation 1.35 in the text is:

$$\varphi = \frac{f(ct - r)}{r}$$

To verify that the expression obtained above for φ is a solution to the wave equation we substitute it into the wave equation and calculate the result term by term.

$$r^2 \frac{\partial^2\varphi}{\partial r^2} = -f(ct - r) - r f'(ct - r)$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \varphi}{\partial r} \right) = \frac{f'(ct-r)}{r^2} + \frac{f''(ct-r)}{r} - \frac{f'(ct-r)}{r^2}$$

$$\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \varphi}{\partial \theta} \right) = 0$$

$$\frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \psi^2} = 0$$

$$-\frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = -\frac{f''(ct-r)}{r}$$

Adding all the above terms together gives 0, so equation 1.35 in the text is a solution.

Problem 5.9

The wavenumber, k , is equal to $(2\pi f/c) = (2\pi \times 250/343) = 4.58$.

The source strength, Q , of the monopole may be calculated using equation 5.12 in the text. That is:

$$Q = \sqrt{\frac{4\pi W}{k^2 \rho c}} = \sqrt{\frac{4\pi \times 0.5}{4.58^2 \times 1.206 \times 343}} = 2.69 \times 10^{-2} \text{ m}^3/\text{s}$$

The dipole acoustic power, W_D , may be calculated using equation 5.29 in the text to give:

$$\begin{aligned} W_D &= \rho c \frac{k^4 h^2 Q^2}{3\pi} \\ &= \frac{1.205 \times 343 \times 4.58^4 \times (0.08/2)^2 \times 2.691^2 \times 10^{-4}}{3\pi} \\ &= 0.0224 \text{ watts} \end{aligned}$$

Sound power level = $10\log_{10}W + 120 = 103.5\text{dB}$.

Problem 5.10

For a single source:

$$p(r) = \frac{j\omega\rho q e^{-jkr}}{4\pi r}$$

For 2 sources separated by a distance $2h$, the total pressure, p , is the sum of the pressures p_1 and p_2 from each source. Thus:

$$p(r) = p_1 + p_2 = \frac{j\omega\rho}{4\pi} \left(\frac{q_1}{r_1} e^{-jkr_1} + \frac{q_2}{r_2} e^{-jkr_2} \right)$$

As shown on page 179 in the text, for phase accuracy purposes, the following approximations are adequate:

$$r_1 \approx r + h \cos\theta \quad \text{and} \quad r_2 \approx r - h \cos\theta$$

where it has been assumed that $h \ll r$.

Noting that for amplitude purposes, $r_1 \approx r_2 \approx r$

The above equation may be rewritten as:

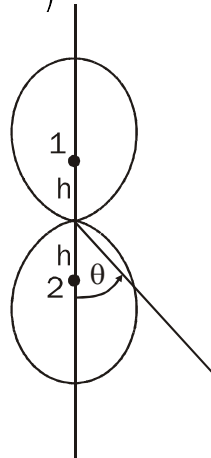
$$\begin{aligned} p(r, \theta) &= \frac{j\omega\rho}{4\pi r} e^{-jkr} \left(q_1 e^{-jkh \cos\theta} + q_2 e^{jkh \cos\theta} \right) \\ &= p_1(r, \theta) \left(1 + \frac{q_2}{q_1} e^{2jkh \cos\theta} \right) \end{aligned}$$

When $\theta = \theta_0$, $p = 0$, or

$$1 + \frac{q_2}{q_1} e^{2jkh \cos\theta_0} = 0.$$

$$\text{Thus, } \frac{q_2}{q_1} = -e^{-2jkh \cos\theta_0}.$$

Substituting this into the preceding equation for p gives:



$$p(r, \theta) = p_1(r, \theta) \left(1 - e^{-2jkh(\cos\theta_0 - \cos\theta)} \right)$$

$$\text{If } \theta_0 = 90^\circ, \text{ then } p(r, \theta) = p_1(r, \theta) \left(1 - e^{2jkh \cos\theta} \right)$$

Taking the modulus of the preceding equation gives:

$$|p| = 2 |p_1| (1 - \cos(2kh \cos\theta))$$

This function contains the directivity information and is plotted in the figure above.

Problem 5.11

The wavenumber, k , is equal to $(2\pi f/c) = (2\pi \times 500/343) = 9.16$.

The source strength, Q , of each monopole making up the dipole source may be calculated using equation 5.12 in the text. That is:

$$Q = \sqrt{\frac{4\pi W}{k^2 \rho c}} = \sqrt{\frac{4\pi \times 0.01}{9.16^2 \times 1.206 \times 343}} = 1.903 \times 10^{-3} \text{ m}^3/\text{s}$$

(a) The dipole intensity at $\theta = 45^\circ$ is given by equation 5.28 in the text and is:

$$\begin{aligned} I_D &= \rho c \frac{k^4 h^2 Q^2}{(2\pi r)^2} \cos^2\theta \\ &= \frac{1.205 \times 343 \times 9.16^4 \times (0.005/2)^2 \times 1.903^2 \times 10^{-6}}{(2\pi \times 0.5)^2} \times 0.707^2 \\ &= 3.34 \mu \text{ Watts/m}^2 \end{aligned}$$

(b)

$$\langle p^2 \rangle = \rho c I = 1.205 \times 343 \times 3.33 \times 10^{-6} = 1.38 \times 10^{-3} \text{ Pa}^2$$

$$L_p = 10 \log_{10} \frac{\langle p^2 \rangle}{p_{ref}^2} = 65.4 \text{ dB}$$

- (c) Required driving force can be calculated by taking the mean square value calculated using equation 5.39 in the text. Thus:

$$F_{rms} = \frac{4\pi a A}{3k\sqrt{2}} \sqrt{1 + \frac{1}{(ka)^2}}$$

From equation 5.35 in the text:

$$\frac{A}{\sqrt{2}} = \frac{\rho c h Q k^3}{2\pi}$$

Substituting in the previously calculated values for k and Q gives:

$$\frac{A}{\sqrt{2}} = \frac{1.206 \times 343 \times (0.005/2) \times 1.903 \times 10^{-3} \times 9.16^3}{2\pi} = 0.241$$

As ka is very small, equation 5.40 in the text may be used. Thus:

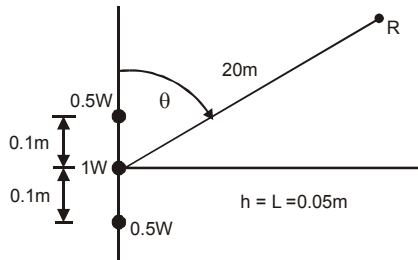
$$F_{rms} = \frac{A}{\sqrt{2}} \frac{4\pi}{3k^2} = \frac{0.241 \times 4\pi}{3 \times 9.16^2} = 12.0 \text{ mN}$$

Problem 5.12

The arrangement is illustrated in the figure.

As R is 1m off the floor, the distance to it is

$$\sqrt{20^2 + 1^2} = 20.02 \text{ m}$$



- (a) The strength of each source may be calculated using equation 5.12 in the text:

$$Q^2 = \frac{W_M 4\pi}{\rho c k^2}$$

In this case, $W_M = 0.5W$ and $k = 2\pi \times 125/343 = 2.290$. Thus:

$$Q^2 = \frac{0.5 \times 4\pi}{1.206 \times 343 \times 2.29^2} = 2.897 \times 10^{-3}$$

The arrangement shown is a longitudinal quadrupole and the mean square sound pressure at any location may be calculated using equations 5.54 and 5.55 in the text. Note that the equation for the mean square pressure is multiplied by 2 in this case because the radiation is into half space. Thus:

$$\begin{aligned} \langle p^2 \rangle &= \frac{5\rho c W_{long} \cos^4\theta}{4\pi r^2} \times 2 = Q^2 \left(\frac{\rho c k^3 h L \cos^2\theta}{\pi r} \right)^2 \times 2 \\ &= 2.897 \times 10^{-3} \left(\frac{1.206 \times 343 \times 2.29^3 \times 0.05^2 \times \cos^2(30)}{\pi \times 20.02} \right)^2 \times 2 \\ &= 1.271 \times 10^{-4} \text{ Pa}^2 \end{aligned}$$

The sound pressure level is then:

$$L_p = 10 \log_{10} \frac{1.271 \times 10^{-4}}{4 \times 10^{-10}} = 55.0 \text{ dB}$$

- (b) 125Hz random noise will make the sources act independently and the power radiated will be the arithmetic sum of the individual sources. Thus $W = 1.5W$ and

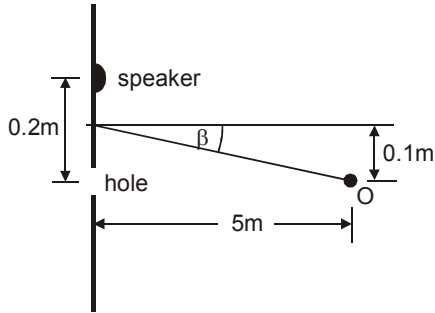
$$L_w = 10 \log_{10}(1.5/10^{-12}) = 121.8 \text{ dB}.$$

Using equation 5.108 in the text, the sound pressure level may be calculated using $S = 2\pi r^2$ and $\rho c = 413.6$. Thus:

$$L_p = 121.8 - 10 \log_{10}(2\pi \times 20.02^2) - 10 \log_{10}(400/413.66) = 87.9 \text{ dB}$$

Problem 5.13

- (a) The arrangement approximates a simple dipole and is illustrated in the figure. The angle $\beta = \sin^{-1} 0.1/5 = 1.145^\circ$. The azimuthal angle given in the problem is irrelevant.



The wavenumber, $k = (2\pi \times 250)/343 = 4.58$.

The sound pressure levels radiated by the hole alone (monopole) and hole + speaker (dipole) may be calculated using equations 5.13a, 5.30b and 5.29 in the text.

Using these equations, the ratio of the mean square pressures (dipole/monopole) may be written as:

$$\frac{\langle p_D^2 \rangle}{\langle p_M^2 \rangle} = 4(kh)^2 \cos^2(90 - 1.145)$$

$$= 4(4.58 \times 0.1)^2 (0.02)^2 = 3.36 \times 10^{-4}$$

Thus the reduction in sound pressure level due to the presence of the speaker is:

$$\text{Reduction} = 10 \log_{10} (3.36 \times 10^{-4})^{-1} = 34.7 \text{ dB}$$

- (b) If a speaker is placed below the hole as well, a longitudinal quadrupole is formed with $L = h = 0.1$. In this case, $\beta = 0$, and as can be seen from equation 5.55 in the text (where $\theta = 90 - \beta$), the theoretical mean square sound pressure will be zero, implying a reduction of infinity dB.

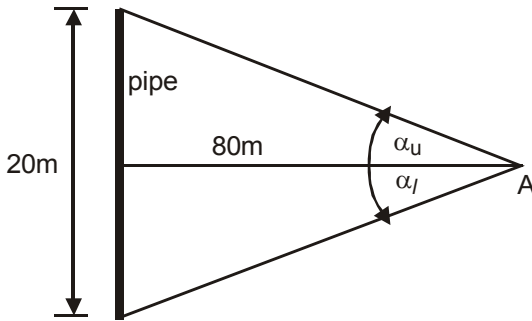
Problem 5.14

In equation 5.62, (W/b) is effectively the power per unit length of source (as W is the power of each source separated by b) and in equation 5.70, W/D is the same quantity. Thus the difference between the finite length and infinite

length source is the quantity $(\alpha_u - \alpha_l)/\pi$ which is the ratio of the angle subtended by the source at the observer in each case. Thus, by logical argument, equation 5.65 can be rewritten as follows for a **finite** coherent line source.

$$\langle p^2 \rangle = \rho c \frac{W}{2\pi^2 r_0 D} [\alpha_u - \alpha_l]$$

Problem 5.15



The arrangement is as shown in the figure.

$$\alpha_u = -\alpha_l = \tan^{-1}(10/80) = 0.124^c$$

Finite length pipe, $D = 20\text{m}$, $r_0 = 80\text{m}$ and $L_w = 130\text{dB}$. Turbulent flow, so assume an incoherent source. Also assume incoherent addition of the direct and ground reflected waves. Taking logs of equation 5.70 in the text gives for the direct wave:

$$\begin{aligned} L_p &= L_w - 10 \log_{10}(4\pi r_0 D) + 10 \log_{10}(\alpha_u - \alpha_l) + 10 \log_{10}(\rho c / 400) \\ &= 130 - 10 \log_{10}(4\pi \times 80 \times 20) + 10 \log_{10}(0.248) + 10 \log_{10}(1.0342) \\ &= 81.1 \text{ dB} \end{aligned}$$

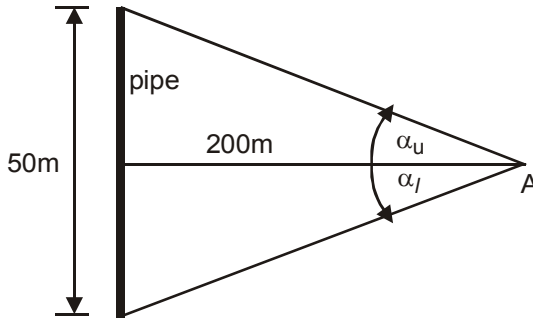
The ground reflected wave level is then $81.1 - 3 = 78.1\text{dB}$. Thus the total

level at the receiver is:

$$L_p = 10 \log_{10} (10^{8.11} + 10^{7.81}) = 82.9 \text{ dB}$$

Problem 5.16

The situation is as shown in the figure below. Equation 5.70 in the text may be used to calculate the sound pressure level. The equation must be multiplied by the directivity factor (2 in this case).



Thus:

$$\langle p^2 \rangle = [W\rho c / 4\pi r_0 D] [\alpha_u - \alpha_l] \times 2$$

$$\alpha_u = \alpha_l = \tan^{-1} \frac{25}{200} = 0.124 \text{ radians.}$$

$W = 2$, $r_0 = 200$, $D = 50$. Thus:

$$\begin{aligned} \langle p^2 \rangle &= [2 \times 1.206 \times 343 / (4\pi \times 200 \times 50)] \times 0.249 \times 2 \\ &= 3.275 \times 10^{-3} \text{ Pa}^2 \end{aligned}$$

$$L_p = 10 \log_{10} \frac{3.275 \times 10^{-3}}{4 \times 10^{-10}} = 69.1 \text{ dB}$$

From table 5.3 in the text, $A_a = 19.3\text{dB}$ per 1000m, so for 200m, $A_a = 19.3/5 = 3.9\text{dB}$. Sound intensity loss due to ground reflection is 2dB. Thus the ground effect is given by equation 5.175b in the text as:

$$A_g = -10\log_{10}[1 + 10^{-2/10}] = -2.1\text{dB}$$

Thus, $A_g + A_a = 1.8\text{dB}$ and the sound pressure level at the receiver is:

$$L_p = 69.1 - 1.8 = 67.3\text{dB}$$

Problem 5.17

The traffic may be treated as an infinite line source. At 1m $r_0 < b/\pi$ and the mean square sound pressure is related to the source sound power, W , of one vehicle by:

$$\langle p_1^2 \rangle = \rho c \frac{W}{4\pi r_0^2} \times DF$$

where DF is the directivity factor for the source/ground combination.

At 50m, the sound pressure is related to the sound power, W , of one vehicle by:

$$\langle p_2^2 \rangle = \rho c \frac{W}{4br_0} \times DF$$

Thus:

$$\frac{\langle p_1^2 \rangle}{\langle p_2^2 \rangle} = \frac{br_2}{\pi \times r_1^2} = \frac{6 \times 50}{\pi} = 95.5$$

The level at 50m = level at 1m - $10\log_{10} \frac{\langle p_1^2 \rangle}{\langle p_2^2 \rangle} = 88 - 19.8 = 68.2\text{dB(A)}$.

Problem 5.18

From Equation 5.62:

$$\langle p^2 \rangle = \rho c \frac{W}{4\pi r_0^2} = \frac{413 \times 2}{4 \times 7 \times 250} = 0.118 \text{ Pa}^2$$

$$L_p = 10 \log_{10} \frac{0.118}{(2 \times 10^{-5})^2} = 84.7 \text{ dB}$$

Concrete ground, so ground effect is $A_g = -3 \text{ dB}$. Assume 20°C , air absorption ranges from 2.6 to 2.8 dB per 1000 m. For 250 m air absorption - 0.7 dB.

From table 5.3, meteorological influence is +6, -3 dB. Assume no obstacles blocking line of sight to the road from the residence.

So the sound pressure level at the residence is:

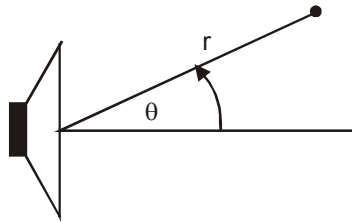
$$L_p = 84.7 - 0.6 + 3.0 = 87 \text{ dB (+6, -3 dB)}$$

Problem 5.19

- (a) The arrangement is shown in the figure. We are given:

$$I = (\bar{p}_o^2 / 3\rho c r^2) (2 + \cos\theta)$$

The sound power is obtained by integrating the intensity over an imaginary hemispherical surface centred at the centre of the speaker.



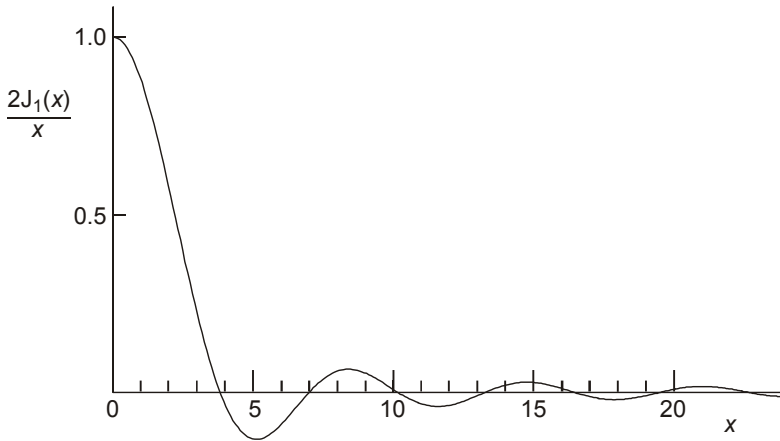
The sound power is then:

$$\begin{aligned}
 W &= \int_S I \, dS = \frac{\bar{p}_0^2}{3\rho c} \int_0^{2\pi} d\psi \int_0^{\pi/2} \frac{(2 + \cos\theta) r^2 \sin\theta}{r^2} d\theta \\
 &= \frac{\bar{p}_0^2}{3\rho c} 2\pi \int_0^{\pi/2} (2 \sin\theta + \cos\theta \sin\theta) d\theta \\
 &= \frac{\bar{p}_0^2}{3\rho c} 2\pi \left\{ \int_0^{\pi/2} [-2 \cos\theta - 0.25 \cos 2\theta] d\theta \right\} \\
 &= \frac{\bar{p}_0^2}{3\rho c} 2\pi [2 + 1/2] = 0.0127 \bar{p}_0^2 \text{ Watts}
 \end{aligned}$$

- (b)& (c) As the speaker only radiates into a hemispherical space, the presence of a baffle will have no influence on the power radiated, regardless of whether the source is constant volume or constant pressure.

Problem 5.20

- (a) If the piston is assumed to be made up of an infinite number of point monopole sources, all pulsating in phase, then the sound pressure at any location can be calculated by summing the contributions from each source. This effectively means that an expression for the sound pressure at some distance, r , due to a monopole on the piston surface must be integrated over the piston surface. If a baffle is present, the monopole source must be replaced with a hemispherical source which radiates twice the pressure.
- (b) The function $2J_1(x)/x$ vs x is plotted out in the figure below, where $x = ka \sin\theta$.

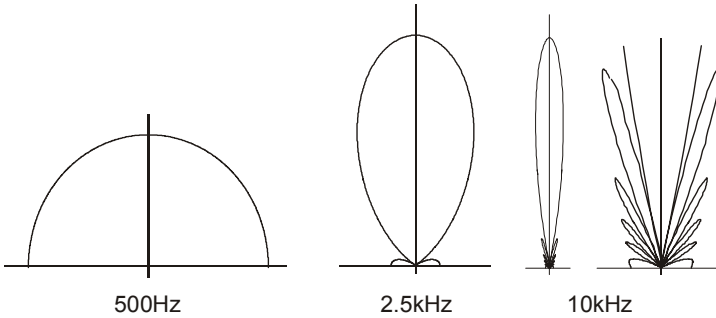


At 500Hz, $ka = 2\pi \times 500 \times 0.1/343 = 0.91$. Nodes in the radiation pattern occur when $0.91 \sin\theta = 3.8, 7, 10.1$, etc. That is, there are no nodes.

At 2,500Hz, $ka = 2\pi \times 2,500 \times 0.1/343 = 4.58$. Nodes in the radiation pattern occur when $4.58 \sin\theta = 3.8, 7, 10.1$, etc. That is, there is only one node at $\theta = 60^\circ$.

At 10,000Hz, $ka = 2\pi \times 10,000 \times 0.1/343 = 18.31$. Nodes in the radiation pattern occur when $18.31 \sin\theta = 3.8, 7, 10.1, 13.3, 16.4$, and 19.6 . That is, there are 5 nodes at $\theta = 12^\circ, 23^\circ, 33^\circ, 47^\circ$ and 64° .

Directivity patterns are shown for each of these cases in the figures on the next page. The side lobes for the 10kHz case have been expanded for clarity. For sketching purposes, the figure shown at the beginning of part (b) (figure 5.7 in the text) may be used with the x-axis crossings representing the nodal locations of each lobe and the peaks in the curve representing the relative amplitude of each lobe. [Note that fig 5.6 will only provide information for the first three lobes so it must be extended for the 10kHz case.]

**Problem 5.21**

- (a) Using equations 5.95b and 5.96 in the text, the radiation resistance for a piston can be written as:

$$R_R = \rho c \pi a^2 \times (2ka)^2 / 8 = \rho c (\pi a^2)^2 4\omega^2 / (8\pi c^2) = \rho c S^2 \omega^2 / (2\pi c^2)$$

where it has been assumed that ka is sufficiently small that all but the first term of equation 5.96 in the text is negligible.

- (b) The radiation efficiency at low frequencies is:

$$\sigma = \frac{R_R}{\rho c S} = \frac{\pi a^2 \omega^2}{2\pi c^2} = (ka)^2 / 2$$

which described the solid line in figure 5.9 in the text for $ka < 0.8$.

- (c) See fig 5.9 in the text.

Problem 5.22

- (a) Piston radiating from an infinite baffle. $k = 2\pi a / \lambda = 2$ and $a = 0.1\text{m}$. From equation 5.98b in the text:

$$W = R_R \pi a^2 \rho c U^2 / 2$$

and from fig 5.8, for $ka = 2$, $R_R = 1$. The piston velocity amplitude, U , is given by:

$$U = \bar{\xi}\omega = 2\bar{\xi}c/\lambda = 2\bar{\xi}c/a = 2 \times 0.0002 \times 343/0.1 = 1.372 \text{ m/s}$$

Thus the radiated power is:

$$W = \pi \times 0.1^2 \times 1.206 \times 343 \times 1.372^2/2 = 12.2 \text{ W}$$

(b) From equation 5.84 in the text, the on-axis intensity is:

$$I = \frac{\rho c k^2}{8\pi^2 r^2} F^2(w)$$

where $w = ka \sin\theta = 0$. Thus, $F(0) = U\pi a^2$.

Thus:

$$\begin{aligned} I &= \frac{\rho c h^2 U^2 \pi^2 a^4}{8\pi^2 r^2} = \frac{\rho c U^2 a^2}{8r^2} \\ &= \frac{1.206 \times 343 \times 1.372^2 \times 0.01}{2 \times 2^2} = 0.97 \text{ W/m}^2 \end{aligned}$$

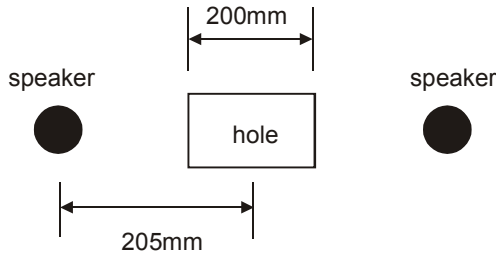
(c) Radiation mass loading $= \pi a^2 \rho c [X(2ka)]$, where $X(2ka) = 0.55$ (see fig 5.8 in the text). Thus the mass loading is
 $\pi \times 0.01 \times 1.206 \times 343 \times 0.55 = 7.1 \text{ kg/s}$

(d) Sound pressure level at 2m:

$$L_p = 10 \log_{10} \frac{\langle p^2 \rangle}{p_{ref}^2} = 10 \log_{10} \frac{\rho c I}{4 \times 10^{-10}} = 120.0 \text{ dB}$$

Problem 5.23

(a) The arrangement is shown in the figure on the next page where it can be seen that $h = L = 102.5 \text{ mm}$.



- (b) Power radiated by original opening (assuming a constant volume velocity source in an infinite baffle) is (see equation 5.12 in the text and multiply by 2 to account for radiation into half space):

$$W_M = (Q_H^2 \rho c k^2 / 4\pi) \times 2$$

The power radiated by the longitudinal quadrupole may be calculated using equation 5.54 in the text. Again the equation in the text must be multiplied by 2. Thus:

$$W_{long} = [(2k^3 h L Q_L)^2 \rho c / 5\pi] \times 2$$

where $Q_L = Q_H/2$. Thus:

$$\begin{aligned} \frac{W_{long}}{W_M} &= \frac{4k^6 h^2 L^2 Q_H^2 \rho c 4\pi}{20\pi Q_H^2 \rho c k^2} = \frac{4}{5} k^4 h^2 L^2 \\ &= \frac{4}{5} \frac{(2\pi)^4 f^4 (0.1025)^4}{343^4} = 9.94 \times 10^{-12} f^4 \end{aligned}$$

Frequency, Hz	dB reduction ($-10\log_{10}(W_{long}/W_m)$)
63	38
125	26
250	14
500	2

- (c) The ratio of the mean square pressures may be obtained using equations 5.13b and 5.55 in the text. Thus:

$$\frac{\langle p_{long}^2 \rangle}{\langle p_M^2 \rangle} = \frac{[5\rho c W_{long} \cos^4 \theta / 4\pi r^2] \times 2}{[W_M \rho c / 4\pi r^2] \times 2} = 5 \cos^4 \theta \frac{W_{long}}{W_M}$$

The reduction in sound pressure level is then:

$$\Delta L_p = -10 \log_{10} [5 \cos^4 \theta W_{long} / W_M]$$

Values of sound pressure reduction (dB re 20μPa) are tabulated below for the required values of θ .

frequency (Hz)	$\theta = 0$	$\theta = \pi/4$	$\theta = \pi/2$
63	31	37	∞
125	19	25	∞

When the speakers are turned on, the sound field amplitude distribution has 2 lobes with maxima at $\theta = 0, \pi$ and minima at $\theta = \pi/2, 3\pi/2$.

Problem 5.24

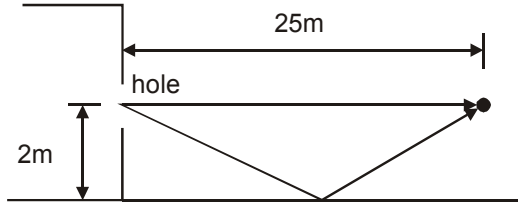
- (a) The directivity index due to the hard floor is 3dB.
- (b) The expected sound pressure level due to an omni directional source on a hard floor is:

$$L_p = L_w - 10 \log_{10} [4\pi r^2] = 120 - 10 \log_{10} [2\pi \times 100] = 92 \text{ dB}$$

The actual sound pressure level is 110dB so the directivity due to the source characteristics is -18dB.

Problem 5.25

The radiated sound power is $W = IS = 0.01 \times S = 0.01S$ Watts. The arrangement is shown in the figure.



With no ground reflection, the on-axis sound pressure level may be calculated using equation 5.105 in the text (as we may assume that the opening behaves like an incoherent plane source). Here, $H = L = 0.5$, $S = HL$ and $r = 25$. Thus:

$$\begin{aligned}
 \langle p^2 \rangle &= \frac{2\rho c W}{\pi H L} \tan^{-1} \left[\frac{H L}{2r \sqrt{H^2 + L^2 + 4r^2}} \right] \\
 &= \frac{2 \times 1.206 \times 343 \times 0.01 \times S}{S \times \pi} \tan^{-1} \left[\frac{0.25}{2 \times 25 \sqrt{0.25 + 0.25 + 4 \times 25^2}} \right] \\
 &= 2.63 \times 10^{-4} \text{ Pa}^2
 \end{aligned}$$

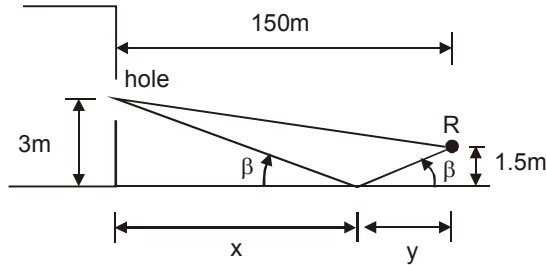
Assuming a similar travel distance for the ground reflected wave, the total sound pressure level at the receiver is:

$$L_p = 10 \log_{10} \frac{2 \times 2.633 \times 10^{-4}}{4 \times 10^{-10}} = 61.2 \text{ dB}$$

Interestingly, in this case, the receiver is sufficiently far from the source for the source to appear as a point source and the same result would have been obtained if equation 5.106 had been used instead of equation 5.105.

Problem 5.26

- (a) Sound power level, $L_w = 10\log_{10}W + 120 = 123\text{dB}$.



- (b) The arrangement for calculating the ground effect is shown in the figure.

From similar triangles, $x = 2y$. Thus $x = 100$ and $y = 50$.

$\tan\beta = 3/100$, so $\beta = 1.72^\circ$.

For grass covered ground, $R_1 = 2.25 \times 10^5$ (middle of range). Thus:

$$\frac{\rho f}{R_1} = \frac{1.206 \times 2000}{2.25 \times 10^5} = 0.011$$

$$\beta \left[\frac{R_1}{\rho f} \right]^{1/2} = 1.72 \left[\frac{1}{0.011} \right]^{1/2} = 16.6$$

From figure 5.20 in the text, $A_R = 1.3\text{dB}$. The ground effect is then:

$$A_g = -10\log_{10} \left[1 + 10^{-A_R/10} \right] = -2.4\text{dB}$$

Thus the effect of the ground is to increase the level at the receiver by 2.4dB.

- (c) Loss due to atmospheric absorption. From table 5.3 in the text, $A_a = 15.5\text{dB}$ per 1000m (25% RH and 20°C). So for a distance of 150m, $A_a = 15.5 \times 0.15 = 2.3\text{dB}$.

- (d) The opening should be treated as an incoherent plane source and equation 5.105 in the text used to calculate the sound pressure level. However as we found in the previous problem, the receiver is far enough from the source for it to appear as a point source (see figure 5.11) and we may use equation 5.106. Thus:

$$\langle p^2 \rangle = \frac{1.206 \times 343 \times 2}{2\pi \times 150^2} = 5.852 \times 10^{-3} \text{ Pa}^2$$

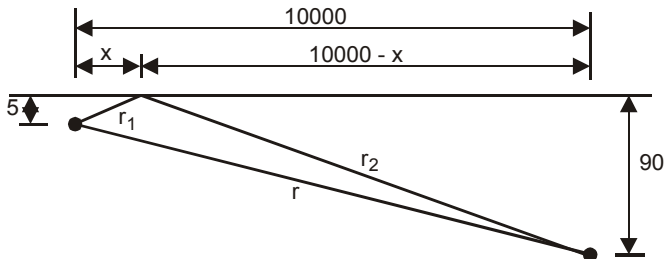
The sound pressure level is then:

$$L_p = 10 \log_{10} \frac{5.852 \times 10^{-3}}{4 \times 10^{-10}} = 71.7 \text{ dB}$$

$A_a + A_g = -0.1$, thus the sound pressure level at the receiver is equal to 71.8dB.

- (e) Adding a second opening will add 3dB to the sound pressure levels at the receiver as it may be assumed that the sound fields from the two sources are incoherent. Thus the sound pressure level at the community location of (b) above would be 74.8dB.

Problem 5.27



The arrangement is illustrated in the figure. Using similar triangles:

$$\frac{x}{5} = \frac{10000 - x}{90}$$

$$90x = 50000 - 5x$$

$$x = 50000/95 = 526.31579 \text{ m}$$

$$r_1 = \sqrt{5^2 + (526.315)^2} = 526.33954$$

$$r_2 = \sqrt{90^2 + (10000 - 526.31579)^2} = 9474.111701$$

$$r = \sqrt{85^2 + 10000^2} = 10000.36124$$

$$r_1 + r_2 - r = 0.0900 \text{ m}$$

Destructive interference will occur if $\lambda/2 = 0.0900 \text{ m}$. This occurs at a frequency, f , given by:

$$f = \frac{c}{\lambda} = \frac{1500}{2 \times 0.0900} = 8.3 \text{ kHz}$$

Problem 5.28

We would **NOT** expect to measure 84dB(A) at the operator's position due to reflected energy from the nearby wall. As the sound is predominantly in the 500Hz to 2000Hz band, the operator is not in the hydrodynamic near field of the machine (although he/she could be in the geometric near field). Also the machine is far enough from the wall for its sound power to be unaffected by the wall. Assuming incoherent addition of direct and reflected waves, assuming that geometric near field effects are negligible assuming that the machine does not act as a barrier to the reflected sound and assuming that the loss on reflection from the wall is negligible, the level at the operator's position may be calculated as the logarithmic sum of the direct and reflected waves. The reflected wave path length is 6m and the direct path length is 2m. Thus the sound pressure level of the reflected wave is

$84 - 20\log_{10}(6/2) = 74.5 \text{ dB}$. Thus the total sound pressure level expected at the operator's position is:

$$L_p = 10\log_{10}(10^{8.4} + 10^{7.45}) = 84.5 \text{ dB(A)}$$

Problem 5.29

- (a) Specific acoustic impedance is a complex quantity characterised by an amplitude and a phase and is the complex ratio of acoustic pressure to acoustic particle velocity at any point in an acoustic medium, including the interface between two different media. Characteristic impedance is equal to ρc . For an acoustic medium where the viscous and thermal losses are small (such as air or water), it is a real quantity and is equal to the specific acoustic impedance of a plane wave propagating in an acoustic medium of infinite size.
- (b) The absorption coefficient (assuming plane incident waves) is defined in terms of the reflection coefficient, R_p , as $\alpha = 1 - |R_p|^2$. Also, the normal specific acoustic impedance of the surface of an acoustic medium of infinite extent is the characteristic impedance of the medium, for an infinitely thick medium. Thus equation 5.129 in the text may be used with $\theta = 0$ to give:

$$R_p = \frac{(Z_s/\rho c) - 1}{(Z_s/\rho c) + 1}$$

If θ is not equal to 0, then

$$R_p = \frac{(Z_s/\rho c)\cos\theta - \cos\psi}{(Z_s/\rho c)\cos\theta + \cos\psi}$$

If it is assumed that the wavenumber in the material is much larger than that in air, $\cos\psi = 1$ and:

$$|R_p| = \frac{|(Z_s/\rho c)\cos\theta - 1|}{|(Z_s/\rho c)\cos\theta + 1|}$$

The maximum absorption coefficient will occur when the modulus squared of the reflection coefficient is a minimum; that is, when

$$|R_p|^2 = \frac{(2\cos\theta - 1)^2 + 9\cos^2\theta}{(2\cos\theta + 1)^2 + 9\cos^2\theta} = \frac{13\cos^2\theta - 4\cos\theta + 1}{13\cos^2\theta + 4\cos\theta + 1}$$

is a minimum. Differentiating the above expression wrt $\cos\theta$ using the chain rule, and setting the result equal to zero, we obtain the minimum value when $\cos\theta = 0.2774$ or $\theta = 74^\circ$. The corresponding maximum value of the absorption coefficient is given by:

$$\alpha = 1 - |R_p|^2 = 1 - \frac{13 \times 0.0769 - 4 \times 0.2774 + 1}{13 \times 0.0769 + 4 \times 0.2774 + 1} = 1 - 0.286 = 0.71$$

Problem 5.30

Power radiated by window = 0.1 W = 110 dB

Concrete ground, so $A_g = -3$ dB

Assume 20 °C temperature, so A_a ranges from 2.6 to 2.8. Use $A_a = 2.7$.

Range due to meteorological conditions is: (+8, -6 dB) from Table 5.10.

Assume no barriers between the source and receiver.

The receiver is far enough away for the window to be treated as a point source in a baffle. Thus:

$$\begin{aligned} L_p &= L_w - 10\log_{10}(2\pi r^2) - A_g - A_a - A_m = 110 - 65.5 + 3 - 2.7 \times 0.75 - A_m \\ &= 45.5 \text{ dB (+8, -6 dB)} \end{aligned}$$

Problem 5.31

The pressure amplitude at any location x is given by:

$$\frac{\bar{p}}{\bar{p}_0} = e^{-\alpha x}$$

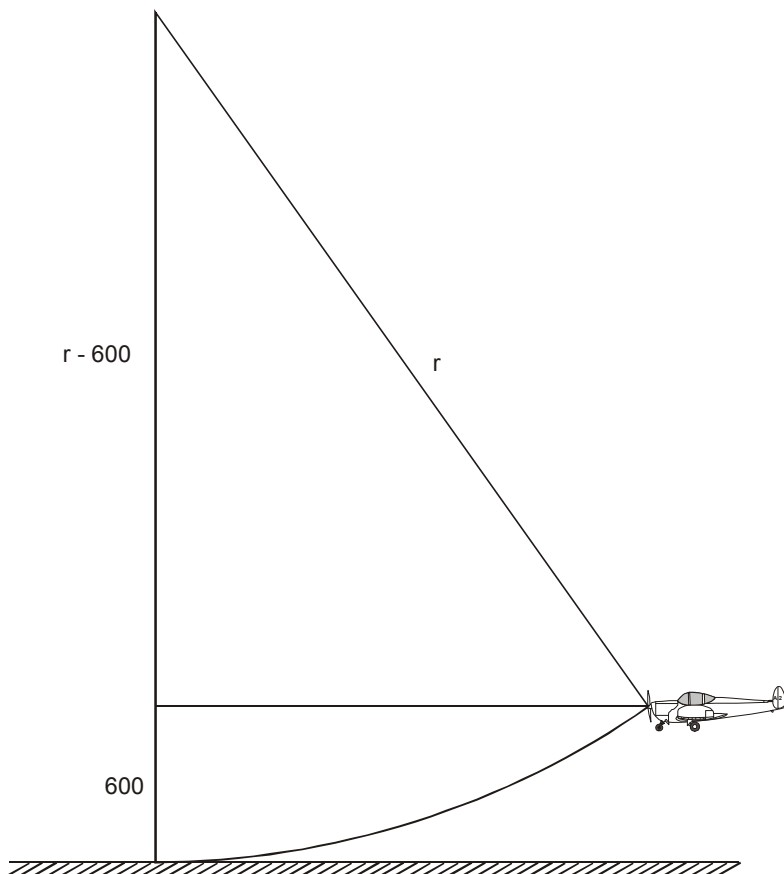
where \bar{p}_0 is the amplitude at $x = 0$. The decibel decay rate per unit distance is $20\log_{10}$ of the reciprocal of the above expression when $x = 1$ and is thus given by:

$$\text{Decay rate} = 20\log_{10}e^\alpha = 20 \times 0.4343 \ln e^\alpha = 8.686\alpha \text{ (dB/m)}$$

Important factors are air temperature and humidity.

Problem 5.32

The situation is illustrated in the figure below. We need to find the radius of curvature, r , of the wave and hence the distance, d . We may use the aircraft as the reference frame. Thus we assume a coordinate system moving horizontally at the speed of the aircraft and then later on calculate the distance that the aircraft travels during the time it takes for the sound to reach the ground (with an assumed aircraft speed). The wave which hits the ground at grazing incidence will be the one heard first. The total sonic gradient is $1/10 = 0.1\text{s}^{-1}$. Using equation 5.190 in the text, the radius of curvature of the wave is thus $343 \times 10 = 3430\text{m}$. The distance d is then:



$$d = \sqrt{r^2 - (r - 600)^2} = \sqrt{3430^2 - 2830^2} = 1940 \text{ m}$$

We now need to take into account the speed of the aircraft and the distance it will travel in the time the sound wave travels to the observer.

Let the angle subtended at the centre of the circular arc shown in the previous figure be θ_0 . The speed of sound as a function of θ (with $\theta = 0$ corresponding to ground level and $\theta = \theta_0$ corresponding to the aircraft level) is given by $c_\theta = c_0 - 0.1h$, where h = height above the ground and $c_0 = 343 \text{ m/s}$ is the speed of sound at ground level. The value of θ_0 is given by

$$\theta_0 = \cos^{-1} \left[\frac{3430 - 600}{3430} \right] = 34.4^\circ$$

Thus the time taken for the sound to travel from the aircraft to the ground is

$$\begin{aligned} t &= \int_0^{\theta_0} \frac{r \, d\theta}{c_0 - 0.1h} = \int_0^{\theta_0} \frac{d\theta}{(c_0/r) - 0.1 + 0.1 \cos\theta} \\ &= 10 \times \int_0^{\theta_0} \frac{d\theta}{\cos\theta} = 10 \log_e [\sec\theta + \tan\theta]_0^{34.4^\circ} = 6.4 \text{ seconds} \end{aligned}$$

If the aircraft travels at 400 km/hour (not given in the question), then it would travel 710 m in 6.4 seconds.

Thus the aircraft emerges from ground shadow $(1940 \text{ m} - 710 \text{ m}) = 1230 \text{ m}$ from the observer.

6

Solutions to problems relating to sound power, its use and measurement

Problem 6.1

- (a) This is discussed in detail on p246–247 in the text.
- (b) For a **constant power** source in the corner of the room, the radiated power would be concentrated over a one eighth sphere instead of a sphere and the sound pressure level in the direct field would thus be increased by 9dB. However, there would be no change in the reverberant field sound pressure level.

For a **constant volume velocity** source in the corner of the room, the radiated power would be increased by a factor of 8 due to there being 3 reflecting surfaces and in addition the power would be concentrated over a one eighth sphere instead of a sphere and the direct field sound pressure level would thus be increased by 18dB. However, the reverberant field sound pressure level would be increased by only 9dB, corresponding to the power increase.

For a **constant pressure** source in the corner of the room, the direct field radiated sound pressure level would be unchanged, but the reverberant field sound pressure level would be reduced by 9dB, corresponding to a reduction of 9dB in the radiated power.

- (c) A good approximation would be a constant pressure source model because the noise is originally generated by a fluctuating pressure, the amplitude of which is controlled by the aerodynamics and not the acoustics of the problem.

Problem 6.2

- (a) Sound power level is a measure of the rate of total energy radiated by an acoustic source while sound pressure level is a measure of the fluctuating sound pressure at a particular location. Sound power level is a source property whereas sound pressure level depends on the measurement location as well as the strength and size of the source.

- (b) Beginning with $W = \langle p^2 \rangle S / \rho c$ and taking logs of both sides gives:

$$10\log_{10} W = 10\log_{10} \langle p^2 \rangle + 10\log_{10} S - 10\log_{10} (\rho c)$$

Dividing both sides by 10^{-12} and remembering that the reference sound power level is 10^{-12} W and the reference sound pressure level is 2×10^{-5} Pa, the preceding equation may be written as:

$$\begin{aligned} 10\log_{10} \frac{W}{W_{ref}} \\ = 10\log_{10} \frac{\langle p^2 \rangle}{p_{ref}^2} + 10\log_{10} S + 10\log_{10}(400) - 10\log_{10}(\rho c) \end{aligned}$$

If the quantity ρc is approximated as 400, then the preceding equation becomes:

$$L_w = L_p + 10\log_{10} S$$

- (c) Using the above equation, the sound power level would be:

$$\begin{aligned} L_w &= L_p + 10\log_{10} S = 85 + 10\log_{10}(2\pi \times 2^2) \\ &= 85 + 14 = 99 \text{ dB} \end{aligned}$$

Assumptions: $\rho c = 400$ and the sound pressure level measurements were made in the acoustic far field of the machine.

- (d) The desirable quantity is usually sound power level as it indicates the amount of acoustic energy which will be added to an environment and allows the increase in sound pressure level to be calculated at

any location in the far field of the source as a result of the introduction of the machine (provided the sound radiation is omnidirectional or directivity information is also given). Sound power is also independent of the presence of nearby reflecting surfaces, provided that these are more than a quarter of a wavelength from the acoustic centre of the machine or noise source.

On the other hand, the sound pressure level at a specified location is affected by the presence of reflecting surfaces and also does not necessarily allow the sound pressure level at other locations to be calculated. However, if only the noise exposure of the operator of a machine is of concern, then perhaps it is better to specify sound pressure level at the operator's location measured in the presence of specified reflecting surfaces than sound power level as the operator may not be in the acoustic far field of the source.

Problem 6.3

- (a) This is discussed in detail on pages 249-251 in the text.
- (b) Assuming that "negligible" is a factor of 10, the frequency would be given by $\gamma = 10/\kappa$ (see figure 6.1 in the text).
and therefore $2r/\ell = 10\lambda/(\pi\ell) = 10 \times 343/(\pi\ell f)$
 $f = 10 \times 343/(2\pi r) = 3430/(2 \times \pi \times 1) \approx 550 \text{ Hz}$
- (c) $r = \ell = 1$, so $\gamma = 2$ and $\kappa = \pi \times 550/343 \approx 5$. From figure 6.1, for the far field to be dominant, $\gamma = 15$. Thus $r = 7.5\text{m}$ is the distance at which the far field would be dominant.

Problem 6.4

- (a) The anechoic room should be sufficiently large that sound pressure measurements can be made in the far field of the source. Thus the following criteria should be satisfied (see eq. 6.5, p.250 in text).

$$r > 3\lambda/2\pi; \quad r > 3\ell; \quad r > 3\pi\ell^2/2\lambda$$

The lowest frequency of interest is 44Hz (see table 1.2 on p43 in the text) and the highest frequency is 11,300Hz.

frequency (Hz)	λ (m)	$3\lambda/2\pi$	3ℓ	$3\pi\ell^2/2\lambda$		
				$\ell = 1.5$	$\ell = 0.4$	$\ell = 0.6$
44	7.8	3.72	4.5, 1.2, 1.8	1.36	0.1	0.22
11,300	0.03	0.015		349	24.8	55.9

The last criterion in the above table for r represents the transition from the geometric near field to the far field. Thus for high frequencies it is not practical to take measurements in the far field; the geometric near field will suffice. As all measurements must be taken at least $\lambda/4$ from the room walls, the following minimum interior room dimensions are needed.

$$\begin{aligned}
 & 2 \times \{(4.5 + 0.75 + 7.8/4) \text{ by } (3.72 + 0.2 + 7.8/4)\} \\
 & \quad \text{by } (3.72 + 0.6 + 7.8/4)\} \\
 & = 14.4 \text{ m} \times 11.7 \text{ m} \times 6.3 \text{ m}
 \end{aligned}$$

As standard measurements use a hemispherical surface, the required room dimensions are $14.4 \text{ m} \times 14.4 \text{ m} \times 7.2 \text{ m}$ high.

- (b) For measurements in octave bands, the required room volume is:

$$1.3\lambda^3 = 1.3 \left(\frac{343}{63} \right)^3 = 210 \text{ m}^3$$

Optimum dimensions are in the ratios 2:3:5. Thus $2x \times 3x \times 5x = 210$, or $x = 1.91 \text{ m}$. Thus the required dimensions are $3.8 \text{ m} \times 5.7 \text{ m} \times 9.6 \text{ m}$.

Problem 6.5

The sound power is related to the average mean square pressure by:

$$W = \frac{\langle p^2 \rangle S}{\rho c} = \frac{\langle p^2 \rangle 2\pi r^2}{\rho c}$$

or in terms of sound power level, L_w :

$$\begin{aligned}
 L_w &= L_p + 10 \log_{10}(2\pi r^2) + 10 \log_{10} 400/\rho c \\
 &= 70 + 10 \log_{10}(8\pi) + 10 \log_{10} 400/413.6 \\
 &= 83.9 \text{ dB}
 \end{aligned}$$

The sound power, W , is thus:

$$W = 10^{-12} \times 10^{L_w/10} = 10^{-12 + 8.39} = 0.245 \text{ mW}$$

Problem 6.6

(a) r.m.s. acoustic pressure:

$$p_{rms} = 2 \times 10^{-5} \times 10^{L_p/20} = 2 \times 10^{3.3-5} = 0.04 \text{ Pa}$$

(b) Source dimension, $\ell = 0.1 \text{ m}$; wavelength, $\lambda = 1481/250 = 5.92 \text{ m}$; $r = 2 \text{ m}$.

The distance r must be larger than the following quantities for the location to be in the far field:

$$3\lambda/2\pi; \quad 3\ell; \quad \text{and} \quad 3\pi\ell^2/2\lambda$$

Substituting values for λ and ℓ into the preceding expressions and evaluating gives 2.82, 0.3 and 0.008 respectively, so the location is not in the far field but rather in the transition between the hydrodynamic near field and the far field.

(c) Intensity:

$$I = \frac{p_{rms}^2}{\rho c} = \frac{0.04^2}{998 \times 1481} = 1.08 \text{ nano-watts/m}^2$$

(d) Power, $W = IS = 1.08 \times 10^{-9} \times 2 \times \pi \times 2^2 = 0.027 \mu\text{W}$

(e) The mean square sound pressure is related to the source volume velocity, Q , for a spherical source by equation 5.13a in the text as:

$$\langle p^2 \rangle = \frac{(Qk\rho c)^2}{(4\pi r)^2}$$

In this case, the source is a hemisphere and it is radiating into hemispherical space. Thus the same result is obtained as for a spherical source radiating into spherical space. The volume velocity, Q , is related to the surface displacement, d , by:

$$d = \frac{\sqrt{2} Q}{2\pi f \times 4\pi a^2} = 1.791 Q/f$$

Combining the above two equations gives:

$$d = \frac{3.582 r p_{rms}}{\rho f^2}$$

Substituting values for the variables gives:

$$d = \frac{3.582 \times 2 \times 0.04}{998 \times 250^2} = 4.6 \times 10^{-9} \text{ m}$$

Problem 6.7

Equation 6.13 in the text is:

$$L_w = L_p + 10\log_{10} V - 10\log_{10} T_{60} + 10\log_{10} \left(1 + \frac{S\lambda}{8V} \right) - 13.9 \text{ dB}$$

From problem 6.4 the room size is $3.8\text{m} \times 5.7\text{m} \times 9.6\text{m}$. The volume, V , is 207.9m^3 and the surface area, S , is $2(3.8 \times 5.7 + 3.8 \times 9.6 + 5.7 \times 9.6) = 225.7\text{m}^2$.

Using the above equation, the following table may be constructed.

f	L_p	$10\log_{10} T_{60}$	λ	$10\log_{10}(1+S\lambda/8V)$	L_w
63	85	9.1	5.44	2.4	87.6
125	105	8.8	2.74	1.4	106.9
250	100	7.4	1.37	0.7	102.6
500	90	6.5	0.69	0.4	93.2
1000	95	5.4	0.34	0.2	99.1
2000	98	4.0	0.17	0.1	103.4
4000	90	3.0	0.086	0.1	96.4
8000	88	1.8	0.043	0.0	95.5
Overall					110.3

Problem 6.8

Average $L_p = 10 \log_{10} \{(1/5)[10^{8.5} + 10^{8.3} + 10^{8.0} + 10^{8.7} + 10^{8.6}]\} = 84.8\text{dB}$

Area of measurement surface $= 2(2 \times 4 + 2 \times 3) + 3 \times 4 = 40\text{m}^2$.

Thus radiated power, $L_w = L_p + 10\log_{10}S - \Delta_1 - \Delta_2$ (eq. 6.25 in text).

Ratio of area of measurement surface to machine surface $= 40/8 = 5$, so $\Delta_2 = 0$.

Assuming that the machine is in a large enclosure, $\Delta_1 = 0$ as well. Thus $L_w = 84.8 + 10\log_{10}40 = 100.8\text{dB}$.

Problem 6.9

The data may be used to construct the following table.

measured	85	88	86	90	84	85	87	88	89	90	90	88	87	88	89	85
background	80	82	80	81	80	79	81	79	80	81	83	83	82	80	80	79
machine only	83.3	86.7	84.7	89.4	81.8	83.7	85.7	87.4	88.4	89.4	89	86.3	85.3	87	88.4	83.7

$$L_p(av) = 10\log_{10}\left(\frac{1}{N}\sum_i 10^{L_{pi}/10}\right) = 86.8\text{dB}$$

The sound power level may be calculated using equation 6.25 in the text which is:

$$L_w = L_p + 10\log_{10}S - \Delta_1 - \Delta_2$$

where $S = 40\text{m}^2$, and the factory volume $V = 20 \times 20 \times 5 = 2000\text{m}^3$. Thus $V/S = 50$ and from table 6.4, p. 266 in the text, $\Delta_1 = 2.5\text{dB}$. Machine surface area is the area of a cube of dimensions 1m smaller than the test cube. Let the side of the test cube $= x$. Then $5x^2 = 40$ and $x = 2.828\text{m}$. Thus the machine size is $0.828\text{m} \times 0.828\text{m} \times 1.828\text{m}$ high (as the machine is resting on the floor). The machine surface area is then $S_m = 1.828 \times 0.828 \times 4 + 0.828 \times 0.828 =$

6.7m². Thus $S/S_m = 40/6.7 = 5.9$ and from table 6.3, p266 in the text, $\Delta_2 = 0$.

Assuming $\rho c = 400$:

$$L_w = 86.8 + 10\log_{10} 40 - 2.5 = 100.3 \text{ dB re } 10^{-12} \text{ W}$$

Problem 6.10

Second surface average = $86.8 - 2 = 84.8 \text{ dB}$. Area ratio (surface1 to surface2) = $40/120 = 0.3333$. From figure 6.3 or equation 6.27 in the text, $\Delta_1 = 2.6 \text{ dB}$ and from problem 6.8, $\Delta_2 = 0$. Thus:

$$L_w = 86.8 + 10\log_{10} 40 - 3 \approx 100 \text{ dB re } 10^{-12} \text{ W}$$

Problem 6.11

Machine surface area = $2(8 \times 3 + 4 \times 3) + 8 \times 4 = 104 \text{ m}^2$. Area of test surface 1m from machine = $2(10 \times 4 + 6 \times 4) + 10 \times 6 = 188 \text{ m}^2 = S_1$. Area of test surface 3m from machine = $2(14 \times 6 + 10 \times 6) + 14 \times 10 = 428 \text{ m}^2 = S_2$. Using equation 6.22 in the text:

$$\frac{4}{R} = \frac{(1/188) - (1/428)[10^{2/10}]}{10^{2/10} - 1} = 2.763 \times 10^{-3}$$

Thus $R = 1448 \text{ m}^2$.

Sound power level, L_w is calculated using equation 6.24 in the text which is:

$$L_w = L_{p2} - 10\log_{10}[S_1^{-1} - S_2^{-1}] + 10\log_{10}[10^{(L_{p1} - L_{p2})/10} - 1] \\ - 10\log_{10}\left(\frac{\rho c}{400}\right)$$

Thus:

$$L_w = 84 - 10\log_{10}(188^{-1} - 428^{-1}) + 10\log_{10}[10^{2/10} - 1] - 10\log_{10}\left[\frac{413.7}{400}\right] \\ = 84 + 25.25 - 2.33 - 0.15 = 106.8 \text{ dB re } 10^{-12} \text{ W}$$

However, this excludes the near field correction term of equation (6.25) in the text, which for this case is -1 dB. So if we used equation 6.25 and also applied the correction due to pc not equal to 400, the result would be $L_w = 105.8$ dB.

Problem 6.12

- (a) The average noise level measured on the larger surface is calculated by logarithmically averaging the given values. Thus:

$$\begin{aligned} L_{p2} &= 10 \log_{10} \frac{1}{11} (10^{8.7} + 10^{8.75} + 10^{8.6} + 10^{8.5} + 10^{8.65} \\ &\quad + 10^{8.8} + 10^{8.68} + 10^{8.72} + 10^{8.6} + 10^{8.58} + 10^{8.53}) \\ &= 86.6 \text{ dB} \end{aligned}$$

The background level may be subtracted from the overall averaged levels as this will give the same result as subtracting it from the individual levels and then averaging.

Thus the noise level on test surface 1 due only to the machine is:

$$L_{p1} = 10 \log_{10} (10^{90/10} - 10^{80/10}) = 89.5 \text{ dB}$$

and the noise on test surface 2 due only to the machine is:

$$L_{p2} = 10 \log_{10} (10^{86.6/10} - 10^{80/10}) = 85.5 \text{ dB}$$

- (b) Reverberant field correction, Δ_1 , use equation 6.27 in the text. First calculate the areas of the test surfaces.

Surface 1, $S_1 = 2(6 \times 2.5 \times 2) + 6 \times 6 = 96 \text{ m}^2$.

Surface 2, $S_2 = 2(10 \times 4.5 \times 2) + 10 \times 10 = 280 \text{ m}^2$. Thus:

$$\begin{aligned} \Delta_1 &= 89.5 - 85.5 - 10 \log_{10} [10^{0.4} - 1] + 10 \log_{10} [1 - 96/280] \\ &= 0.4 \text{ dB} \end{aligned}$$

- (c) Correction for non-normal sound propagation is Δ_2 . Machine surface area, $S_m = 2(5 \times 2 \times 2) + 5 \times 5 = 65 \text{ m}^2$. $S_1/S_m = 96/65 = 1.48$. Thus from table 6.3 in the text, $\Delta_2 = 1$.

- (d) Sound power level, L_w is calculated using equation 6.25 in the text which is:

$$\begin{aligned} L_w &= L_p + 10 \log_{10} S - \Delta_1 - \Delta_2 \\ &= 89.5 + 10 \log_{10}(96) - 0.4 - 1 = 107.9 \text{ dB} \end{aligned}$$

Problem 6.13

- (a) sound power level:

$$L_w = L_p + 10 \log_{10}(2\pi r^2) + 10 \log_{10} \frac{400}{\rho c} = 75 + 17.5 - 0.1 = 92.4 \text{ dB(A)}$$

- (b) Sound power level of existing machinery is $92.4 + 82 - 87 = 87.4 \text{ dB(A)}$.

- (c) Maximum allowable total reverberant sound pressure level generated by the three new machines is:

$$L_p = 10 \log_{10} (10^{85/10} - 10^{82/10}) = 82 \text{ dB(A)}$$

Thus the maximum allowable sound power level is 87.4 dB(A) .

- (d) If all new machines emit the same sound power level and the total allowed is 87.4 dB(A) , the upper bound on the level generated by each machine is:

$$L_w = 87.4 - 10 \log_{10} 3 = 82.6 \text{ dB(A)}$$

Problem 6.14

- (a) Advantages of sound intensity for sound power measurement:
- reduces errors arising from presence of reflecting surfaces;
 - reduces errors from near field effects resulting from measurements taken close to the source;
 - reduces errors caused by background noise generated by sources other than the one under test;
 - Allows good results to be obtained at low frequencies.

Disadvantages of sound intensity for sound power measurement:

- Usually more time consuming;
- instrumentation is more expensive;
- When the radiated sound field is complex, sound intensity measurements can provide too much data which is time consuming to analyse and can be confusing.

(b) Advantages of sound intensity for transmission loss measurement:

- reduces errors arising from flanking path transmission;
- only requires a single reverberant room rather than two;
- Allows good results to be obtained at low frequencies.

Disadvantages of sound intensity for transmission loss measurement:

- Usually more time consuming;
- instrumentation is more expensive;

(c) Advantages of sound intensity for localisation and identification of noise sources:

- more reliable than a directional microphone due to greater spatial resolution and the ability to measure very close to a source;
- thus contamination from other nearby sources is reduced;

Disadvantages of sound intensity for localisation and identification of noise sources:

- instrumentation is more expensive;

Problem 6.15

(a) "radiation efficiency" is a measure of the amount of sound power radiated by a vibrating surface compared to that carried by a plane wave having the same mean square acoustic velocity (as the vibrating surface) and equal area. It can be expressed as:

$$\sigma = \frac{W}{S\rho c\langle v^2 \rangle}$$

(b) Referring to the above equation, it can be seen that the sound power radiated by a surface of area S , mean square velocity $\langle v^2 \rangle$ and radiation efficiency σ is:

$$W = \sigma S \rho c \langle v^2 \rangle$$

To identify possible paths of sound transmission between rooms, the radiation efficiencies of all walls ceiling and floor can be used together with their measured mean square velocities and the above equation to calculate the relative contributions of each surface to the overall sound power transmitted into the room, thus allowing the flanking paths to be identified and ranked.

- (c) The radiation efficiency of a surface is close to one when the bending wavelength of bending waves in the surface is less than or equal to the wavelength of the radiated acoustic waves; that is, at frequencies equal to or above (and in practice just below, for finite size surfaces) the critical frequency of the surface. In this case there will always be some radiation angle at which the bending waves on the surface will match the trace acoustic wavelength in the surrounding medium with a resulting strong coupling between the surface vibration field and the radiated acoustic field.

Problem 6.16

- (a) Equation 6.31 in the text is:

$$L_w = 10 \log_{10} \langle v^2 \rangle + 10 \log_{10} S + 10 \log_{10} \sigma + 146 \text{ dB re } 10^{-12} \text{ W}$$

The critical frequency,

$$f_c = 0.551 c^2 / (c_L h) = 0.551 \times 343^2 / (5400 \times 0.003) = 4001 \text{ Hz}$$

$$10 \log_{10} S = 0.$$

$$Ph/S = 4 \times 0.003/1 = 0.012.$$

Thus the following table may be constructed.

Octave band centre frequency (Hz)	ff_c	$10\log_{10}\sigma$	$10\log_{10}\langle v^2 \rangle$	L_w (dB re 10^{-12}W)
250	0.06	-19.8	-54.0	72.2
500	0.12	-18.2	-46.0	81.8
1000	0.24	-16.0	-54.0	76.0

- (b) From figure 5.11 in the text we can see that for $r/\sqrt{HL} = 10$, the panel will appear as a point source and the sound pressure level is given by:

$$L_p = L_w - 10\log_{10}(2\pi r^2) + 10\log_{10}\frac{\rho c}{400}$$

The quantity $10\log_{10}2\pi r^2 = 28$ and $10\log_{10}(\rho c/400) = 0.1$.

Thus the following table can be constructed.

Octave band centre frequency (Hz)	L_w (dB re 10^{-12}W)	L_p (dB re $20\mu\text{Pa}$)
250	72.2	44.3
500	81.8	53.9
1000	76.0	48.1

The overall sound pressure level is:

$$L_p = 10\log_{10}(10^{4.43} + 10^{5.39} + 10^{4.81}) = 55.3 \text{ dB re } 20\mu\text{Pa}$$

- (c) The A-weighted sound pressure level is:

$$L_{pA} = 10\log_{10}(10^{(4.43 - 0.86)} + 10^{(5.39 - 0.3)} + 10^{4.81}) = 52.8 \text{ dB(A)}$$

- (d) r.m.s. acceleration levels (approximate).

$a = 2\pi f v$ (see following table)

f (Hz)	v (mm/s)	a (m/s ²)
250	2	3.1
500	5	15.7
1000	2	12.6

Solutions to problems relating to sound in enclosed spaces

Problem 7.1

Direct field

The direct field of a sound source is defined as that part of the sound field which has not suffered any reflection from any room surfaces or obstacles.

Reverberant field

The reverberant field of a source is defined as that part of the sound field radiated by a source which has experienced at least one reflection from a boundary of the room or enclosure containing the source.

Problem 7.2

250Hz octave band, bandwidth from table 1.2 on p43 in the text is $353 - 176 = 177\text{Hz}$. Thus the modal overlap, M , is (from equation 7.24 in the text) given by:

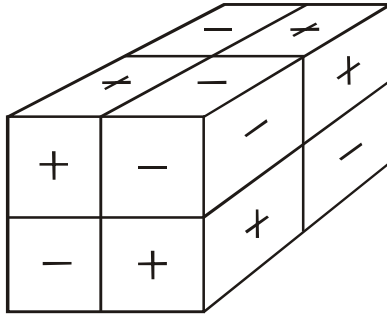
$$M = (20 + 25 + 30 + 32) / 177 = 0.6$$

Problem 7.3

- (a) If $n_z = 0$, the pressure distribution in the room is uniform and the monopole source will excite the mode. For $n_z = 1$ and $n_z = 3$, there will be a node at $L_z/2$ and the monopole will not excite the mode. For $n_z = 2$, there will be an antinode at $L_z/2$ and the mode will be excited by a monopole source.
- (b) A dipole will be ineffective at exciting the $n_z = 0$ and $n_z = 2$ modes

because the phase of each part of the dipole is opposite to the other and the modal response is not. On the other hand, one would expect good excitation of the $n_z = 3$ mode because the phase on one side of a node is 180° different to that on the other side and this can be matched by the dipole.

- (c) At a rigid wall, the particle velocity is zero. The required derivation is described on pages 278 and 279 in the text.
- (d) The required shape is shown in the figure below where the nodes are represented by lines and the relative phases of acoustic pressure are



shown by plus and minus signs.

Using equation 7.17 in the text:

$$f_{1,1,1} = \frac{343}{2} \sqrt{\frac{1}{10^2} + \frac{1}{5^2} + \frac{1}{2^2}} = 93.9 \text{ Hz}$$

Problem 7.4

- (a) Cut-on frequency is the frequency at which the wavenumber, κ becomes real and the mode begins to propagate down the duct without decaying in amplitude (assuming a rigid, hard walled duct).
- (b) From the equation given in the problem, for a single mode the acoustic

pressure amplitude is given by $\bar{p} = \bar{p}_0 e^{-j\kappa_{mn}x}$. Thus the dB decay per unit distance is:

$\Delta = 20 \log_{10} \frac{\bar{p}_0}{\bar{p}_1} = 20 \times 0.4343 \times j\kappa_{mn}$, where \bar{p}_1 is the sound pressure amplitude at 1m. For $m = 3$, $n = 2$, and $L_y = L_z/3$:

$$\kappa_{3,2} = \sqrt{(\omega/c)^2 - [45\pi^2/L_z^2]}$$

Cut-on frequency is thus given by:

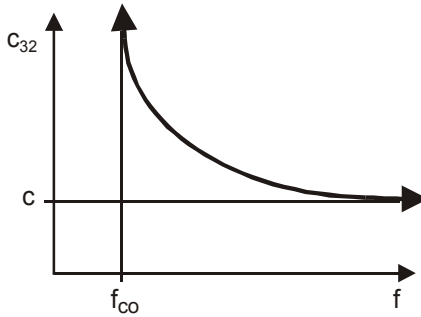
$$\omega = c[\sqrt{45}\pi/L_z]$$

Thus at 1/3 of the cut-on frequency, $\kappa_{3,2}$ is given by:

$$\kappa_{3,2} = \sqrt{[5\pi^2/L_z^2] - [45\pi^2/L_z^2]} = \pm j\sqrt{40}\pi/L_z$$

Thus $\Delta = 172/L_z$ (dB/m).

$$\begin{aligned} \text{(c) Phase speed, } c_{3,2} &= \omega/\kappa_{3,2} = \frac{\omega}{\sqrt{(\omega^2/c^2) - 45\pi^2/L_z^2}} \\ &= \frac{c}{\sqrt{1 - 45\pi^2 c^2/(\omega^2 L_z^2)}} = \frac{c}{\sqrt{1 - \text{const}/\omega^2}} \end{aligned}$$



The variation of $c_{3,2}$ as a function of frequency is shown in the figure. Near the mode cut-on frequency it can be seen that the phase speed approaches infinity and at high frequencies it approaches the speed of sound in free space.

Problem 7.5

- (a) Room dimensions are $4.6\text{m} \times 6.2\text{m} \times 3.5\text{m}$. The lowest resonance frequency is the axial mode corresponding to the longest room dimension. Thus:

$$f = \frac{c}{2} \frac{1}{6.2} = \frac{343}{2} \times \frac{1}{6.2} = 27.7\text{Hz}$$

- (b) Pressure distribution follows a half cosine wave in the 6.2m direction as shown in the figure and calculated from equation 7.19 in the text. The pressure is uniform across a given cross section defined by the $4.6\text{m} \times 3.5\text{m}$ dimensions. Energy density is given by equation 7.32 in the text as:

$$\psi = \frac{\langle p^2 \rangle}{\rho c^2}$$

The sound pressure level in a room corner is 80dB and this corresponds to the maximum sound pressure. Thus:

$$\langle p_{\max}^2 \rangle = (2 \times 10^{-5})^2 \times 10^{80/10} = 0.04\text{Pa}^2$$

The sound pressure at any other location is (from equation 7.19 in the text):

$$\langle p^2 \rangle = 0.04 \times \cos^2\left(\frac{\pi x}{L}\right)$$

Thus the energy in the room is given by:

$$\begin{aligned}
 E &= \int_V \psi \, dV = 4.6 \times 3.5 \int_0^L \frac{\langle p^2 \rangle}{\rho c^2} dL \\
 &= \frac{4.6 \times 3.5 \times 0.04}{1.206 \times 343^2} \int_0^L \cos^2\left(\frac{\pi x}{L}\right) dx \\
 &= 4.54 \times 10^{-6} \left[\frac{x}{2} + \frac{L \sin(2\pi x/L)}{4\pi} \right]_0^L
 \end{aligned}$$

Substituting $L = 6.2$ gives $E = 1.4 \times 10^{-5}$ Joules.

- (c) The analysis procedure used on p290 and 291 in the text may be used here. Alternatively, we may start with the given equation and write:

$$\frac{dE}{E} = -\frac{Sc\bar{a}}{V} dt$$

Integrating gives:

$$\int_{E_0}^E \frac{dE}{E} = -\int_0^t \frac{Sc\bar{a}}{V} dt$$

which gives:

$$\log_e E - \log_e E_0 = -\frac{Sc\bar{a}t}{V}$$

or

$$E = E_0 e^{-Sc\bar{a}t/V}$$

As $E \propto \langle p^2 \rangle$, we can write:

$$\langle p^2 \rangle = \langle p_0^2 \rangle e^{-Sc\bar{a}t/V}$$

Thus:

$$L_p - L_{p0} = 4.343 c\bar{a}tS/V = 4.343 c\bar{a}t/L$$

and

$$60 = 4.343 c \bar{\alpha} T_{60} / L$$

Therefore:

$$\bar{\alpha} = \frac{60}{4.343} \times \frac{L}{c T_{60}} = \frac{60}{4.343} \times \frac{6.2}{343 \times T_{60}} = \frac{0.250}{T_{60}}$$

- (d) $T_{60} = 5$ seconds, so $\bar{\alpha} = 0.25/5 = 0.05$.

Power consumption, $W_a = 2\bar{\alpha}S_w$, and $S_w = 4.6 \times 3.5 = 16.1 \text{ m}^2$. The quantity, I , is the sound intensity in the direction of one wall. As shown on p287 in the text, the effective intensity, I , in one direction is related to the acoustic pressure in a diffuse field by:

$$\frac{\langle p^2 \rangle}{4\rho c} = I$$

Substituting 0.04Pa^2 for the maximum mean square pressure and the above equation into the equation for W_a gives for the absorbed power:

$$W_a = \frac{2 \times 0.04}{4 \times 1.206 \times 343} \times 0.05 \times 16.1 = 38.9 \mu\text{W}$$

Alternatively, the expression given in part (c) may be used to give:

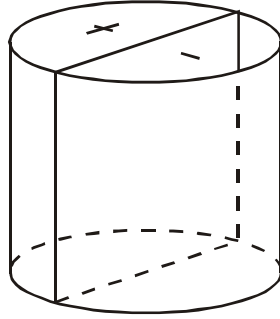
$$W_a = \frac{dE}{dt} = \frac{ESc\bar{\alpha}}{V} = \frac{1.407 \times 10^{-5} \times 343 \times 0.05}{6.2} = 38.9 \mu\text{W}$$

Problem 7.6

- (a) Substituting $L = 2.7$, $c = 343$, $a = 5.5$ and the values of the characteristic function ψ given in the problem table into the equation for f given in the problem gives for the lowest order resonance frequency ($n = 0$, $m = 1$ and $n_z = 0$) gives:

$$f = \frac{343}{2} \times \frac{0.5861}{5.5} = 18.3 \text{ Hz}$$

- (b) The lowest order mode pressure distribution is shown in the figure at right, where the nodal plane which runs the full length of the cylinder is indicated. The sound field is in opposite phase from one side of the nodal line to the other. The sound pressure will be at a maximum along two axial lines at the surface of the cylinder furthest from the nodal plane.



- (c) The air particles are oscillating with a velocity in-quadrature with the local acoustic pressure as the mode is characterised by a standing wave generated by the interference of two acoustic waves travelling in opposite directions across the nodal plane.
- (d) This problem may be answered by inspection of the given table and equation. The modes are listed in the table below in order of ascending frequency, column 1 followed by column 2 etc.

1,0	4,0	0,2	7,0	5,1	4,2	5,2	4,3	3,4	5,4
2,0	1,1	6,0	4,1	3,2	2,3	3,3	2,4	6,3	6,4
0,1	5,0	3,1	2,2	1,3	7,1	1,4	7,2	4,4	7,4
3,0	2,1	1,2	0,3	6,1	0,4	6,2	5,3	7,3	

- (e) $f = 343/(2 \times 2.7) = 63.5\text{Hz}$.
- (f) Axial modes have 2 zeroes in the subscripts n_z , n , and m . Tangential modes have one zero and oblique modes have none. These criteria can be used to list any number of axial, tangential and oblique modes.

The resonance frequency of the first oblique mode is:

$$f_{1,1,1} = \frac{343}{2} \sqrt{\frac{1}{2.7^2} + \left(\frac{1.697}{5.5}\right)^2} = 82.7\text{Hz}$$

Modes with resonances below this with $n_z = 1$ are found from the table in the problem as those with ψ less than the value of ψ corresponding to

$n = 1$ and $m = 1$. For $n_z = 0$, the value of ψ must be below that given by:

$$\psi < 82.7 \times 2 \times 5.5 / 343 = 2.651$$

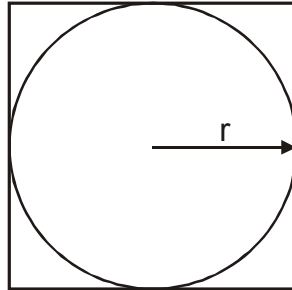
The modes with resonance frequencies below the (1,1,1) mode are listed below in the form (n_z, m, n) .

1,0,0	1,4,0	0,3,0	0,0,1	0,0,2
1,1,0	1,0,1	0,4,0	0,1,1	
1,2,0	0,1,0	0,5,0	0,2,1	
1,3,0	0,2,0	0,6,0	0,3,1	

That is, there are 17 modes with resonance frequencies below that of the first oblique mode. Of these, 9 are axial and 8 are tangential modes.

Problem 7.7

- (a) A 2-D space would be one where the dimensions in 2 directions were very much larger than in the third direction. An example would be a large factory with a low ceiling in the frequency range below the first floor/ceiling axial resonance frequency.



Following the procedure for a 3-D space described on p285-287 in the text, a 2-D square region enclosing a circle is considered as shown in the figure.

$$\frac{\text{area of circular region}}{\text{area of square region}} = \frac{\pi r^2}{4r^2} = \frac{\pi}{4}$$

Time for sound to travel through the circular region (length of encompassing square) = $2r/c$. Energy in the square region of length $2r$ (and unit thickness) as a result of a wave travelling normally to any of the sides is $I \frac{2r}{c}$, where I is the incident wave intensity. The energy per unit perimeter in the circular region as a result of an incident wave $2r$ wide is:

$$\Delta E = \frac{\pi}{4} I \frac{2r}{c}$$

The total energy in the circular region due to waves from all directions is found by integrating the preceding expression over the perimeter of the circle. Thus:

$$E = \int_{2\pi r} \Delta E \, dx = \int_0^{2\pi} \Delta E r \, d\theta = \frac{\pi}{2} \frac{Ir^2}{c} \int_0^{2\pi} d\theta = \frac{Ir^2\pi^2}{c}$$

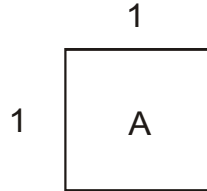
Energy density, ψ , is given by:

$$\psi = \frac{E}{S} = \frac{Ir^2\pi^2}{c\pi r^2} = \frac{\pi I}{c}$$

Thus:

$$I = \frac{\psi c}{\pi}$$

Consider a plane wave travelling unit distance. The energy in unit area is the intensity multiplied by the time it is present which is, $E = I/c$. The energy is also equal to the energy density multiplied by unit area. Thus $\psi = E$. Thus for a plane wave:



$$\psi = \frac{I}{c} = \frac{\langle p^2 \rangle}{\rho c^2}$$

Equation 7.32 in the text also gives the relation between sound pressure and energy density. Thus for a diffuse 2-D field:

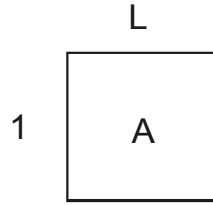
$$\psi = \frac{\pi I}{c} = \frac{\langle p^2 \rangle}{\rho c^2}$$

Rearranging gives:

$$I = \frac{\langle p^2 \rangle}{\pi \rho c}$$

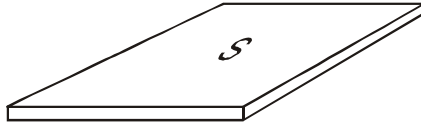
- (b) An example of a 1-D field would be the inside of a rigid tube closed at both ends at frequencies below the first higher order mode cut-on

frequency. Assuming unit cross-sectional area and a wave travelling from left to right over a distance of L . The time for a wave to travel a distance, L , is L/c . The energy in the wave travelling from left to right is IL/c . The total energy (due to left and right travelling waves) is thus $2IL/c$. The total energy in the volume is also ψL . Thus, $I = \psi c/2$ and $\psi = 2I/c$. Using this relation and equation 7.32 in the text gives for a 1-D field:



$$I = \frac{\langle p^2 \rangle}{2\rho c}$$

- (c) Let S = area of 2-D room of unit height as shown in the figure. Rate of energy absorbed, W_a , around the perimeter, of length P and absorption coefficient $\bar{\alpha}$ is given by:



$$W_a = IP\bar{\alpha} = \frac{\psi c}{\pi} P\bar{\alpha}$$

Rate of change of energy in the reverberant field = rate of supply, W_0 - rate absorbed, W_a . Thus:

$$W = S \frac{\partial \psi}{\partial t} = W_0 - \frac{\psi c}{\pi} P\bar{\alpha}$$

Introducing the dummy variable:

$$X = [\pi W_0 / P c \bar{\alpha}] - \psi$$

into the preceding equation, we may write:

$$\frac{dX}{dt} = -\frac{\partial \psi}{\partial t} = -\frac{1}{S} \left(W_0 - \frac{\psi c}{\pi} P\bar{\alpha} \right)$$

and

$$\frac{1}{X} = \frac{P\bar{\alpha}c}{\pi} \left(W_0 - \frac{\psi c}{\pi} P\bar{\alpha} \right)^{-1}$$

Thus:

$$\frac{1}{X} \frac{dX}{dt} = -\frac{P\bar{\alpha}c}{\pi S}$$

Integrating gives:

$$\int_{X_0}^X \frac{dX}{X} = \int_0^t -\frac{P\bar{\alpha}c}{\pi S}$$

Thus:

$$\log_e X - \log_e X_0 = -\frac{P\bar{\alpha}ct}{\pi S}$$

and

$$X = X_0 e^{-P\bar{\alpha}ct/\pi S}$$

For a decaying sound field, $W_0 = 0$ at $t = 0$. Using the preceding equation, $X = X_0$ when $t = 0$. Also the definition of the dummy variable, X , above can be used to show that when $W_0 = 0$, $X = X_0 = -\psi_0$. The same equation may be used to show that as $W_0 = 0$, then at any time t , $X = -\psi$. From the preceding discussion and the equation above we may write:

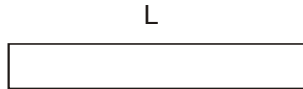
$$\psi = \psi_0 e^{-P\bar{\alpha}ct/\pi S}$$

We know that $\psi \propto \langle p^2 \rangle$. Thus:

$$\langle p^2 \rangle = \langle p_0^2 \rangle e^{-P\bar{\alpha}ct/\pi S}$$

- (d) Let L be the length of the 1-D tube of unit cross sectional area.

Rate of energy absorbed, W_a , at the ends of the tube of length L and absorption coefficient $\bar{\alpha}$ is given by:



$$W_a = 2I\bar{\alpha} = 2\frac{\psi^2 c}{2}\bar{\alpha}$$

Rate of change of energy in the reverberant field = rate of supply, W_0 -

rate absorbed, W_a . Thus:

$$W = L \frac{\partial \psi}{\partial t} = W_0 - \psi c \bar{\alpha}$$

Introducing the dummy variable:

$$X = [W_0 / c \bar{\alpha}] - \psi$$

into the preceding equation, we may write:

$$\frac{dX}{dt} = -\frac{\partial \psi}{\partial t} = -\frac{1}{L}(W_0 - \psi c \bar{\alpha})$$

and

$$\frac{1}{X} = \bar{\alpha} c (W_0 - \psi c \bar{\alpha})^{-1}$$

Thus:

$$\frac{1}{X} \frac{dX}{dt} = -\frac{\bar{\alpha} c}{L}$$

Integrating gives:

$$\int_{X_0}^X \frac{dX}{X} = \int_0^t -\frac{\bar{\alpha} c}{L}$$

Thus:

$$\log_e X - \log_e X_0 = -\frac{\bar{\alpha} c t}{L}$$

and

$$X = X_0 e^{-\bar{\alpha} c t / L}$$

For a decaying sound field, $W_0 = 0$ at $t = 0$. Using the preceding equation, $X = X_0$ when $t = 0$. Also the definition of the dummy variable, X , above can be used to show that when $W_0 = 0$, $X = X_0 = -\psi_0$. The same equation may be used to show that as $W_0 = 0$, then at any time t , $X = -\psi$. From the preceding discussion and the equation above we may write:

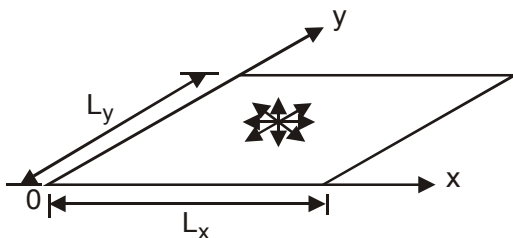
$$\psi = \psi_0 e^{-\bar{a}ct/L}$$

We know that $\psi \propto \langle p^2 \rangle$. Thus:

$$\langle p^2 \rangle = \langle p_0^2 \rangle e^{-\bar{a}ct/L}$$

- (e) Mean free path, 2-D space. Sound propagation in any direction in the 2-D plane is equally likely (see figure). Due to symmetry, we need to consider only one of the 4 quadrants and find an average path length between reflections for sound propagating in these directions.

Consider a sound wave propagating with speed c , in an angular direction of ψ from the x -axis as shown in the figure. Resolving into x and y components, we have:



$$c_x = c \cos \theta \quad \text{and} \quad c_y = c \sin \theta$$

The number of reflections per unit time for a wave travelling in the θ direction is given by:

$$N_r = \frac{c \cos \theta}{L_x} + \frac{c \sin \theta}{L_y}$$

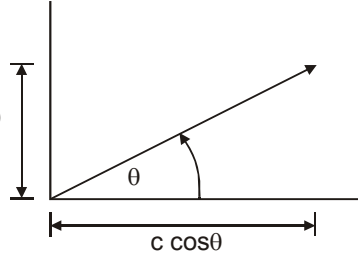
Averaging over all directions in one quadrant (the average for the other three quadrants would be the same) we find that the average number of reflections per unit time is:

$$\begin{aligned} N_{av} &= \frac{1}{\pi/2} \int_0^{\pi/2} N_r d\theta = \frac{2c}{\pi} \int_0^{\pi/2} \left(\frac{\cos \theta}{L_x} + \frac{\sin \theta}{L_y} \right) d\theta \\ &= \frac{2c}{\pi} \left[\frac{\sin \theta}{L_x} - \frac{\cos \theta}{L_y} \right]_0^{\pi/2} = \frac{2c}{\pi} \left[\frac{1}{L_x} + \frac{1}{L_y} \right] \end{aligned}$$

But $N_{av} = c/\Lambda$, where Λ is the mean free path. Thus:

$$\Lambda = \frac{\pi}{2} \left[\frac{L_x L_y}{L_x + L_y} \right] = \frac{\pi S}{P}$$

For a 1-D space (see figure), the mean free path, which is the distance between reflections is equal to L , the $c \sin \theta$ length of the space.



Problem 7.8

- (a) Room size = $7\text{m} \times 5\text{m} \times 3\text{m}$. Volume = 105m^3 , surface area = $2(7 \times 5 + 7 \times 3 + 5 \times 3) = 142\text{m}^2$. Wavelength of sound, $\lambda = 343/500 = 0.686\text{m}$. The sound power in the reverberation room is related to the sound pressure level by equation 6.13 in the text. Thus:

$$\begin{aligned} L_w &= L_p + 10\log_{10} V - 10\log_{10} T_{60} + 10\log_{10} (1 + S\lambda/8V) - 13.9 \\ &= 95 + 10\log_{10}(105) - 10\log_{10}(2.5) + 10\log_{10} \left(1 + \frac{142 \times 0.686}{8 \times 105} \right) - 13.9 \\ &= 95 + 20.2 - 4.0 + 0.5 - 13.9 = 97.8\text{dB re } 10^{-12}\text{W} \end{aligned}$$

Alternatively, we could assume that the correction term, $(1 + S\lambda/8V)$ is already included in L_p . In this case, $L_w = 97.8 - 0.5 = 97.3\text{dB}$.

Power conversion efficiency is given by:

$$\eta = \frac{\text{acoustic power}}{\text{electrical power}} = \frac{10^{-12} \times 10^{97.8/10}}{10} = 6 \times 10^{-4} = 0.06\%$$

- (b) Fire box dimensions are: $10 \times 12 \times 20\text{m}$.

Axial resonances - first mode in each direction given by:

$$f = \frac{c}{2L} = \frac{864}{2 \times 10}, \frac{864}{2 \times 12}, \frac{864}{2 \times 20}$$

$$= 43.2 \text{ Hz}, 36 \text{ Hz}, 21.6 \text{ Hz}$$

- (i) As the frequency of instability is 36Hz, it seems likely that it is associated with the axial mode across the width in the 12m direction.
- (ii) The acoustic pressure will be largest at the two side walls normal to the 12m dimension, because on reflection the pressure amplitude is doubled.
- (iii) Amplitude of cyclic force acting on walls.
Acoustic pressure is 155dB. Amplitude is then:

$$\bar{p} = \sqrt{2} p_{rms} = 2\sqrt{2} \times 10^{-5} \times 10^{155/20} = 1.59 \text{ kPa}$$

Side wall area = $10 \times 20 = 200 \text{ m}^2$.

Force on each wall = $1.59 \times 200 = 318 \text{ kN}$.

- (iv) Power absorbed by 2 side walls.
Pressure at wall is the sum of the incident and reflected pressures.
Thus, $(p_i + p_r)_{rms} = 1.125 \times 10^3$. However, $p_r = \sqrt{(1 - \bar{\alpha})} p_i$.
So, $[1 + \sqrt{(1 - \bar{\alpha})}] p_i = 1.125 \times 10^3$
The absorbed power is controlled by the incident sound intensity which is:

$$I = \frac{p_i^2}{\rho c} = \frac{(1.125 \times 10^3)^2}{(1 + \sqrt{1 - \bar{\alpha}})^2 \rho c}$$

The absorbed power is $W_a = 2\bar{\alpha} I A_w$, where A_w = area of one wall.

For a 1-D sound field, $\langle p^2 \rangle = \langle p_0^2 \rangle e^{-c\bar{\alpha}t/L}$. Thus:

$$L_{p0} - L_p = 4.343 c \bar{\alpha} t / L$$

$$\text{and } T_{60} = \frac{(60/4.343) \times L}{c\bar{\alpha}}. \text{ But}$$

$$T_{60} = \frac{2.2Q}{f_n} = \frac{2.2 \times 30}{36} = 1.833 \text{ secs}$$

Thus:

$$\bar{\alpha} = \frac{(60/4.343) \times 12}{864 \times 1.833} = 0.105$$

The absorbed power is then:

$$W_a = \frac{2 \times 0.1047 \times (1.1246 \times 10^3)^2 \times 200}{(1 + \sqrt{1 - 0.1047})^2 \times 1.16 \times 864} = 14 \text{ kW}$$

(v) Power conversion efficiency is:

$$\eta = \frac{13953}{800 \times 10^3} = 1.7\%$$

Problem 7.9

(a) Taking logs of the equation given in the problem for a 1-D field, we have:

$$L_{p0} - L_p = 4.343 c \bar{\alpha} t / L$$

Thus:

$$60 = 4.343 \times 343 \times 0.05 \times T_{60} / 5$$

which gives, $T_{60} = 4.0$ seconds.

(b) Again taking logs of the equation given in the problem gives:

$$L_{p0} - L_p = 4.343 P \bar{\alpha} c t / (\pi S)$$

$P = 4 \times 5 = 20 \text{ m}$ and $S = 5 \times 5 = 25 \text{ m}^2$. Thus:

$$60 = 4.343 \times 20 \times 0.25 \times 343 \times T_{60} / (\pi \times 25)$$

which gives, $T_{60} = 0.63$ seconds.

(c) $S = 2(5 \times 5 + 5 \times 5 + 5 \times 5) = 150 \text{ m}^2$

$$V = 5 \times 5 \times 5 = 125 \text{ m}^3$$

$$\bar{\alpha} = \frac{50 \times 0.05 + 100 \times 0.25}{150} = 0.183$$

Using equation 7.50 in the text, we obtain:

$$60 = 1.086 \times 150 \times 343 \times 0.1833 \times T_{60} / 125$$

which gives, $T_{60} = 0.73$ seconds.

Problem 7.10

In a reverberant field the sound intensity in any particular direction is equal to that in any other direction resulting in a net active intensity (averaged over all directions) of zero and a large reactive intensity. Thus in the situation under consideration here, the active intensity will only be contributed to by the direct field of the source and the reactive intensity field will dominate the active field by a large amount, especially at large distances from the source. Due to the dominance by the reactive field together with limitations on the phase accuracy between the two microphones of any measurement system, accurate measurements of the active field will not be generally feasible in a reverberant room.

Problem 7.11

- (a) Energy density in a reverberant field is $\langle p^2 \rangle / \rho c^2$ and the given SPL is 95 dB.

Thus, $\langle p^2 \rangle = (2 \times 10^{-5})^2 \times 10^{9.5} = 1.265 \text{ Pa}^2$ and the energy density is then: Energy density $= 1.265 / (413 \times 343) = 8.9 \times 10^{-6} \text{ J/m}^3$

- (b) Sound power incident on a wall is given by:

$$W = IA = \frac{\langle p^2 \rangle}{4\rho c} A = \frac{1.265}{4 \times 413} \times 30 = 0.023 \text{ W} = 103.6 \text{ dB re } 10^{-12} \text{ W}$$

Problem 7.12

- (a) Room $10\text{m} \times 10\text{m} \times 4\text{m}$, $S = 2(10 \times 10 + 10 \times 4 \times 2) = 360\text{m}^2$,
 $V = 10 \times 10 \times 4 = 400\text{m}^3$. Using equation 7.51 in the text for reverberation time, we may write:

$$T_{60} = \frac{55.25 \times 400}{343 \times 360 \times 0.1} = 1.8\text{seconds}$$

- (b) $L_p = 60\text{dB}$ corresponds to $\langle p^2 \rangle = 4 \times 10^{-10} \times 10^{60/10} = 4 \times 10^{-4} \text{Pa}^2$. Using equation 7.41 in the text we obtain:

$$W = \frac{360 \times 0.1 \times 4 \times 10^{-4}}{4 \times 1.206 \times 343 \times (1 - 0.1)} = 9.67 \mu\text{W}$$

Sound power level, $L_w = 10\log_{10}(9.67 \times 10^{-6}) + 120 = 69.9\text{dB re } 10^{-12}\text{W}$.

- (c) Using equation 7.33 in the text:

$$I = \frac{4 \times 10^{-4}}{4 \times 1.206 \times 343} = 2.42 \times 10^{-7} \text{W/m}^2$$

- (d) From equations 7.40 to 7.42 in the text, the reverberant field level is equal to the direct field level when $D_\theta/(4\pi r^2) = 4/R = 4(1 - \bar{\alpha})/(S\bar{\alpha})$. Assuming the acoustic centre of the source is within a quarter wavelength from the hard floor, $D_\theta = 2$ and the distance, r , at which the fields are equal is:

$$r = \left(\frac{S\bar{\alpha}}{8\pi(1 - \bar{\alpha})} \right)^{1/2} = \left(\frac{36}{8\pi \times 0.9} \right)^{1/2} = 1.26\text{m}$$

Problem 7.13

- (a) Room $3.05\text{m} \times 6.1\text{m} \times 15.24\text{m}$,
 $S = 2(3.05 \times 6.1 + 3.05 \times 15.24 + 6.1 \times 15.24) = 316.1\text{m}^2$,
 $V = 3.05 \times 6.1 \times 15.24 = 283.54\text{m}^3$. Using equation 7.51 in the text for reverberation time, we may write:

$$\bar{\alpha} = \frac{55.25 V}{ScT_{60}} = \frac{55.25 \times 283.54}{316.1 \times 343 \times 2} = 0.072$$

Using equation 7.42, the sound power output may be written as:

$$\begin{aligned} L_w &= L_p + 10 \log_{10} \left(\frac{4(1 - \bar{\alpha})}{S\bar{\alpha}} \right) + 10 \log_{10} \left(\frac{\rho c}{400} \right) \\ &= 74 - 10 \log_{10} \left(\frac{4 \times 0.928}{316.1 \times 0.072} \right) - 0.15 = 81.7 \text{ dB re } 10^{-12} \text{ W} \end{aligned}$$

- (b) To lower the reverberant field by 10dB, we need to increase R by a factor of 10. Old $S\bar{\alpha}/(1 - \bar{\alpha}) = 24.61 \text{ m}^2$, thus required new $S\bar{\alpha}/(1 - \bar{\alpha}) = 246.1 \text{ m}^2$. Thus in the new situation, expanding the above equation gives:

$$316.1\bar{\alpha} = 246.1 - 246.1\bar{\alpha}$$

which results in a new required value of $\bar{\alpha} = 0.438$. The old value of $\bar{\alpha}$ is 0.0722, so the increase in absorption needed is:

$$\Delta S\bar{\alpha} = 316.1(0.438 - 0.0722) = 115.5 \text{ m}^2.$$

- (c) Using the value of $\bar{\alpha} = 0.438$ obtained above and equation 7.51 in the text, we obtain:

$$T_{60} = \frac{55.25 V}{Sc\bar{\alpha}} = \frac{55.25 \times 283.54}{316.1 \times 343 \times 0.438} = 0.33 \text{ seconds}$$

Problem 7.14

- (a) Sound power level = $10 \log_{10} W + 120$
 $= 10 \log_{10} 3.1 + 120 = 124.9 \text{ dB re } 10^{-12} \text{ W}$

- (b)

$$c = \sqrt{\frac{\gamma R T}{M}} = \sqrt{\frac{1.4 \times 8.314 \times 1473}{0.035}} = 700 \text{ m/s}$$

Surface area, $S = \pi dL + \pi d^2/2 = \pi \times 4 \times 6 + \pi \times 16/2 = 32\pi = 100 \text{ m}^2$

Problem 7.15

$$\begin{aligned}
 \rho &= \frac{\gamma P}{c} = \frac{1.4 \times 101.4 \times 10^3}{340} = 0.29 \text{ kg/m}^3 \\
 L_p &= L_w + 10 \log_{10} \left[\frac{Q}{4\pi r^2} + \frac{4(1 - \bar{\alpha})}{S\bar{\alpha}} \right] + 10 \log_{10} \left[\frac{\rho c}{400} \right] \\
 &= 124.9 + 10 \log_{10} \left[\frac{4 \times 0.95}{100 \times 0.05} \right] + 10 \log_{10} \left[\frac{700 \times 0.29}{400} \right] = 120.8 \text{ dB}
 \end{aligned}$$

$$L_p = 10 \log_{10} (10^{9.5} + 10^{9.7} + 10^{9.9}) = 102.1 \text{ dB(A)}$$

$$L_p = L_w + 10 \log_{10} \left(\frac{Q}{4\pi r^2} + \frac{4(1 - \bar{\alpha})}{S\bar{\alpha}} \right)$$

$$\text{Thus, } 102.1 - 110 = 10 \log_{10} \left(\frac{2}{4\pi r^2} + \frac{3.64}{320 \times 0.09} \right)$$

$$\text{and, } 10^{-7.9/10} - \frac{3.64}{320 \times 0.09} = \frac{1}{6.28r^2}$$

$$\text{Thus, } r = \sqrt{\frac{1}{6.28 \times 0.0358}} = 2.1 \text{ m}$$

Problem 7.16

Room $10\text{m} \times 10\text{m} \times 5\text{m}$, $S = 2(10 \times 10 + 10 \times 5 \times 2) = 400\text{m}^2$,
 $V = 10 \times 10 \times 5 = 500\text{m}^3$. The room constant before treatment is

$$R_b = \frac{S\bar{\alpha}}{(1 - \bar{\alpha})} = \frac{400 \times 0.08}{0.92} = 34.78\text{m}^2$$

Adding sound absorbing treatment to the walls and ceiling results in a new mean absorption coefficient calculated as follows:

$$\bar{\alpha}_n = \frac{100 \times 0.08 + 300 \times 0.5}{400} = 0.395$$

Thus the new room constant after treatment is:

$$R_n = \frac{S\bar{\alpha}}{(1 - \bar{\alpha})} = \frac{400 \times 0.395}{0.605} = 261.2 \text{ m}^2$$

Using equation 7.42 in the text allows the difference in sound pressure level (assuming that the sound power is constant) to be calculated as follows (assuming the acoustic centre of the machine is within a quarter wavelength of the hard floor):

$$\begin{aligned} \Delta L_p &= 10 \log_{10} \left(\frac{D_\theta}{4\pi r^2} + \frac{4}{R} \right)_{old} - 10 \log_{10} \left(\frac{D_\theta}{4\pi r^2} + \frac{4}{R} \right)_{new} \\ &= 10 \log_{10} \left(\frac{2}{4\pi 3^2} + \frac{4}{34.78} \right) - 10 \log_{10} \left(\frac{2}{4\pi 3^2} + \frac{4}{261.2} \right) \\ &= 6.0 \text{ dB} \end{aligned}$$

Problem 7.17

- (a) Room $8\text{m} \times 6\text{m} \times 3\text{m}$, $S = 2(8 \times 6 + 8 \times 3 + 6 \times 3) = 180\text{m}^2$.
The mean Sabine absorption coefficient may be calculated using equation 7.78 in the text to give:

$$\bar{\alpha} = \frac{48 \times 0.15 + 132 \times 0.05}{180} = 0.0767$$

Using equation 7.41, we can write:

$$\langle p_R^2 \rangle = \frac{4 \times 25 \times 10^{-3} \times 1.206 \times 343 (1 - 0.0767)}{180 \times 0.0767} = 2.766 \text{ Pa}^2$$

The reverberant field sound pressure level is then:

$$L_{pR} = 10 \log_{10}(2.766) + 94 = 98.4 \text{ dB}$$

- (b) Direct and reverberant fields equal (see equation 7.42 in the text) when

$D_0/4\pi r^2 = 4/R$. Assuming that the acoustic centre of the source is well above the floor, $D_0 = 1$ and the distance, r , at which the two fields are equal is:

$$r = \left(\frac{S\bar{\alpha}}{16\pi(1 - \bar{\alpha})} \right)^{1/2} = \left(\frac{180 \times 0.0767}{16\pi \times (1 - 0.0767)} \right)^{1/2} = 0.55 \text{ m}$$

Problem 7.18

- (a) This is a flat room as on pages 314-319 in the text. Pressure reflection coefficient amplitude = 0.7, $\beta = 0.7^2$ and $\bar{\alpha} = 1 - 0.7^2 = 0.51$. Room height, $a = 5$ m and distance $r = 5$ m. Thus $r/a = 1$. From the figure, the reverberant field sound pressure is given by:

$$10\log_{10}\langle p^2 \rangle = 10\log_{10} \left[\frac{W\rho c}{\pi a^2} \right] - 8$$

Thus the reverberant field sound pressure level is:

$$\begin{aligned} L_{p(\text{reverb})} &= L_w + 10\log_{10} \left(\frac{\rho c}{400} \right) - 10\log_{10}(\pi a^2) - 8 \\ &= 130 + 0.14 - 18.95 - 8 = 103.2 \text{ dB} \end{aligned}$$

The direct field sound pressure level is:

$$\begin{aligned} L_{p(\text{direct})} &= L_w + 10\log_{10} \left(\frac{\rho c}{400} \right) - 10\log_{10}(4\pi r^2) \\ &= 130 + 0.14 - 24.97 = 105.2 \text{ dB} \end{aligned}$$

- (b) Sabine room, area, $S = 2[10 \times 10 + 10 \times 5 \times 2] = 400 \text{ m}^2$ and volume, $V = 10 \times 10 \times 5 = 500 \text{ m}^3$

The total sound pressure level (direct plus reverberant) in the room is:

$$\begin{aligned}
 L_p &= L_w + 10 \log_{10} \left[\frac{D}{4\pi r^2} + \frac{4(1 - \bar{\alpha})}{S\bar{\alpha}} \right] \\
 &= 130 + 10 \log_{10} \left[\frac{1}{4\pi \times 25} + \frac{4 \times 0.49}{400 \times 0.51} \right] = 111 \text{ dB}
 \end{aligned}$$

- (c) Direct and reverberant fields equal when $D/4\pi r^2 = 4(1 - \bar{\alpha})/S\bar{\alpha}$.
Thus:

$$r = \sqrt{\frac{DS\bar{\alpha}}{4\pi \times 4(1 - \bar{\alpha})}} = \sqrt{\frac{400 \times 0.51}{16\pi \times 0.49}} = 2.9 \text{ m}$$

- (d)

$$T_{60} = \frac{55.25 V}{Sc\bar{\alpha}} = \frac{55.25 \times 500}{400 \times 343 \times 0.51} = 0.39 \text{ seconds}$$

- (e) Treat the room like an enclosure. Thus, the sound level at the receiver without the enclosure at a distance of 50 + 5 m is:

$$L_p = L_w + 0.14 - 10 \log_{10} 2\pi r^2 = 130 + 0.14 - 42.8 = 87.4 \text{ dB}$$

The enclosure noise reduction is given by $NR = TL - C$, where

$$C = 10 \log_{10} \left[0.3 + \frac{S_E(1 - \bar{\alpha})}{S_i\bar{\alpha}} \right] = 10 \log_{10} \left[0.3 + \frac{300 \times 0.49}{400 \times 0.51} \right] = 0.1 \text{ dB}$$

Thus the noise reduction = 24.9 dB and the SPL at 50 m is:

$$L_p = 87.4 - 24.9 = 62.5 \text{ dB}$$

Problem 7.19

$Q = 2$, $r = 1$, $\alpha = 0.08$, $L_w = 95 \text{ dB}$, $S = 400$. Thus:

$$L_p = L_w + 10 \log_{10} \left(\frac{Q}{4\pi r^2} + \frac{4(1 - \bar{\alpha})}{S\bar{\alpha}} \right) + 0.15$$

$$L_p = 95 + 10 \log_{10} \left(\frac{2}{4 \times \pi} + \frac{4(1-0.08)}{400 \times 0.08} \right) + 0.15 = 89.5 \text{ dB}$$

Problem 7.20

- (a) Reference source on hard asphalt ($DI = 3$), $L_p = 80\text{dB}$ at 3m. Sound power level is:

$$\begin{aligned} L_w &= L_p + 20 \log_{10} r + 10 \log_{10}(4\pi) - 3 + 10 \log_{10} \left(\frac{400}{\rho c} \right) \\ &= 80 + 9.54 + 11.00 - 3.01 - 0.15 = 97.4 \text{ dB} \end{aligned}$$

- (b) Assuming a negligible direct field, the room constant may be calculated using equation 7.42 in the text (with a 0.15dB correction for ρc different from 400). Thus:

$$L_p = L_w + 10 \log_{10}(4/R) + 0.15$$

Substituting in values for the parameters gives:

$$10 \log_{10} R = 97.38 - 85 + 0.15 + 6.02 = 18.55$$

Thus, $R = 71.6$.

- (c) Room $14\text{m} \times 6\text{m} \times 4\text{m}$, $S = 2(14 \times 6 + 14 \times 4 + 6 \times 4) = 328\text{m}^2$. Using equation 7.43 in the text gives:

$$\frac{328 \bar{\alpha}}{(1 - \bar{\alpha})} = 71.62$$

Thus, $\bar{\alpha} = 0.18$.

- (d) Existing $L_p = 75\text{dB}$ and the allowable total = 80dB. Thus the allowable contribution from the three new line printers is

$$10 \log_{10}(10^8 - 10^{7.5}) = 78.3 \text{ dB. The allowable contribution from each}$$

$$\text{line printer is then } 10 \log_{10} \left(\frac{10^{7.83}}{3} \right) = 73.5 \text{ dB. The allowable sound}$$

power level is the obtained using equation 7.42 in the text (using only the reverberant field part) as follows:

$$L_w = 73.6 - 10 \log_{10} \left(\frac{4}{71.6} \right) - 0.15 = 86.0 \text{ dB}$$

- (e) Area of ceiling = $14 \times 6 = 84 \text{ m}^2$, corresponding $\bar{\alpha} = 0.5$.
 Area of floor and walls = $328 - 84 = 244 \text{ m}^2$,
 corresponding $\bar{\alpha} = 0.179$.

$$\text{Average } \bar{\alpha} = \frac{0.5 \times 84 + 0.179 \times 244}{328} = 0.261. \text{ Thus:}$$

$$R = \frac{328 \times 0.261}{(1 - 0.261)} = 116 \text{ m}^2$$

Using equation 7.120 in the text, the reduction in sound pressure level is thus:

$$\Delta L_p = 10 \log_{10} \left(\frac{116}{71.6} \right) = 2.1 \text{ dB}$$

Thus the level in the room before addition of the new printers will now be $75 - 2.1 = 72.9 \text{ dB}$. The allowable sound pressure level of the three new line printers together is now $10 \log_{10}(10^8 - 10^{7.29}) = 79.1 \text{ dB}$. The allowable contribution from each line printer is then

$10 \log_{10} \left(\frac{10^{7.91}}{3} \right) = 74.3 \text{ dB}$. The allowable sound power level is the obtained using equation 7.42 in the text (using only the reverberant field part) as follows:

$$L_w = 74.29 - 10 \log_{10} \left(\frac{4}{116} \right) - 0.15 = 88.8 \text{ dB}$$

Problem 7.21

- (a) Considering only the reverberant field of the machine we may use equation 7.41 in the text. The surface area of the factory is $S = 2(10 \times 10 \times 3) = 600 \text{ m}^2$. An L_p of 83 dB corresponds to a $\langle p^2 \rangle$ of $4 \times 10^{-10} \times 10^{83/10} = 7.98 \times 10^{-2} \text{ Pa}^2$. Using equation 7.41, the required mean absorption coefficient is given by:

$$\frac{\bar{\alpha}}{(1 - \bar{\alpha})} = \frac{4 \times 0.01 \times 1.206 \times 343}{7.981 \times 10^{-2} \times 600} = 0.346$$

Thus $\bar{\alpha} = 0.257$. However this would be the required absorption coefficient if the floor were lined as well. If we assume that the concrete floor has an absorption coefficient of 0.01, and we let the required absorption coefficient of the walls and ceiling be x , then we can use equation 7.78 in the text to write:

$$600 \times 0.257 = 100 \times 0.01 + 500x$$

which gives $x = 0.306$. Thus the required absorption coefficient for the walls and ceiling is 0.31.

- (b) The room constant, R , is from part (a) $0.346 \times 600 = 208\text{m}^2$.
 The sound power level, $L_w = 10\log_{10}(0.01) + 120 = 100\text{dB}$.
 For a total L_p of 90dB we may write (assuming a directivity factor, $D_\theta = 2$ as the source is assumed close to a hard floor and other surfaces are more absorptive):

$$90 = 100 + 10\log_{10}\left(\frac{2}{4\pi r^2} + \frac{4}{208}\right)$$

Solving the above gives $r = 1.40\text{m}$ as the radius around the machine within which the sound pressure level will exceed 90dB.

Problem 7.22

- (a) From Table 4.10 in the text, the allowable community noise level to ensure minimal risk of complaints is $40 + 15 - 10 = 45\text{dB(A)}$. If the only noise is in the 500Hz octave band then the -3.2dB A-weighting at this frequency, results in an allowable level of 48.2dB in that band. The existing level is 44dB so the allowed increase is 4.2dB. A reverberant field sound pressure level corresponds to a sound power level (see equation 7.42 in the text) of:

$$L_w = 88 - 10 \log_{10} \left(\frac{4(1 - \bar{\alpha})}{S\bar{\alpha}} \right) - 0.15 \text{ dB}$$

Room $25\text{m} \times 20\text{m} \times 5\text{m}$,

$$S = 2(25 \times 20 + 25 \times 5 + 20 \times 5) = 1450\text{m}^2.$$

$$V = 25 \times 20 \times 5 = 2500\text{m}^3$$

$T_{60} = 2.1$ secs. From equation 7.51 in the text:

$$\bar{\alpha} = \frac{55.25 \times 2500}{343 \times 2.1 \times 1450} = 0.132$$

Thus:

$$L_w = 88 - 10 \log_{10} \left(\frac{4(1 - 0.132)}{1450 \times 0.132} \right) - 0.15 = 105.3 \text{ dB}$$

This is the sound power level of the existing equipment. Thus the allowable total sound power of existing + new equipment is $105.3 + 4.2 = 109.5\text{dB}$. Thus the allowed power level for the new equipment is:

$$L_w = 10 \log_{10} (10^{109.5/10} - 10^{105.3/10}) = 107.4 \text{ dB}$$

As there are 5 new machines, the allowed power level for each is

$$L_w = 107.4 - 10 \log_{10}(5) = 100.4 \text{ dB}$$

Assumptions

- Only absorption is due to floor, walls and ceiling
- Air temperature of 20°C .
- The relative contribution of direct and reverberant sound energy to the community noise levels will be the same for the new machines as for the old machines.

(b) If ceiling tile with $\bar{\alpha} = 0.5$ were added then the new $S\bar{\alpha}$ is:

$$S\bar{\alpha} = 500 \times 0.5 + 950 \times 0.132 = 375.4\text{m}^2$$

corresponding to $\bar{\alpha} = 0.259$

$$\text{Old } R = (1450 \times 0.132)/(1 - 0.132) = 220.5\text{m}^2 \text{ and}$$

$$\text{new } R = (1450 \times 0.259)/(1 - 0.259) = 506.8\text{m}^2.$$

Thus the allowed increase in sound power level for the same reverberant

field sound pressure level is:

$$\Delta L_w = 10 \log_{10}(506.8) - 10 \log_{10}(220.5) = 3.6 \text{ dB}$$

Assuming that the reverberant field dominates the direct field, the new allowed power level of each machine is $100.4 + 3.6 = 104 \text{ dB}$.

Problem 7.23

- (a) Energy density, $\psi = \langle p^2 \rangle / \rho c^2$

$$\text{Thus } \psi = (2 \times 10^{-5})^2 \times 10^{140/10} / (1.205 \times 343^2) = 0.28 \text{ J/m}^3$$

- (b) Enclosure surface area,

$$S = 2(0.2 \times 0.15 + 0.2 \times 0.12 + 0.15 \times 0.12) = 0.144 \text{ m}^2.$$

Power flow into walls (equation 7.39a) is given by

$$W_a = \psi c S \bar{\alpha} / 4 = 0.282 \times 343 \times 0.144 \times 0.1 / 4 = 0.35 \text{ W}$$

- (c) The power generated is equal to the power absorbed by the walls. Thus the power required to drive the source is $0.348 / 0.2 = 1.7 \text{ W}$.

Problem 7.24

- (a) Reverberant field L_p from taking logs of equation 7.41 in the text.

$$L_p = L_w + 10 \log_{10} \left(\frac{4(1 - \bar{\alpha})}{S \bar{\alpha}} \right) + 10 \log_{10} \left(\frac{\rho c}{400} \right)$$

$$S = 2(10 \times 6 + 10 \times 3 + 6 \times 3) = 216 \text{ m}^2, \bar{\alpha} = 0.15, \text{ so:}$$

$$L_p = 84 + 10 \log_{10} \left(\frac{4 \times 0.85}{216 \times 0.15} \right) + 0.15 = 74.4 \text{ dB}$$

- (b) Compare $\frac{D_\theta}{4\pi r^2}$ with $\frac{4(1 - \alpha)}{S \bar{\alpha}}$ As the acoustic centre of the source is well above the hard floor, $D_\theta = 1$; so:

$$\frac{D_\theta}{4\pi r^2} = \frac{1}{4\pi \times 1.5^2} = 0.035$$

$$\frac{4(1 - \alpha)}{S\bar{\alpha}} = \frac{4 \times 0.85}{216 \times 0.15} = 0.105$$

Thus the reverberant field dominates.

- (c) Using equation 7.42 in the text (and allowing for ρc not equal to 400) the sound pressure level corresponding to $\bar{\alpha} = 0.15$ is:

$$L_p = 84 + 10 \log_{10} \left(0.0354 + \frac{4 \times 0.85}{216 \times 0.15} \right) + 0.15 = 75.6 \text{ dB(A)}$$

and the sound pressure level corresponding to $\bar{\alpha} = 0.5$ is:

$$L_p = 84 + 10 \log_{10} \left(0.0354 + \frac{4 \times 0.5}{216 \times 0.5} \right) + 0.15 = 71.5 \text{ dB(A)}$$

This corresponds to a reduction of 4.1 dB.

- (d) From equation 7.42 in the text, when each machine is running separately, the fields are equal when $\frac{D_0}{4\pi r^2} = \frac{4(1 - \alpha)}{S\bar{\alpha}}$. Thus for each machine:

$$r = \sqrt{\frac{216 \times 0.15}{4 \times 4 \times \pi \times 0.85}} = 0.87 \text{ m}$$

When the machines are running together, the reverberant field contribution will be the sum of the two reverberant fields originating from each machine. The total sound power output is $10 \log_{10}(10^{8.9} + 10^{8.4}) = 90.2 \text{ dB}$. The reverberant field sound pressure level is then:

$$L_p = 90.2 + 10 \log_{10} \left(\frac{4 \times 0.85}{216 \times 0.15} \right) + 0.15 = 80.6 \text{ dB(A)}$$

Thus the required distance is that at which the direct field L_p is equal to 80.6 dB(A). For the original machine:

$$L_p = 80.6 = 84 - 10 \log_{10}(4\pi r^2) + 0.15$$

Thus $r = 0.42\text{m}$ (distance from original machine at which original machine direct field = reverberant field with both machines running).

For the new machine, $L_p = 80.6 = 89 - 10 \log_{10}(4\pi r^2) + 0.15$

Thus $r = 0.75\text{m}$ (distance from new machine at which new machine direct field = reverberant field with both machines running).

- (e) Increase α to 0.5. From equation 7.42 in the text, when each machine is running separately, the fields are equal when

$$\frac{D_0}{4\pi r^2} = \frac{4(1 - \alpha)}{S\alpha}. \text{ Thus for each machine,}$$

$$r = \sqrt{\frac{216 \times 0.5}{4 \times 4 \times \pi \times 0.5}} = 2.07\text{m}$$

The reverberant field sound pressure level when both machines are running is,

$$L_p = 90.2 + 10 \log_{10} \left(\frac{4 \times 0.5}{216 \times 0.5} \right) + 0.15 = 73.0\text{dB(A)}$$

Thus the required distance is that at which the direct field L_p is equal to 73.0dB(A). For the original machine,

$$L_p = 73.0 = 84 - 10 \log_{10}(4\pi r^2) + 0.15$$

Thus $r = 1.02\text{m}$ (distance from original machine at which original machine direct field = reverberant field with both machines running).

For the new machine,

$$L_p = 73.0 = 89 - 10 \log_{10}(4\pi r^2) + 0.15$$

Thus $r = 1.81\text{m}$ (distance from new machine at which new machine direct field = reverberant field with both machines running).

- (f) Assumptions:

- Each machine is radiating omni-directional sound.
- Both machines exhibit similar frequency spectra.
- The frequency averaged absorption coefficient is obtained using a sound source with a frequency spectrum similar to that of the machines under test.

Problem 7.25

- (a) Room
- $10\text{m} \times 15\text{m} \times 6\text{m}$
- ,

$$S = 2(10 \times 15 + 10 \times 6 + 15 \times 6) = 600\text{m}^2.$$

$V = 10 \times 15 \times 6 = 900\text{m}^3$. The room reverberation time can be calculated using equation 7.51 in the text. Thus:

$$T_{60} = \frac{55.25 \times 900}{600 \times 343 \times 0.1} = 2.4\text{secs}$$

- (b) Room constant from equation 7.43 in the text:

$$R = \frac{600 \times 0.1}{1 - 0.1} = 66.67\text{m}^2$$

- (c) For each machine,
- $L_w = 94\text{dB}$
- . Thus for 4 new machines,
-
- $L_w = 94 + 10\log_{10}4$
- . Thus the reverberant field sound pressure level due to the 4 new machines is:

$$\begin{aligned} L_p &= L_w + 10\log_{10}(4/R) + 10\log_{10}4 + 0.15 \\ &= 94 + 10\log_{10}(4/66.7) + 10\log_{10}(4) + 0.15 \quad (87.95) \\ &= 94 - 12.22 + 6.02 + 0.15 = 88.0\text{dB} \end{aligned}$$

The existing reverberant field level is 75dB prior to installation of the new machines. Thus the total level after installation is:

$$L_p = 10\log_{10}(10^{7.5} + 10^{8.80}) = 88.2\text{dB} \quad (88.16)$$

- (d) When quiet machines are installed, the sound power and the sound pressure level contribution due to the new machines is reduced by
- 10dB
- to
- 94
- and
- 78.0
- respectively. Thus the total reverberant field level after installation of quiet machines is:

$$L_p = 10\log_{10}(10^{7.5} + 10^{7.795}) = 79.7\text{dB}$$

- (e) The design goal is
- 85dB
- . Thus the required reduction due to ceiling tile if untreated machines are used is:

$$\Delta L_p = 88.16 - 85 = 3.16\text{dB} = 10\log_{10}(R_f/R_i)$$

Thus $R_f/R_i = 10^{3.16/10} = 2.07$. The initial room constant is 66.67m^2 . Thus the new requirement is $R = 138\text{m}^2$. The required average absorption coefficient is then given by $600 \times \bar{\alpha} = 138 - 138 \times \bar{\alpha}$. Thus $\bar{\alpha} = 0.187$. To achieve this, let x square metres of room surface be covered by tile. Then:

$$0.187 = \frac{0.1(600 - x) + 0.6x}{600}$$

which gives $x = 105\text{m}^2$. Area of ceiling = $10 \times 15 = 150\text{m}^2$. So covering the ceiling with ceiling tile would be adequate. This would cost $\$50 + 3 \times 150 = \500 which is much less than the machine noise control and is thus the preferred option.

Problem 7.26

- (a) Distance at which direct and reverberant fields are equal is given by:

$$\frac{D}{4\pi r^2} = \frac{4(1 - \bar{\alpha})}{S\bar{\alpha}} \quad \text{or} \quad r = \sqrt{\frac{DS\bar{\alpha}}{16\pi(1 - \bar{\alpha})}}$$

The room surface area, S , is given by

$$S = 2(20 \times 25 + 20 \times 5 + 25 \times 5) = 1450\text{m}^2$$

$$\text{So } r = \sqrt{\frac{2 \times 1450 \times 0.1}{16\pi \times 0.9}} = 2.5\text{m}$$

- (b) In this case, $D=4$ instead of 2 and $r = 2.53 \times 2^{1/2} = 3.6\text{m}$
- (c) If the ceiling were covered with ceiling tiles, the new sabine absorption coefficient would be:

$$\bar{\alpha} = \frac{(1450 - 500) \times 0.1 + 500 \times 0.5}{1450} = 0.238$$

$$\text{So } r = \sqrt{\frac{2 \times 1450 \times 0.238}{16\pi \times 0.762}} = 4.2\text{m}$$

- (d) The operator is only 0.5m from the machine so he/she is in the direct field. The reverberant field contribution at this distance is small so the ceiling tiles will have only a very small effect. Could calculate this (but not necessary).

$$\begin{aligned}
 L_{p1} - L_{p2} &= 10\log_{10}\left(\frac{D}{4\pi r^2} + \frac{4(1 - \bar{\alpha}_1)}{S\bar{\alpha}_1}\right) - 10\log_{10}\left(\frac{D}{4\pi r^2} + \frac{4(1 - \bar{\alpha}_2)}{S\bar{\alpha}_2}\right) \\
 &= 10\log_{10}\left(\frac{2}{4\pi \times 0.5^2} + \frac{4 \times 0.9}{1450 \times 0.1}\right) - 10\log_{10}\left(\frac{2}{4\pi \times 0.5^2} + \frac{4 \times 0.762}{1450 \times 0.238}\right) \\
 &= -1.795 + 1.901 = 0.1 \text{ dB}
 \end{aligned}$$

Problem 7.27

- (a) Room $5\text{m} \times 5.5\text{m} \times 3\text{m}$,

$$S = 2(5 \times 5.5 + 5 \times 3 + 5.5 \times 3) = 118\text{m}^2.$$

$$V = 5 \times 5.5 \times 3 = 82.5\text{m}^3. \text{ Perimeter, } L = 4(5.5 + 5 + 3) = 54\text{m}.$$

Assuming that measurements of L_p are made far enough from the machine that the direct field is negligible compared to the reverberant field, the sound power level is given by:

$$L_w = L_p - 10\log_{10}(4/R) - 0.15$$

This allows the following table to be constructed.

Octave band centre frequency (Hz)	L_p	$\bar{\alpha}$	R	L_w
63	75	0.01	1.192	69.6
250	85	0.02	2.408	82.6
1000	84	0.02	2.408	81.6
4000	70	0.03	3.649	69.5
Overall				85.4

- (b) The reverberation time may be calculated using equation 7.51 in the text and the modal density may be calculated using equation 7.21. Equations

7.23 and 7.24 are then used to calculate the modal overlap. The following table may be constructed.

Octave band centre frequency (Hz)	$\bar{\alpha}$	T_{60}	Δf	$\frac{dN}{df}$	M
63	0.01	11.26	0.195	0.22	0.04
250	0.02	5.63	0.391	2.0	0.79
1000	0.02	5.63	0.391	27.3	10.6
4000	0.03	3.75	0.590	417.4	246.3

The reverberation time may be assumed constant between 250 and 1000Hz, so we need to calculate the frequency where $dN/df = 3/0.391 = 7.67$. That is:

$$2.569 \times 10^{-5} f^2 + 1.575 \times 10^{-3} f + 0.0197 = 7.673$$

Thus:

$$\begin{aligned}
 f &= -\frac{1.575 \times 10^{-3}}{2 \times 2.569 \times 10^{-5}} \\
 &\quad + \frac{\sqrt{(1.575 \times 10^{-3})^2 + 4 \times 2.569 \times 10^{-5} \times 7.673}}{2 \times 2.569 \times 10^{-5}} \\
 &= -31 + 547 = 516 \text{ Hz}
 \end{aligned}$$

Thus the modal overlap is greater than 3 for frequencies above 520Hz.

- (c) The effect of acoustic tile may be calculated using equation 7.42. The quantity L_w will remain the same in each case so we need to calculate the change in the second term in the equation.

Distance from room corner to centre is given by:

$$d = \sqrt{\left(\frac{5.5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = 4.01 \text{ m}$$

Assuming that the acoustic centre of the source is within a quarter of a wavelength from the hard corner, $D_\theta = 8$ and so

$\frac{D_\theta}{4\pi r^2} = \frac{8}{4\pi \times 4.01^2} = 0.0396$. The following two tables (one with no tile and the other with 25m² of acoustic tile) may be constructed.

Octave band centre frequency (Hz)	$\bar{\alpha}$ (no tile)	R	$\frac{D_\theta}{4\pi r^2} + \frac{4}{R}$	$10 \log_{10} \left(\frac{D_\theta}{4\pi r^2} + \frac{4}{R} \right)$
63	0.01	1.192	3.4	5.3
250	0.02	2.408	1.7	2.3
1000	0.02	2.408	1.7	2.3
4000	0.03	3.649	1.14	0.6

In the following table (which includes the effects of acoustic tile, the overall absorption coefficient is calculated using equation 7.78 in the text.

Octave band centre frequency (Hz)	$\bar{\alpha}$ (wall)	$\bar{\alpha}$ (tile)	$\bar{\alpha}$ (overall)	R	$\frac{D_\theta}{4\pi r^2} + \frac{4}{R}$	$10 \log_{10} \left(\frac{D_\theta}{4\pi r^2} + \frac{4}{R} \right)$	Improvement (dB)
63	0.01	0.08	0.025	3.0	1.37	1.4	3.9
250	0.02	0.15	0.048	5.95	0.72	-1.5	3.7
1000	0.02	0.20	0.058	7.27	0.59	-2.3	4.6
4000	0.03	0.25	0.077	9.84	0.45	-3.5	4.1

The noise reductions in each octave band are given by the last column in the above table.

- (d) Assuming that the overall space average level will be reduced by the same amount as the level at the centre of the room, the difference between new and old overall levels is:

$$\begin{aligned}
 & 10 \log_{10} \{ 10^{7.5} + 10^{8.5} + 10^{8.4} + 10^7 \} \\
 & - 10 \log_{10} \{ 10^{(7.5 - 0.39)} + 10^{(8.5 - 0.38)} + 10^{(8.4 - 0.46)} + 10^{(7 - 0.41)} \} \\
 & = 87.8 - 83.7 = 4.1 \text{ dB}
 \end{aligned}$$

Problem 7.28

- (a) Using equation 6.13 in the text and substituting in the appropriate values gives:

$$L_w = 95 - 8.3 + 22.5 + 0.2 - 13.9 = 95.5 \text{ dB re } 10^{-12} \text{ W}$$

- (b) Using equation 7.42 in the text, (and allowing for $\rho c \neq 400$) we can write:

$$\begin{aligned} L_p &= 95.5 + 10 \log_{10} \left(\frac{2}{4\pi \times 0.5^2} + \frac{4(1 - 0.022)}{193.2 \times 0.022} \right) + 0.15 \\ &= 97.6 \text{ dB re } 20 \mu\text{Pa} \end{aligned}$$

Problem 7.29

- (a) Room $6.84\text{m} \times 5.565\text{m} \times 4.72\text{m}$,
 $S = 2(6.84 \times 5.565 + 6.84 \times 4.72 + 5.565 \times 4.72) = 193.2\text{m}^2$.
 $V = 6.84 \times 5.565 \times 4.72 = 179.7\text{m}^3$. The room reverberation time can be calculated using equation 7.51 in the text. Thus:

$$T_{60} = \frac{55.25V}{Sc\bar{\alpha}} = \frac{55.25 \times 179.7}{193.2 \times 343 \times \bar{\alpha}} = \frac{0.150}{\bar{\alpha}}$$

The results for each third octave band are given in the following table.

One third octave band centre frequency (Hz)	$\bar{\alpha}$	T_{60}
63	0.010	15.0
80	0.010	15.0
100	0.011	13.6
125	0.011	13.6
160	0.013	11.5
200	0.015	10.0
250	0.017	8.8
315	0.017	8.8
400	0.018	8.3
500	0.018	8.3
630	0.019	7.9
800	0.020	7.5
1000	0.022	6.8
1250	0.025	6.0
1600	0.028	5.4
2000	0.031	4.8
2500	0.034	4.4
3150	0.037	4.0
4000	0.040	3.7
5000	0.044	3.4
6300	0.047	3.2
8000	0.050	3.0

- (b) Lowest 1/3 octave band given by $V = 4.6\lambda^3$ (see p259 in text). Thus:

$$\lambda = (179.7/4.6)^{1/3} = 3.393 \text{ m}$$

This corresponds to a frequency, $f = 343/3.393 = 101\text{Hz}$. Thus the room is suitable for measurements down to and including the 100Hz 1/3 octave band.

- (c) Lowest octave band given by $V = 1.3\lambda^3$ (see p259 in text). Thus:

$$\lambda = (179.7/1.3)^{1/3} = 5.171 \text{ m}$$

This corresponds to a frequency, $f = 343/5.171 = 66\text{Hz}$. Thus the room may just be suitable for measurements down to and including the 63Hz

octave band.

- (d) For pure tone noise, the lowest acceptable frequency is given by $f = (T_{60}/V)^{1/2}$ (see p259 in text). We need to solve for the frequency by trial and error as illustrated in the table below.

One third octave band centre frequency (Hz)	T_{60}	corresponding lowest acceptable frequency
63	15.0	578
80	15.0	578
100	13.6	550
125	13.6	550
160	11.5	506
200	10.0	472
250	8.8	443
315	8.8	443
400	8.3	430
500	8.3	430
630	7.9	419

From the table, it can be seen that the lowest acceptable frequency for tonal noise is 430Hz.

Problem 7.30

- (a) $L_w = 92.5\text{dB}$, $L_{pr} = 87\text{dB}$ for reference source. The room constant is calculated using only the reverberant part of equation 7.42 in the text and allowing for $\rho c = 413.6$. Thus:

$$L_w = L_p - 10\log_{10}(4/R) + 10\log_{10}(400/\rho c)$$

$$92.5 = 87 - 10\log_{10}(4/R) - 0.15$$

$$(4/R) = 10^{(87 - 92.5 - 0.15)/10} = 0.272$$

and so $R = 14.7\text{m}^2$.

- (b) Existing L_p is 81dB(A). Allowed total $L_p = 85$ dB(A). 4 new machines, so allowed L_p from each new machine is:

$$\begin{aligned} L_p &= 10 \log_{10} (10^{8.5} - 10^{8.1}) - 10 \log_{10}(4) \\ &= 76.8 \text{ dB(A) re } 20 \mu\text{Pa} \end{aligned}$$

The corresponding allowed sound power level of each machine is then:

$$\begin{aligned} L_w &= 76.77 - 10 \log_{10}(4/14.7) - 0.15 \\ &= 82.3 \text{ dB(A) re } 10^{-12} \text{ W} \end{aligned}$$

Assumptions:

- Direct field small compared to reverberant field at measurement locations.
- $\rho c = 413.6$
- Spectral content of noise from new machines is similar to that of existing noise. If not, then calculations should be done in octave bands.

- (c) Doubling room constant gives the allowed sound power level of

$$L_w = 76.77 - 10 \log_{10}(4/29.4) - 0.15 = 85.3 \text{ dB(A) re } 10^{-12} \text{ W}$$

Same assumptions as for part (b).

Problem 7.31

Room $15\text{m} \times 15\text{m} \times 5\text{m}$,

$$S = 2(15 \times 5 \times 2 + 15 \times 15) = 750 \text{ m}^2.$$

Using equations 7.42 and 7.43, the noise reduction is given by:

$$10 \log_{10} \left(\frac{D_\theta}{4\pi r^2} + \frac{4(1 - \bar{\alpha}_1)}{S\bar{\alpha}_1} \right) - 10 \log_{10} \left(\frac{D_\theta}{4\pi r^2} + \frac{4(1 - \bar{\alpha}_2)}{S\bar{\alpha}_2} \right)$$

The mean absorption coefficient before treatment is calculated using equation 7.78 and is:

$$\bar{\alpha}_1 = \frac{15 \times 15 \times 0.01 + 525 \times 0.1}{750} = 0.073$$

and after treatment it is:

$$\bar{\alpha}_2 = \frac{15 \times 15 \times 0.01 + 525 \times 0.7}{750} = 0.493$$

Assuming that the acoustic centre of the machine is within a quarter of a wavelength of the floor, $D_\theta = 2$; thus the noise reduction at the specified location is:

$$\begin{aligned} \Delta L_p &= 10 \log_{10} \left(\frac{2}{4 \times \pi \times 16} + \frac{4(1 - 0.073)}{750 \times 0.073} \right) \\ &\quad - 10 \log_{10} \left(\frac{2}{4 \times \pi \times 16} + \frac{4(1 - 0.493)}{750 \times 0.493} \right) \\ &= -11.1 + 18.1 = 7.0 \text{ dB} \end{aligned}$$

Problem 7.32

Room: $L_w = 105 \text{ dB}$, $V = 100 \text{ m}^3$, $S = 130 \text{ m}^2$, $T_{60} = 1.5 \text{ secs}$.

Office: $L_w = 85 \text{ dB}$, $V = 80 \text{ m}^3$, $S = 100 \text{ m}^2$, $T_{60} = 0.75 \text{ secs}$.

Partition area: 15 m^2 .

For the room and the office, the quantity $S\bar{\alpha}$ is calculated using equation 7.51 in the text and the reverberant sound pressure level existing in each space is calculated by combining equations 7.42 and 7.43 with the direct field term of equation 7.42 omitted. Thus the following table may be constructed.

	Room	Office
$S\bar{\alpha}$	10.739	17.182
$\bar{\alpha}$	0.0826	0.1718
$L_p = L_w + 10 \log_{10} \left(\frac{4(1 - \bar{\alpha})}{S\bar{\alpha}} \right) + 0.15$	100.5dB	78.0dB

Allowable level in office = $78 + 1 = 79\text{dB}$. Allowable level due to new machine is:

$$L_p = 10 \log_{10}(10^{7.9} - 10^{7.8}) = 72.1 \text{ dB}$$

The level in the room is 100.5dB , so the noise reduction required of the wall is $100.5 - 72.1 = 28.4\text{dB}$ which should be rounded up to 30dB for specification purposes.

Problem 7.33

- (a) Room volume $V = 30\text{m}^3$, surface area, $S = 50\text{m}^2$
Reverberant field mean square pressure is:

$$\langle p^2 \rangle = 4 \times 10^{-10} \times 10^{116/10} = 159.2 \text{ Pa}^2$$

Using equation 7.41 in the text we obtain:

$$\frac{1 - \bar{\alpha}}{\bar{\alpha}} = \frac{159.2 \times 50}{4 \times 1.206 \times 343 \times 1} = 4.812$$

Thus $\bar{\alpha} = 0.172$

- (b) Using equation 7.51 in the text:

$$T_{60} = \frac{55.25 \times 30}{343 \times 50 \times 0.172} = 0.56 \text{ secs}$$

Thus, $T_{10} = 0.56/6 = 0.094\text{secs}$.

- (c) New $\bar{\alpha}$ is given by:

$$\bar{\alpha} = \frac{10 \times 0.8 + 40 \times 0.172}{50} = 0.298$$

Using equation 7.41, we obtain for the new mean square sound pressure:

$$\langle p^2 \rangle = \frac{4 \times 1 \times 1.206 \times 343 \times (1 - 0.298)}{50 \times 0.298} = 77.96 \text{ Pa}^2$$

The corresponding sound pressure level is then:

$$L_p = 94 + 10 \log_{10}(77.96) = 112.9 \text{ dB}$$

which corresponds to a reduction of $116 - 112.9 = 3.1 \text{ dB}$.

- (d) Adding 3 more sources increases the existing number by a factor of 4. Providing all sources produce the same sound pressure level, the increase in total sound pressure level over that corresponding to one source would be $\Delta L_p = 10 \log_{10}(4) = 6 \text{ dB}$.

Problem 7.34

porous acoustic fibrous material (fibreglass, rockwool, ceramic fibre, steel wool):

Absorption

mechanisms: viscous friction losses due to difference in velocities of air particles adjacent to fibres and those in the centre of the gap between fibres.

Absorption

characteristics: good at mid to high frequencies but at low frequencies need a large thickness of material to be effective.

Applications: air handling duct mufflers, pipe lagging, reverberant space absorption.

Advantages: Inexpensive, high absorption coefficient in mid to high frequency range.

Disadvantages: fibre loss can be a health hazard so care has to be taken to properly contain the material; susceptible to powdering in the presence of vibration; not oil, water or chemical resistant.

Porous plastic and rubber (polyurethane foam etc.):

Absorption

mechanisms: viscous friction losses due to difference in velocities of air particles adjacent to capillary walls and those in the centre of the capillaries.

Absorption

characteristics:	good at mid to high frequencies but at low frequencies need a large thickness of material to be effective.
Applications:	vehicle cabins, pipe lagging, reverberant space absorption.
Advantages:	High absorption coefficient in mid to high frequency range; no health risk due to fibres.
Disadvantages:	not oil, water or chemical resistant; expensive, fire and smoke hazard; will not tolerate high temperatures.

Helmholtz resonators:

Absorption mechanisms:	viscous friction losses around neck of resonator associated with large air particle motion in the centre of the neck and zero motion at the walls of the neck at resonance.
Absorption characteristics:	good in a narrow band of frequencies around the design frequency.
Applications:	electrical transformer enclosures, vehicle mufflers, mufflers for tonal noise generated by large industrial fans.
Advantages:	can be made to be immune to moisture, oil and chemicals. Can withstand high temperatures; no health risk.
Disadvantages:	Narrow frequency range of effective absorption.

Resonant panels

Absorption mechanisms:	panel and backing cavity damping losses.
Absorption characteristics:	good in a narrow band of frequencies around the design frequency.
Applications:	electrical transformer enclosures, auditoria, concert halls, reverberant spaces.
Advantages:	can be made to be immune to moisture, oil and chemicals. Can withstand high temperatures; no health risk.
Disadvantages:	Narrow frequency range of effective absorption.

Problem 7.35

We require $\bar{\alpha} = 0.8$ at 125Hz. Referring to figure 7.8 in the text (p.307) it is clear that we need to use curve "D" which implies the use of sound absorbing material behind the panel. From figure 7.9, it can be seen that the 125Hz line crosses curve "D" when the cavity depth is 110mm and the panel mass is 2kg/m^2 . Thus this is the required design. Note that the guide notes in the caption of figure 7.8 should also be included as design specifications.

Problem 7.36

Following equation 7.78 in the text:

$$\begin{aligned}\bar{\alpha} &= \frac{\sum S_i \bar{\alpha}_i}{\sum S_i} \\ &= \frac{2 \times 6.84 \times 5.565 \times 0.02 + 2 \times 5.565 \times 4.72 \times 0.05}{2 \times 6.84 \times 5.565 + 2 \times 5.565 \times 4.72 + 2 \times 6.84 \times 4.72} \\ &= 8.023/193.2 = 0.042\end{aligned}$$

Problem 7.37

Room volume = $\pi \times 7^2 \times 2.5 = 384.85\text{m}^3$. The optimum reverberation times are calculated using equation 7.121 in the text with $K = 5$. The calculated values are listed in the following table.

Octave band centre frequency (Hz)	T_{60}
125	1.45
250	1.06
500	0.96
1000	0.96
2000	0.96
4000	0.96
8000	0.96

- (b) Using equation 7.51 in the text, the recommended $S\bar{\alpha}$ is 65m^2 at 500Hz and above, 59m^2 at 250Hz and 43m^2 at 125Hz.
- (c) Area ceiling = $\pi \times 49 = 154\text{m}^2$. At 125Hz, the Sabine absorption coefficient of tile is 0.2. Thus if all the ceiling were covered the maximum $S\bar{\alpha}$ would be $154 \times 0.2 = 30.8\text{m}^2$ (assuming that the floor and walls contributed a negligible amount). Alternatively, if it is assumed that the floor and walls are of concrete with $\bar{\alpha} = 0.01$, then the total $\bar{\alpha} = 30.8 + 0.01 \times (154 + 2\pi \times 7 \times 2.5) = 33.4\text{m}^2$. The recommended $S\bar{\alpha}$ at 125Hz is 43m^2 (from part (b)), so the ceiling tile would NOT be adequate.
- (d) A compromise would be to use sufficient tiles to achieve as closely as possible the required absorption over the range 500 to 1000Hz and then design a panel absorber with a maximum absorption at 125Hz and no absorption at 500Hz. The required absorption in the range 500 to 4000Hz is 65m^2 . Thus the optimum amount of tile is $65/0.8 = 81\text{m}^2$, which will be OK at 500 and 2000Hz, a bit much at 1000Hz and a bit little at 4000Hz, but nevertheless a good compromise. We are then left with an area of ceiling of $154 - 81 = 73\text{m}^2$ for panel absorbers. The amount of absorption needed at 125Hz is $43 - 3 - 81 \times 0.2 = 24\text{m}^2$. So the panel absorber must have an absorption coefficient of $24/73 = 0.33$ at 125Hz and nothing at 500Hz.

Choosing curve H from figure 7.8 in the text and allowing for the fact that the 125Hz band includes the peak, the average $\bar{\alpha}$ for the 125Hz band is approximately 0.35. The absorption coefficient of the panel at 500 and above is likely to be 0.08 and at 250Hz it will be 0.12. To optimise the required absorption coefficients, we can vary the relative areas. The area of floor and walls is 264m^2 . Thus the amount of absorption due to the floor and walls is 2.6m^2 in the 125, 250 and 500Hz bands and 5.3m^2 in the 1000, 2000 and 4000Hz octave bands. Including this and with an area of 70m^2 of tile and 65m^2 of panel, the amount of absorption in the octave bands from 125Hz to 4000Hz is 41, 53, 64, 70, 67, and 63m^2 which is a little low at 125Hz and 250Hz (optimum is 43 and 59 respectively) and a little high in the other bands (optimum at 500Hz and higher frequencies is 62m^2). Choosing the area of panel = 75m^2 and the area of tile = 65m^2 , gives the following amounts of absorption: 42, 51, 61, 67, 63, 60m^2 which is close enough. Note that there are many other adequate solutions to this problem.

Problem 7.38

- (a) sound power of the source is given by:

$$L_w = L_p + 10\log_{10}(2\pi r^2) - 0.15 = L_p + 21.85$$

Thus the following table may be constructed.

Octave band centre frequency (Hz)	63	125	250	500	1k	2k
L_p (dB re 20 μ Pa)	90	85	78	73	70	65
L_w (dB re 10 ⁻¹² W)	112	107	100	95	92	87

- (b) Room dimensions 5m
- \times
- 3m
- \times
- 2m. Using equation 7.17 in the text, we have:

$$f_{1,0,0} = \frac{343}{2} \times \frac{1}{5} = 34.3 \text{ Hz}$$

$$f_{0,1,0} = \frac{343}{2} \times \frac{1}{3} = 57.2 \text{ Hz}$$

$$f_{0,0,1} = \frac{343}{2} \times \frac{1}{2} = 85.8 \text{ Hz}$$

$$f_{2,0,0} = \frac{343}{2} \times \frac{2}{5} = 68.6 \text{ Hz}$$

$$f_{1,1,0} = \frac{343}{2} \times \sqrt{(1/5)^2 + (1/3)^2} = 66.7 \text{ Hz}$$

Thus the 3 lowest order modes are in the 31.5Hz and 63Hz third octave bands.

Room volume, $V = 5 \times 3 \times 2 = 30\text{m}^3$

Area, $S = 2(5 \times 3 + 5 \times 2 + 3 \times 2) = 62\text{m}^2$.

Perimeter, $L = 4(5 + 2 + 3) = 40$

The modal density is given by equation 7.21 in the text. Thus:

$$\frac{dN}{df} = \frac{4 \times \pi \times 125^2 \times 30}{343^3} + \frac{\pi \times 125 \times 62}{2 \times 343^2} + \frac{40}{8 \times 343} = 0.264$$

From table 1.2 on p43 in the text, the 125Hz third octave bandwidth is $141 - 113 = 28\text{Hz}$, so the number of modes in the band is $28 \times 0.264 = 7$ to 8 modes.

Alternatively equation 7.20 could be used to calculate the number of modes occurring below 141Hz and 113Hz and then take the difference. The number of modes below 141Hz is:

$$N = \frac{4 \times \pi \times 141^3 \times 30}{3 \times 343^3} + \frac{\pi \times 141^2 \times 62}{4 \times 343^2} + \frac{40 \times 141}{8 \times 343} = 19 \text{ modes}$$

The number of modes below 113Hz is:

$$N = \frac{4 \times \pi \times 113^3 \times 30}{3 \times 343^3} + \frac{\pi \times 113^2 \times 62}{4 \times 343^2} + \frac{40 \times 113}{8 \times 343} = 11.4 \text{ modes}$$

Thus the number in the 125Hz octave band is between 7 and 8 modes.

- (c) Equations 7.51, 7.43 and 7.42 (with a correction for $\rho c = 413.6$) in the text, and the knowledge that $S = 62\text{m}^2$, may be used to construct the following table.

Octave band centre frequency (Hz)	63	125	250	500	1k	2k	Overall
L_w (dB re 10^{-12}W)	112	107	100	95	92	87	
T_{60}	5.5	5	4	3	2	1.5	
$S\bar{\alpha}$	0.877	0.964	1.206	1.608	2.411	3.215	
$\bar{\alpha}$	0.0141	0.015 6	0.019 4	0.025 9	0.038 9	0.0519	
$R = \frac{S\bar{\alpha}}{1 - \bar{\alpha}}$	0.890	0.979	1.230	1.650	2.509	3.391	
$L_p(\text{reverb})$	118.7	113.3	105.3	99.0	94.2	87.9	
A-weighting	-26.2	-16.1	-8.6	-3.2	0.0	1.2	
L_p (dB(A))	92.5	97.2	96.7	95.8	94.2	89.1	102.8

Assumptions:

- A-weighting assumed uniform across each octave band when in fact it varies continuously with frequency.
 - Direct field contribution assumed negligible.
 - Reflections from and absorption of surfaces other than room boundaries is excluded.
- (d) If 2 more generators were added, the sound pressure level would increase by $10 \log_{10}(2 + 1) = 4.8 \text{ dB(A)}$.
- (e) Ceiling tiles added with area = 15 m^2 . Remaining room surface area = $62 - 15 = 47 \text{ m}^2 = S_{\text{floor, walls}}$. The following table may be constructed, where:

$$\bar{\alpha}_{\text{overall}} = \bar{\alpha}_{\text{new}} = \frac{S \bar{\alpha}_{\text{floor walls}} + S \bar{\alpha}_{\text{ceiling}}}{S_{\text{floor walls}} + S_{\text{ceiling}}}.$$

Octave band centre frequency (Hz)	63	125	250	500	1k	2k	Over-all
L_w (dB re 10^{-12} W)	112	107	100	95	92	87	
$S \bar{\alpha}_{\text{floor walls}}$	0.66	0.73	0.91	1.22	1.83	2.44	
$\bar{\alpha}_{\text{ceiling}}$	0.15	0.25	0.55	0.85	1.0	1.0	
$S \bar{\alpha}_{\text{ceiling}}$	2.25	3.75	8.25	12.75	15	15	
$\bar{\alpha}_{\text{overall}} = \bar{\alpha}_{\text{new}}$	0.0469	0.0722	0.1477	0.2253	0.2715	0.2813	
$R_{\text{new}} = \frac{S \bar{\alpha}_{\text{new}}}{1 - \bar{\alpha}_{\text{new}}}$	3.05	4.83	10.75	18.03	23.10	24.27	
L_p (reverb)	113.3	106.3	95.9	88.6	84.5	79.3	114.2
A-weighting	-26.2	-16.1	-8.6	-3.2	0.0	1.2	
L_p (dB(A))	87.1	90.2	87.3	85.4	84.5	80.5	94.5

Similar assumptions as made in part (c).

Problem 7.39

Room $20\text{m} \times 15 \times 4\text{m}$,

$$S = 2(20 \times 15 + 20 \times 4 + 15 \times 4) = 880\text{m}^2.$$

$$V = 20 \times 15 \times 4 = 1200\text{m}^3.$$

The desired reverberation times are calculated using equation 7.121 on p. 329 in the text, with 10% increase at 250Hz, 50% increase at 125Hz and 100% increase at 63Hz. The existing mean statistical absorption coefficient may be calculated using equation 7.56 in the text rearranged to give:

$$\alpha_{st} = 1 - e^{-55.25V/ScT_{60}}$$

Thus the following table may be constructed.

Octave band centre frequency (Hz)	Existing T_{60}	Existing mean α_{st}	Desired T_{60}	Desired mean α_{st}	Required increase in α_{st}
63	3.0	0.0706	1.86	0.111	0.040
125	2.6	0.0810	1.40	0.145	0.064
250	2.3	0.0911	1.02	0.194	0.103
500	2.1	0.0993	0.93	0.211	0.112
1000	2.0	0.1040	0.93	0.211	0.107
2000	2.0	0.1040	0.93	0.211	0.107
4000	2.0	0.1040	0.93	0.211	0.107
8000	1.8	0.1149	0.93	0.211	0.096

There are many possible solutions to achieve the desired mean absorption coefficients. One alternative is to look for the frequency at which the additional absorption required is the largest and choose a material which has a maximum absorption coefficient at this frequency. In this case, the maximum increase in absorption is needed at 500Hz, so a material thickness of 25mm should be chosen. The amount of material required is then calculated on the basis of achieving the optimum reverberation time in the octave bands most important for speech; namely, 500Hz to 2000Hz. In this case, it would seem that the required increase in mean absorption coefficient is 0.107 which would satisfy the requirements at and above 1000Hz, with a compromise of a little less than desired at 500Hz. let x be the fraction of room surface area to be covered with absorbing material. Using equation 7.58, we

have:

$$(1 - 0.211) = (1 - 0.85)^x \times (1 - 0.104)^{(1-x)}$$

Taking logs of both sides, we obtain:

$$\log_{10}(0.79) = x \log_{10}(0.15) + (1-x) \log_{10}(0.896)$$

which gives $x = 0.0711$. So the required area of 25mm thick acoustic material is 63m^2 .

Problem 7.40

Assume that the house is approximately in a direction along the normal axis from the window. The power incident on the window is the intensity in the direction of the window multiplied by the area of the window. Thus

$$W = \frac{\langle p_i^2 \rangle S}{4\rho c}$$

The power radiated through the window is then

$$W = \frac{\tau \langle p_i^2 \rangle S}{4\rho c}$$

where $\tau = 10^{-TL/10} = T 10^{-2.7} = 1.995 \times 10^{-3}$ the reverberant sound pressure level is 88dB. Thus

$$\langle p_i^2 \rangle = 4 \times 10^{-10} \times 10^{88/10} = 0.252 \text{Pa}^2$$

The power radiated through the window is then

$$W = \frac{1.995 \times 10^{-3} \times 0.252 \times 1.5}{4 \times 1.206 \times 343} = 0.457 \mu\text{W}$$

For an incoherent plane source, the on-axis sound pressure at the receiver is given by equation 5.106. The quantity $r/\sqrt{HL} = 60/1.5 = 40$. Thus from figure 5.11, it is clear that we can treat it as a hemispherically radiating point source producing a sound pressure described by equation 5.106 in the text. Thus:

$$\langle p^2 \rangle = \frac{1.206 \times 343 \times 4.565 \times 10^{-7}}{2 \times \pi \times 60^2} = 8.349 \times 10^{-9} \text{Pa}^2$$

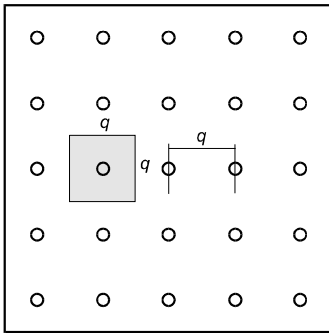
This corresponds to a sound pressure level of :

$$L_p = 10 \log_{10}(8.349 \times 10^{-9}) + 94 = 13.2 \text{ dB re } 20 \mu\text{Pa}$$

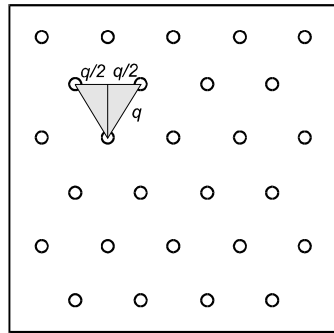
As the ground is hard asphalt, we may add 3dB to the level to account for the effect of ground reflection. Thus the expected level at the house is 16dB.

Problem 7.41

- (a) First calculate distance between holes. Could assume parallel or staggered holes as shown in the two figures to follow.



parallel holes



staggered holes

Let q be the distance between holes as shown in the figures. Choosing a segment of plate as shown in the figure we can calculate the ratio of holes to total area of the segment and set this equal to 0.07. This gives a hole spacing for the parallel holes of:

$$q = 1000 \times \sqrt{\frac{\pi \times 0.002^2}{4 \times 0.07}} = 6.7 \text{ mm}$$

and for the staggered holes,

$$q = 1000 \times \sqrt{\frac{\pi \times 0.002^2}{2 \times 0.07 \times 2\sqrt{3}}} = 5.1 \text{ mm}$$

For the purposes of this problem we will use $q = 6.7 \text{ mm}$. Using equation E.7 in the 2nd. edn. of the text on p528, we have:

$$f_{\max} \left(\frac{2 \times \pi \times 0.1}{343} \right) \tan \left(f_{\max} \times \frac{2 \times \pi \times 0.1}{343} \right)$$

$$= \frac{7 \times 0.1 / 100}{0.003 + 0.85 \times 0.002 \times (1 - 0.22 \times 0.002 / 0.0067)}$$

Rewriting gives:

$$0.00183 \times f_{\max} \tan(f_{\max} \times 0.00183) = 1.5256$$

Can solve by trial and error, choosing values of f_{\max} until the LHS = 1.5256 as illustrated in the table below.

$f_{\max}(\text{Hz})$	LHS
100	0.0339
500	1.1892
600	2.1467
550	1.5902
540	1.5000
543	1.5263
542.9	1.5254

Thus the frequency of maximum absorption is 543Hz.

If we used Equation 7.77 in the 3rd edn. we would get 691 Hz, but the error is greater than 15% because the condition, $fL/c < 0.1$ is not satisfied.

- (b) Specific normal impedance is given by equation C.43. To evaluate this equation we need to use equation C.41 and to evaluate that we need equations C.3 and C.4. Referring to equation C.15, $X = 1.206 \times 543/10000 = 0.0655$. Thus, $T_1 = 1.63259$, $T_2 = 0.06590$, $T_3 = 3.1696$, $T_4 = 0.1663$. Referring to equation C.9:

$$a(X) = \frac{3.1696(1.63259 - 3.1696) \times 0.0659^2 - 0.1663^2 \times 1.63259^2}{3.1696^2 \times 0.0659^2 + 0.1663^2 \times 1.63259^2}$$

$$= -0.80849$$

and

$$b(X) = \frac{1.63259^2 \times 0.0659 \times 0.1663}{3.1696^2 \times 0.0659^2 + 0.1663^2 \times 1.63259^2} = 0.24893$$

$X_1 = 0.856 \times 1.206 \times 543/10000 = 0.05605$. Thus,
 $T_1 = 1.54150$, $T_2 = 0.05636$, $T_3 = 3.07850$, $T_4 = 0.16526$. Referring to equation C.9:

$$\begin{aligned} a(X_1) &= \frac{3.0785(1.5415 - 3.07850) \times 0.05636^2 - 0.16526^2 \times 1.5415^2}{3.0785^2 \times 0.05636^2 + 0.16526^2 \times 1.5415^2} \\ &= -0.84133 \end{aligned}$$

and

$$b(X_1) = \frac{1.5415^2 \times 0.05636 \times 0.16526}{3.0785^2 \times 0.05636^2 + 0.16526^2 \times 1.5415^2} = 0.23297$$

Thus $\tau = -0.84133 \times 0.592 + j0.23297$ and $\sigma = -0.80849 + j0.24893$. The quantities κ and ρ_m may be calculated using equations C.5 and C.6 as follows:

$$\kappa = (1 - 0.4 \times (-0.49807 + j0.23297))^{-1} = (1.19923 + j0.093188)^{-1}$$

and

$$\rho_m = (1 - 0.8085 + j0.2489)^{-1} = (0.1915 + j0.24893)^{-1}$$

Using equations C.3 , we obtain:

$$\begin{aligned} \frac{Z_m}{\rho c} &= \sqrt{\rho_m \kappa} \\ &= \sqrt{\frac{1}{(0.1915 + j0.24893) \times (1.19923 - j0.093188)}} \\ &= \sqrt{\frac{1}{0.25285 + j0.28068}} \\ &= \sqrt{1.7717 - j1.9667} = \sqrt{2.6471 e^{-j0.8375}} \\ &= 1.6270 e^{-j0.4188} = 1.4864 - j0.6616 \end{aligned}$$

and using equation C.4b we obtain:

$$\begin{aligned}
 k_m &= \frac{2\pi f_{\max}}{c} \sqrt{\frac{\rho_m}{\kappa}} = \frac{2\pi \times 543}{343} \sqrt{\frac{1.19923 - j0.93188}{0.1915 + j0.24893}} \\
 &= 9.9468 \sqrt{\frac{(1.19923 - j0.093188) \times (0.1915 - j0.24893)}{0.098638}} \\
 &= 9.9468 \sqrt{2.09319 - j3.2074} = 9.9468 \sqrt{3.8300 e^{-j0.99259}} \\
 &= 19.4663 e^{-j0.49629} = 17.1178 - j9.2692
 \end{aligned}$$

Before continuing, it will be useful to evaluate the quantity, $\tan(k_m \ell)$. Using the previous result for k_m , and setting $\ell = 0.1$, we can write:

$$\begin{aligned}
 j \tan(k_m \ell) &= \frac{e^{0.9269}(\cos(1.7118) + j\sin(1.7118)) - e^{-0.9269}(\cos(1.7118) - j\sin(1.7118))}{e^{0.9269}(\cos(1.7118) + j\sin(1.7118)) + e^{-0.9269}(\cos(1.7118) - j\sin(1.7118))} \\
 &= \frac{-0.2995 + j2.8934}{-0.4107 + j2.1097} = 1.3480 - j0.1205
 \end{aligned}$$

Thus:

$$\tan(k_m \ell) = -0.1205 - j1.3480$$

Using the previous results and equation C.41 (assuming a rigid backing for the porous material), we can write:

$$\begin{aligned}
 \frac{Z_N}{\rho c} &= \frac{(0.6616 + j1.4864)}{0.1205 + j1.3480} = \frac{2.0834 - j0.7127}{1.8316} \\
 &= 1.1375 - j0.3891
 \end{aligned}$$

To calculate the overall impedance, we use equation C.43, but first we need to evaluate ℓ and $\tan(k\ell)$.

From equation 9.25 in the text, the effective length of the holes in the perforated sheet is:

$$\ell = 0.003 + \frac{16 \times 0.001}{3 \times \pi} (1 - 0.43 \times 0.001 / 0.0067) = 0.004588$$

Thus:

$$\tan(k\ell) = \tan(2 \times \pi \times 543 \times 0.004588 / 343) = 0.04567$$

We also need to calculate the acoustic resistance of the holes using equation 9.29 in the text. To evaluate this equation we need the following quantities:

$$k = (2 \times \pi \times 543) / 343 = 9.9467,$$

$$A = \pi \times 0.002^2 / 4 = 3.1416 \times 10^{-6} \text{ m}^2; \quad D = \pi \times 0.002 = 0.006283$$

$$t = \sqrt{\frac{2 \times 1.8 \times 10^{-5}}{1.206 \times 2 \times \pi \times 543}} = 9.3538 \times 10^{-5}$$

$\varepsilon = 1.0$ as radiation from a baffle.

The quantity, h , is the largest of the half plate thickness or t . Thus:
 $h = w/2 = 0.0015\text{m}$.

The above quantities may be inserted into equation 9.29 to give:

$$\begin{aligned}
\frac{R_a A}{\rho c} &= \frac{9.9467 \times 9.3538 \times 10^{-5} \times 6.283 \times 10^{-3} \times 0.003}{2 \times 3.1416 \times 10^{-6}} \\
&\times \left[1 + (1.4 - 1) \sqrt{\frac{5}{3 \times 1.4}} \right] \\
&+ 0.288 \times 9.9467 \times 9.3538 \times 10^{-5} \times \log_{10} \left[\frac{4 \times 3.1416 \times 10^{-6}}{\pi \times 0.0015^2} \right] \\
&+ \frac{3.1416 \times 10^{-6} \times 9.9467^2}{2 \times \pi} \\
&= 4.00923 \times 10^{-3} + 6.69555 \times 10^{-5} + 4.9468 \times 10^{-5} \\
&= 4.1256 \times 10^{-3}
\end{aligned}$$

We can now use equation C.43 to evaluate the overall impedance. The second term on the right is the impedance due to the perforated sheet and is:

$$\begin{aligned}
\frac{Z_P}{\rho c} &= \frac{(100/7)(0.04567j + 4.1256 \times 10^{-3})}{1 + \frac{100 \times 1.206 \times 343}{2 \times \pi \times 543 \times 21.762 \times 7} \times [0.04567 - 4.1256j \times 10^{-3}]} \\
&= \frac{0.05894 + 0.6524j}{1.00363 - 3.2836j \times 10^{-4}} \\
&= 0.05894 + 0.6548j
\end{aligned}$$

Thus the total impedance is:

$$\begin{aligned}
\frac{Z_N}{\rho c} + \frac{Z_P}{\rho c} &= 1.1375 - 0.3891j + 0.05894 + 0.6548j \\
&= 1.1964 + 0.2657j = 1.2255 e^{0.21854j}
\end{aligned}$$

$$\cos\beta = 0.9762; \cos 2\beta = 0.9060; \sin\beta = 0.2168 \text{ and } \zeta = 1.2255.$$

Using the above data, the statistical absorption coefficient may be calculated using equation C.37 in the text as follows:

$$\begin{aligned}
\alpha_{st} &= \left\{ \frac{8 \times 0.9762}{1.2255} \right\} \left\{ 1 - \left[\frac{0.9762}{1.2255} \right] \right. \\
&\quad \times \log_e (1 + 2 \times 1.2255 \times 0.9762 + 1.2255^2) \\
&\quad \left. + \left[\frac{0.9060}{1.2255 \times (0.2168)} \right] \times \tan^{-1} \left[\frac{1.2255 \times (0.2168)}{1 + 1.2255 \times 0.9762} \right] \right\} \\
&= 6.373 \times (1 - 0.7966 \times 1.5881 + 3.410 \times \tan^{-1} [0.12097]) \\
&= 6.373 \times (1 - 1.2651 + 0.4105) = 0.93
\end{aligned}$$

Problem 7.42

The NRC is given by:

$$NRC = \frac{\bar{\alpha}_{250} + \bar{\alpha}_{500} + \bar{\alpha}_{1000} + \bar{\alpha}_{2000}}{4} = \frac{0.6 + 0.8 + 1.0 + 1.0}{4} = 0.85$$

So the material is adequate for the purpose.

Problem 7.43

Truck emits 110 dB. This is equal to: $W = 10^{-12} \times 10^{110/10} = 0.1$ watts.
 $r/a = 60/6 = 10$.

From Fig 7.14, reverberant field pressure squared is:

$$10 \log_{10} \langle p_R^2 \rangle - 10 \log_{10} \left[\frac{W \rho c}{\pi a^2} \right] = -4 \text{ dB}$$

$$\text{Thus, } 10 \log_{10} \langle p_R^2 \rangle = 10 \log_{10} \left[\frac{0.1 \times 413.6}{\pi \times 36} \right] - 4 = -18.4 \text{ dB}$$

so reverberant field pressure is, $L_{pR} = -18.4 + 94 = 75.6 \text{ dB}$

$$\text{Direct field pressure: } 10 \log_{10} \langle p_D^2 \rangle = 10 \log_{10} \left[\frac{0.1 \times 413.6}{4\pi \times 60^2} \right] = -30.4 \text{ dB}$$

So direct field pressure is, $L_{pD} = -30.4 + 94 = 63.6$ dB

Total pressure, $L_p = 10\log_{10}[10^{75.4/10} + 10^{63.6/10}] = 75.7$ dB

Assumptions:

- Effective acoustic source location is in the centre of the cross section
- Specularly reflecting surfaces
- Ambient temperature of 20 °C

Solutions to problems relating to sound transmission loss, acoustic enclosures and barriers

Problem 8.1

- (a) The Transmission Loss of a partition is an inverse decibel measure (bigger TL means a smaller amount of transmitted energy) of the amount of incident energy which is transmitted to the space on the side opposite that on which the energy is incident. It is defined in terms of the transmission coefficient, τ , which is the fraction of transmitted to incident energy, as follows:

$$TL = -10 \log_{10} \tau$$

It may be measured using two reverberant rooms with the panel to be measured acting as a partition between the two rooms with the space around the panel of high transmission loss construction so that all of the significant acoustic energy transmitted between the two rooms passes through the panel under test. The test is conducted by exciting the one of the reverberant rooms with 1/3 octave band noise and then measuring the difference in the space averaged sound pressure level in the two rooms. The appropriate mathematical analysis is embodied in equations 8.13 to 8.16 in the text (page 342).

- (b) Measurements often do not agree with theoretical calculations because the latter do not take into account the size of the panel exactly. Also the experimental determination of space average sound pressure level is often characterised by errors, especially at low frequencies when the sound fields in the two rooms are not sufficiently diffuse. Sometimes, energy is transmitted from one room to the next by way of paths not through the panel (called "flanking"), resulting in Transmission Loss measurements which are too small.

Problem 8.2

Mass Law Transmission Loss is obtained by combining equations 8.34 (with $\theta = 0^\circ$) and equation 8.35b, which gives:

$$TL = 10 \log_{10} \left(1 + \left[\frac{\pi f m}{\rho c} \right]^2 \right) - 5.5 \quad (\text{dB})$$

Substituting in values for the variables gives:

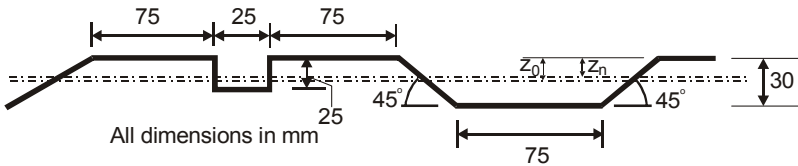
$$4 = 10 \log_{10} \left(1 + \left[\frac{\pi \times f \times 7800 \times 0.01}{988 \times 1481} \right]^2 \right) - 5.5$$

Rearranging gives:

$$f = \sqrt{\frac{10^{0.95} - 1}{2.8046 \times 10^{-8}}} = 16,800 \text{ Hz}$$

The Transmission Loss in air at this frequency is much greater because the impedance of the panel compared to the characteristic impedance in the propagating medium is much larger for air than water.

Problem 8.3



- (a) First find location of neutral axis by taking moments about an axis through the centre of the angled section and shown as z_0 in the figure. In the following equations b_i is the length of the i th section. If the neutral axis is denoted as z_n where z is the vertical coordinate on the figure, then:

$$\begin{aligned}
 z_0 - z_n &= \frac{\sum_i b_i z_{i0}}{\sum_i b_i} \\
 &= \frac{75 \times 15 + 2 \times 25 \times 2.5 - 25 \times 10 + 75 \times 15 - 75 \times 15}{75 + 3 \times 25 + 75 + 2\sqrt{2} \times 30 + 75} \\
 &= \frac{1000}{384.9} = 2.6 \text{ mm}
 \end{aligned}$$

Thus the neutral axis is 12.4mm from the centre of the top of the section. The section thickness, $h = 1.2\text{mm}$ and the horizontal length, ℓ , before repeating itself is $250 + 60\text{mm} = 0.31\text{m}$.

The bending stiffness in the direction along the ribs may be calculated with $E = 207\text{GPa}$ and $\nu = 0.3$ using equation 8.10 in the text, which is incorrect in the first printing of the text and should be:

$$B = \frac{Eh}{(1 - \nu^2)\ell} \sum_n b_n \left(z_n^2 + \frac{h^2 + b_n^2}{24} + \frac{h^2 - b_n^2}{24} \cos 2\theta_n \right)$$

Thus:

$$\begin{aligned}
 B_1 &= \frac{207 \times 10^9 \times 0.0012}{0.91 \times 0.31} \left[0.15 \left(0.0124^2 + \frac{0.0012^2}{12} \right) \right. \\
 &\quad + 0.05 \left(0.0001^2 + \frac{0.025^2}{12} \right) + 0.025 \left(0.0126^2 + \frac{0.0012^2}{12} \right) \\
 &\quad + 0.075 \left(0.0176^2 + \frac{0.0012^2}{12} \right) \\
 &\quad \left. + 0.06\sqrt{2} \left(0.0026^2 + \frac{0.0012^2 + 2 \times 0.03^2}{24} \right) \right] \\
 &= 8.805 \times 10^8 (2.3082 \times 10^{-5} + 2.6046 \times 10^{-6} + 3.9720 \times 10^{-6} \\
 &\quad + 2.3241 \times 10^{-5} + 6.9427 \times 10^{-6}) \\
 &= 5.27 \times 10^4 \text{ kg m}^2 \text{ s}^{-2}
 \end{aligned}$$

The stiffness in the direction across the ribs may be calculated using equation 8.5 as follows:

$$B_2 = \frac{207 \times 10^9 \times 0.0012^3}{12 \times 0.91} \times \frac{0.385}{0.31} = 40.7 \text{ kg m}^2 \text{ s}^{-2}$$

- (b) The bending wavespeed is calculated using equation 8.1 in the text. The surface mass of the panel is $m = 7800 \times 0.0012 \times 0.385/0.31 = 11.62 \text{ kg m}^{-2}$ and the frequency is 1000 Hz. Thus the lower and upper bending wavespeeds corresponding to waves propagating parallel and perpendicular to the ribs respectively are:

$$c_{B1} = \left(\frac{5.27 \times 10^4 \times 4\pi^2 \times 10^6}{11.62} \right)^{1/4} = 650 \text{ m/s}$$

$$c_{B2} = \left(\frac{40.7 \times 4\pi^2 \times 10^6}{11.62} \right)^{1/4} = 108 \text{ m/s}$$

- (c) The lower and upper critical frequencies for the panel may be calculated using equation 8.3 in the text. Thus:

$$f_{c1} = \frac{343^2}{2\pi} \left(\frac{11.62}{5.27 \times 10^4} \right)^{1/2} = 278 \text{ Hz}$$

$$f_{c2} = \frac{343^2}{2\pi} \left(\frac{11.62}{40.7} \right)^{1/2} = 10,000 \text{ Hz}$$

- (d) Assuming that the enclosure wall edge condition is simply supported (a good approximation in practice for most enclosures), the first resonance frequency of the panel may be calculated using equation 8.22 in the text with $i = n = 1$. Thus:

$$\begin{aligned} B_{ab} &= 0.5 \left(5.27 \times 10^4 \times 0.3 + 40.7 \times 0.3 + \frac{207 \times 10^9 \times 0.0012^3}{3 \times 2.6} \right) \\ &= 7934 \end{aligned}$$

and:

$$\begin{aligned}
 f_{1,1} &= \frac{\pi}{2\sqrt{11.62}} \left(\frac{5.27 \times 10^4}{2^4} + \frac{40.7}{2^4} + \frac{7934}{2^4} \right)^{1/2} \\
 &= 28.4 \text{ Hz}
 \end{aligned}$$

- (e) The sound transmission loss of the panel may be calculated using figure 8.8b in the text. Point A is at 139Hz and the corresponding TL is given by:

$$TL_A = 20 \log_{10}(278 \times 11.62) - 54 = 16.2 \text{ dB}$$

At point B (278Hz), the TL is:

$$\begin{aligned}
 TL_B &= 20 \log_{10}(278) + 10 \log_{10}(11.62) - 10 \log_{10}(278) \\
 &\quad - 20 \log_{10}[\log_e(4)] - 13.2 \\
 &= 19.1 \text{ dB}
 \end{aligned}$$

At point C (5,000Hz), the TL is:

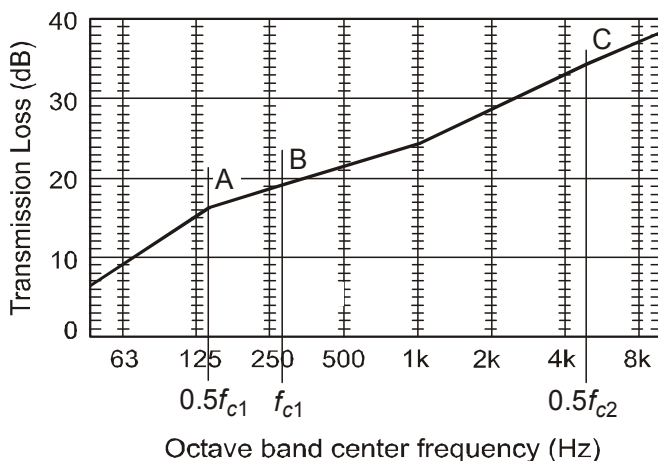
$$\begin{aligned}
 TL_C &= 20 \log_{10}(5000) + 10 \log_{10}(11.62) - 10 \log_{10}(278) \\
 &\quad - 20 \log_{10}[\log_e(20000/278)] - 13.2 \\
 &= 34.4 \text{ dB}
 \end{aligned}$$

At point D (20,000Hz), the TL is:

$$\begin{aligned}
 TL_D &= 10 \log_{10}(11.62) + 15 \log_{10}(10000) - 5 \log_{10}(278) - 17 \\
 &= 41.4 \text{ dB}
 \end{aligned}$$

These points are plotted on the following graph and interpolation is used to find the octave band TL values. Strictly speaking, the curve should only be used to find 1/3 octave values and the octave band levels must then be calculated from the following equation:

$$TL_{oct} = -10 \log_{10} \{ (1/3) [10^{-TL_1/10} + 10^{-TL_2/10} + 10^{-TL_3/10}] \}$$



However, for most practical purposes, the results obtained that way are little different to the results obtained by reading the octave band levels directly from the figure. However, in the case of isotropic panels, care should be taken to avoid errors near the dip in the curve corresponding to the critical frequency. Following the figure, the octave band results are summarised in a table.

Octave band centre frequency (Hz)	Transmission Loss (dB)
63	9
125	15
250	19
500	21
1000	25
2000	29
4000	33
8000	37

Problem 8.4

- (a) Only one half of the sine wave section needs to be considered.
 $y_1 = 20 \sin(15\pi/40) = 18.48\text{mm}.$

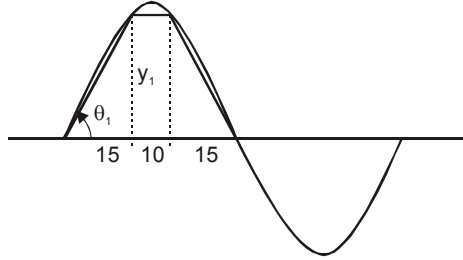
$$b_1 = 23.8\text{mm}, \theta_1 = 50.9^\circ$$

$$b_2 = 10\text{mm}, \theta_2 = 0$$

$$b_3 = 23.8\text{mm}, \theta_3 = 50.9^\circ$$

$$l = 40, E = 207\text{GPa}$$

$$\nu = 0.3$$



Using the corrected form of equation 8.10 in the text, we can write the following for the bending stiffness for waves travelling parallel to the corrugations:

$$\begin{aligned} B_1 &= \frac{207 \times 10^9 \times 0.0016}{0.91 \times 0.04 \times 10^9} \left[23.8 \times 2 \left((18.48/2)^2 + \frac{1.6^2 + 23.8^2}{24} \right. \right. \\ &\quad \left. \left. + \frac{1.6^2 - 23.8^2}{24} \cos(101.8^\circ) \right) + 10 \left(18.48^2 + \frac{1.6^2}{12} \right) \right] \\ &= 8.04 \times 10^4 \text{ kg m}^2 \text{ s}^{-2} \end{aligned}$$

The bending stiffness for waves travelling perpendicular to the corrugations can be calculated using equation 8.11 in the text as:

$$B_2 = \frac{207 \times 10^9 \times 1.6^3 \times 10^{-9}}{12 \times 0.91} \frac{57.6}{40} = 111.8 \text{ kg m}^2 \text{ s}^{-2}$$

The surface mass is $m = 7800 \times 0.0016 \times (23.8 + 23.8 + 10)/40 = 18.0 \text{ kg/m}^2$.

The lower and upper critical frequencies are calculated using equation 8.3 as:

$$f_{c1} = \frac{343^2}{2\pi} \left(\frac{18}{8.04 \times 10^4} \right)^{1/2} = 280 \text{ Hz}$$

$$f_{c2} = \frac{343^2}{2\pi} \left(\frac{18}{111.8} \right)^{1/2} = 7,500 \text{ Hz}$$

Assuming that the enclosure wall edge condition is simply supported, the first resonance frequency may be calculated using equation 8.22 in the text. Thus:

$$\begin{aligned}
 f_{1,1} &= \frac{\pi}{2\sqrt{18}} \left[\frac{8.04 \times 10^4}{3^4} + \frac{111.8}{3^4} \right. \\
 &\quad \left. + 0.5 \left(\frac{8.04 \times 10^4 \times 0.3}{3^4} + \frac{111.8 \times 0.3}{3^4} + \frac{207 \times 10^9 \times 0.0016^3}{3^4 \times 3 \times 2.6} \right) \right]^{1/2} \\
 &= 12.5 \text{ Hz}
 \end{aligned}$$

- (b) The sound transmission loss of the panel may be calculated using figure 8.8b in the text. Point A is at 140Hz and the corresponding TL is given by:

$$TL_A = 20 \log_{10}(280 \times 18.0) - 54 = 20.0 \text{ dB}$$

At point B (280Hz), the TL is:

$$\begin{aligned}
 TL_B &= 20 \log_{10}(280) + 10 \log_{10}(18.0) - 10 \log_{10}(280) \\
 &\quad - 20 \log_{10}[\log_e(4)] - 13.2 \\
 &= 21.0 \text{ dB}
 \end{aligned}$$

At point C (3,750Hz), the TL is:

$$\begin{aligned}
 TL_C &= 20 \log_{10}(3750) + 10 \log_{10}(18.0) - 10 \log_{10}(280) \\
 &\quad - 20 \log_{10}[\log_e(15000/280)] - 13.2 \\
 &= 34.4 \text{ dB}
 \end{aligned}$$

At point D (15,000Hz), the TL is:

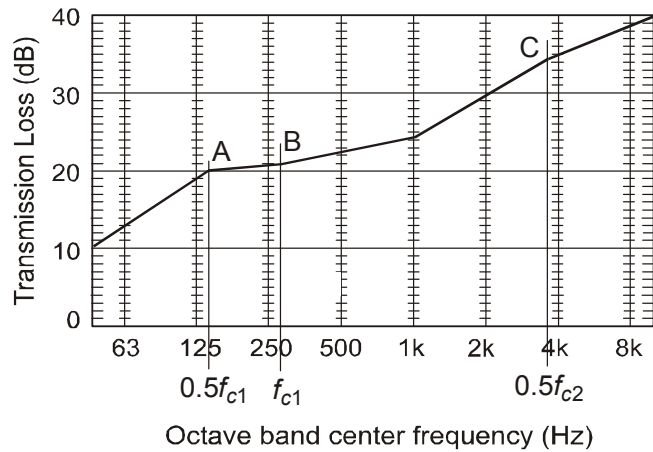
$$TL_D = 10 \log_{10}(18.0) + 15 \log_{10}(7500) - 5 \log_{10}(280) - 17 = 41.4 \text{ dB}$$

These points are plotted on the graph below and interpolation is used to find the octave band TL values. Strictly speaking, the curve should only

be used to find 1/3 octave values and the octave band levels must then be calculated from the following equation:

$$TL_{oct} = -10\log_{10}\{(1/3) [10^{-TL_1/10} + 10^{-TL_2/10} + 10^{-TL_3/10}]\}$$

However, for most practical purposes, the results obtained that way are little different to the results obtained by reading the octave band levels directly from the figure. However, in the case of isotropic panels, care should be taken to avoid errors near the dip in the curve corresponding to the critical frequency. Following the figure, the octave band results are summarised in a table.



Octave band centre frequency (Hz)	Transmission Loss (dB)
63	13
125	19
250	21
500	23
1000	26
2000	30
4000	35
8000	38

- (c) With viscoelastic damping the panel may be treated as isotropic with the surface mass, m , now equal to $2 \times 18 = 36\text{kg/m}^2$. The critical frequency is:

$$f_c = \frac{343^2}{2\pi} \left[\frac{36}{111.8} \right]^{1/2} = 10,600\text{Hz}$$

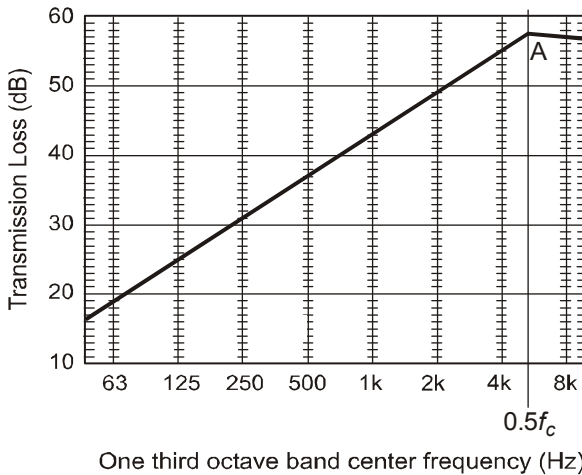
On the isotropic panel curve, point A is at 5,300Hz (10,600/2) and the TL is:

$$TL_A = 20\log_{10}(10,600 \times 36) - 54 = 57.6\text{dB}$$

At point B, the frequency is 10,600Hz and the TL is:

$$TL = 20\log_{10}(10,600 \times 36) + 10\log_{10}(0.1) - 45 = 56.6\text{dB}$$

The results are plotted on the graph below from which can be read TL values as a function of 1/3 octave band centre frequency.



- (d) Second panel, $f_{cl} = 0.55 \times 343^2 / (2000 \times 0.013) = 2500\text{Hz}$. The 1 subscript is used because this critical frequency is smaller than that for the other panel.

$$\text{Surface mass, } m = 0.013 \times 1000 = 13\text{kg/m}^2.$$

Cavity resonance frequency is:

$$f_0 = 80 \sqrt{\frac{36 + 13}{36 \times 13 \times 0.1}} = 82 \text{ Hz}$$

and the corresponding TL is:

$$TL_A = 20 \log_{10}(36 + 13) + 20 \log_{10}(81.86) - 48 = 24 \text{ dB}$$

At point B the frequency is 5300 Hz and the TL is:

$$TL_{B1} = 24 + 20 \log_{10}(2500/82) - 6 = 47.7 \text{ dB}$$

and as there are rubber spacers, one panel may be considered to be point supported. Note that the panel with the higher critical frequency (the damped corrugated panel in this case) must be the one which is point supported to obtain the high TL predicted. The value of TL_{B2} for line-point support is:

$$TL_{B2} = 20 \log_{10}(13 \times 0.6) + 40 \log_{10}(10600) - 99 = 80 \text{ dB}$$

Assuming that there is sound absorbing material in the cavity,
 $TL_B = 80 \text{ dB}$.

At point C, the frequency is 10,600 Hz and the TL is:

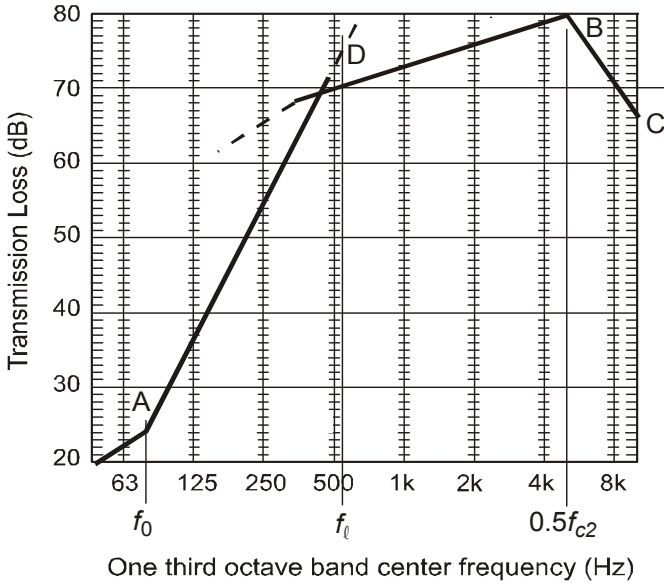
$$TL_C = 80 + 6 + 10 \log_{10}(0.01) = 66 \text{ dB}$$

At point D, the frequency is $f_1 = 55/0.1 = 550 \text{ Hz}$.

The TL data are plotted in the following figure from which the 1/3 octave values can be read directly.

Problem 8.5

A double wall partition may perform more poorly than a single partition of the same weight at resonance frequencies corresponding to acoustic modes in the cavity and also at the critical frequencies of the individual panels (if they are lightly damped).



Problem 8.6

Following the procedure on page 360 in the text, we have at point A:

$$f_0 = 80.4 \sqrt{\frac{1000 \times 0.012 + 7800 \times 0.0016}{0.1 \times 1000 \times 0.012 + 7800 \times 0.0016}} = 103.2 \text{ Hz}$$

and the corresponding Transmission Loss is:

$$TL_A = 20 \log_{10}(12 + 12.48) + 20 \log_{10}(103.2) - 48 = 20.0 \text{ dB}$$

The critical frequencies are:

$$f_{c1} = \frac{0.55 \times 343^2}{2100 \times 0.012} = 2570 \text{ Hz} \quad \text{and} \quad 0.5f_{c1} = 1280 \text{ Hz}$$

$$f_{c2} = \frac{0.55 \times 343^2}{5400 \times 0.0016} = 7490 \text{ Hz}$$

The Transmission Loss at point B is the larger of:

$$TL_{B1} = 20 + 20\log_{10}(2568/102.2) - 6 = 42 \text{ dB}$$

and:

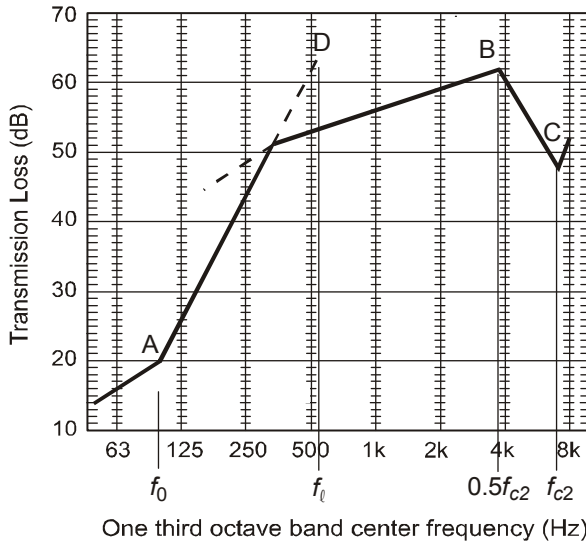
$$\begin{aligned} TL_{B2} &= 20\log_{10}(12) + 10\log_{10}(0.6) + 30\log_{10}(7490) \\ &\quad + 20\log_{10}\left(1 + \frac{12.48\sqrt{2570}}{12\sqrt{7490}}\right) - 78 \\ &= 61.7 \text{ dB} \end{aligned}$$

At point C:

$$TL_C = 61.7 + 6 + 10\log_{10}(0.01) = 47.7 \text{ dB}$$

The frequency $f_\ell = 55/0.1 = 550 \text{ Hz}$.

The Transmission Loss may thus be plotted as shown in the following figure and 1/3 octave (but not octave) values may be read directly from the figure.



Problem 8.7

- (a) $TL_{\text{overall}} = 30\text{dB}$, so $\tau = 10^{-30/10} = 0.001$. If we let the required transmission coefficient of the wall be τ_w , then we can write:

$$0.001 = \frac{\tau_w(18 - 0.25 - 2) + 0.25 \times 10^{-28/10} + 2 \times 10^{-25/10}}{18}$$

From which $\tau_w = 7.161 \times 10^{-4}$. Thus the required wall Transmission Loss is, $TL_w = -10\log_{10}(7.161 \times 10^{-4}) = 31.5\text{dB}$.

- (b) Maximum TL possible corresponds to $\tau_w = 0$. Thus:

$$\tau = \frac{0.25 \times 10^{-28/10} + 2 \times 10^{-25/10}}{18} = 3.73 \times 10^{-4}$$

which corresponds to $TL = 10\log_{10}(3.73 \times 10^{-4}) = 34.3\text{dB}$

- (c) 25mm crack under the door. Effective crack height is 50mm due to reflection in the floor, and $\tau_{\text{crack}} = 1.0$. Thus the overall transmission coefficient is:

$$\tau = \frac{7.16 \times 10^{-4} \times 15.75 + 0.025 \times 1 \times 1 + 0.25 \times 10^{-2.8} + 1.975 \times 10^{-2.5}}{18}$$

$$= 0.00238$$

which corresponds to a Transmission Loss of:

$$TL = -10\log_{10}(2.38 \times 10^{-3}) = 26.2\text{dB}$$

Problem 8.8

When designing an enclosure, always (if possible) include sound absorptive material on the ceiling and walls, thus resulting in an "average" enclosure with the coefficients, C , as listed in the table below (along with the required noise reduction and corresponding wall TL given by $TL = NR + C$).

Octave band centre frequency (Hz)	63	125	250	500	1000	2000	4000	8000
Required NR	14	18	25	35	50	40	40	40
C	13	11	9	7	5	4	3	3
required wall TL	27	29	34	42	55	44	43	43

100mm studs implies gap, $d = 0.1\text{m}$. Assume that the door and window have the same TL as the walls.

To begin, try 3mm steel and 25mm plasterboard.

$$f_c(\text{steel}) = \frac{0.55 \times 343^2}{5400 \times 0.003} \approx 4000\text{Hz} = f_{c2}$$

$$f_c(\text{plaster}) = \frac{0.55 \times 343^2}{1600 \times 0.025} \approx 1600\text{Hz} = f_{c1}$$

$$m_1 = 760 \times 0.025 = 19\text{kg/m}^2 \quad \text{and} \quad m_2 = 7800 \times 0.003 = 23.4\text{kg/m}^2$$

$$f_0 = 80 \sqrt{\frac{19 + 23.4}{0.1 \times 19 \times 23.4}} = 78\text{Hz}$$

$$TL_A = 20\log_{10}(19 + 23.4) + 20\log_{10}(78) - 48 = 22\text{dB at } 78\text{Hz}$$

This is insufficient as we need 27dB at 63Hz. It seems that we need to lower f_0 below 63Hz, to put this point on the 18dB/octave slope.

$$TL_{63} = TL_A + 60\log_{10}(63/f_0) = 27$$

That is:

$$20\log_{10}(m_1 + m_2) + 20\log_{10}f_0 + 60\log_{10}63 - 60\log_{10}f_0 = 27 + 48$$

or

$$20 \log_{10}(m_1 + m_2) - 40 \log_{10} f_0 = -33 \quad (1)$$

Using the equation for f_0 on page 357 in the text and taking logs gives:

$$\begin{aligned} 40 \log_{10} f_0 &= 40 \log_{10} 80 + 20 \log_{10}(m_1 + m_2) - 20 \log_{10}(m_1 \times m_2) + 20 \\ &= 96 + 20 \log_{10}(m_1 + m_2) - 20 \log_{10}(m_1 \times m_2) \end{aligned}$$

Substituting the above expression into equation (1) above gives:

$$20 \log_{10}(m_1 \times m_2) = 63 \text{ dB} \quad \text{or} \quad m_1 m_2 = 1410$$

Try 50mm plasterboard (same as gypsum board), $m = 760 \times 0.05 = 38 \text{ kg/m}^2$. Required steel weight $= 1410/38 = 37 \text{ kg/m}^2$, which is 4.7mm thick. That is, use 50mm thick plasterboard and 5mm thick steel plate.

$$f_{c1} = \frac{0.55 \times 343^2}{1600 \times 0.05} \approx 810 \text{ Hz}$$

$$f_{c2} = \frac{0.55 \times 343^2}{5400 \times 0.005} \approx 2400 \text{ Hz}$$

$$m_1 = 760 \times 0.05 = 38 \text{ kg/m}^2 \quad \text{and} \quad m_2 = 7800 \times 0.005 = 39.0 \text{ kg/m}^2$$

$$f_0 = 80 \sqrt{\frac{38 + 39}{0.1 \times 38 \times 39}} = 58 \text{ Hz}$$

$$TL_A = 20 \log_{10}(38 + 39) + 20 \log_{10}(58) - 48 = 25 \text{ dB at } 58 \text{ Hz}$$

$$TL_{B1} = 24.9 + 20 \log_{10}(810/57.7) - 6 = 41.8 \text{ dB}$$

Assume a stud spacing of 0.6m and line-line support. Thus:

$$TL_{B2} = 20 \log_{10}(38) + 10 \log_{10}(0.6) + 30 \log_{10}(2400)$$

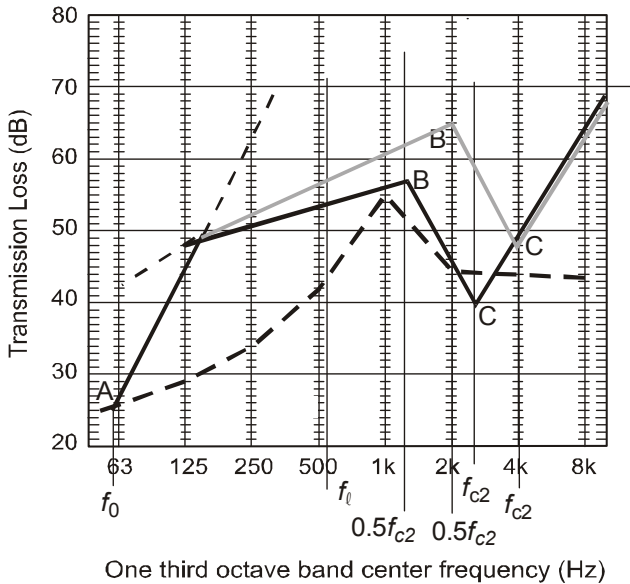
$$+ 20 \log_{10} \left(1 + \frac{39 \times 810^{1/2}}{38 \times 2400^{1/2}} \right) - 78 = 57 \text{ dB}$$

As the cavity is filled with sound absorbing material, $TL_B = 57 \text{ dB}$. Assume a loss factor for the steel of 0.005 (see table on p.609 in text). Then:

$$TL_C = 57 + 6 + 10 \log_{10}(0.005) = 40 \text{ dB}$$

The frequency at point D is $f_t = 55/0.1 = 550 \text{ Hz}$.

The TL for this construction is plotted on the following figure (where the dashed line is the required Transmission Loss and the solid line is the predicted Transmission Loss, calculated using Figure 8.9 in the text), where



it can be seen that the design is deficient between 1000 Hz and 2500 Hz. It is clear that the first critical frequency must be increased or the quantity TL_B (and hence TL_C) must be increased. An easy solution to the problem is to use point supports for the steel panel but this may not be practical. Thus try 75 mm thick plaster board ($m = 760 \times 0.075 = 57 \text{ kg/m}^2$). The required thickness of steel panel is such that steel weight = $1410/57 = 25 \text{ kg/m}^2$, which

is 3.2mm thick. This is an odd thickness, so try 88mm thick plasterboard ($3 \times 25 + 13$) $m = 760 \times 0.088 = 66.9\text{kg/m}^2$. Required steel weight = $1410/66.9 = 21\text{kg/m}^2$, which is 2.7mm thick. That is, use 88mm thick plasterboard and 3mm thick steel plate.

$$f_{c1} = \frac{0.55 \times 343^2}{1600 \times 0.088} \approx 460\text{Hz}$$

$$f_{c2} = \frac{0.55 \times 343^2}{5400 \times 0.003} \approx 4000\text{Hz}$$

$$m_1 = 760 \times 0.088 = 66.9\text{kg/m}^2 \quad \text{and} \quad m_2 = 7800 \times 0.003 = 23.4\text{kg/m}^2$$

$$f_0 = 80 \sqrt{\frac{66.9 + 23.4}{0.1 \times 66.9 \times 23.4}} = 61\text{Hz}$$

$$TL_A = 20\log_{10}(66.9 + 23.4) + 20\log_{10}(61) - 48 = 27\text{dB at } 61\text{Hz}$$

$$TL_{B1} = 26.8 + 20\log_{10}(460/61) - 6 = 38.3\text{dB}$$

Assume a stud spacing of 0.6m and line-line support. Thus:

$$\begin{aligned} TL_{B2} &= 20\log_{10}(66.9) + 10\log_{10}(0.6) + 30\log_{10}(4000) \\ &+ 20\log_{10}\left(1 + \frac{23.4 \times 460^{1/2}}{66.9 \times 4000^{1/2}}\right) - 78 = 65\text{dB} \end{aligned}$$

As the cavity is filled with sound absorbing material, $TL_B = 65\text{dB}$. Assume a loss factor for the steel of 0.005. Then:

$$TL_C = 65 + 6 + 10\log_{10}(0.005) = 48\text{dB}$$

The frequency at point D is $f_t = 55/0.1 = 550\text{Hz}$.

The TL for this construction is plotted on the previous figure as the grey line, where it can be seen that the proposed construction easily meets the noise reduction requirements and is even a little too good.

Problem 8.9

There is a 5mm air gap under door. For the purpose of calculating the transmission coefficient, the reflection in the floor effectively doubles the width of the gap. However, once the transmission coefficient has been determined using figure 8.11 in the text, the area of gap in subsequent calculations is determined without doubling the width.

Area of walls and ceiling = $2(4 \times 2.5 + 3 \times 2.5) + 4 \times 3 = 47\text{m}^2$. Area under door = $0.005 \times 1 = 0.005\text{m}^2$. Effective gap under door = 0.01m . τ_{gap} is calculated using figure 8.11 in the text. We can construct the following table using equations 8.12, 8.65 and 8.66 in the text.

Octave band centre frequency (Hz)	63	125	250	500	1000	2000	4000	8000
TL_{wall}	27	45	51	56.5	60.5	52	49	63
$S_{\text{wall}}\tau_{\text{wall}}$	0.0938	0.00149	3.73×10^{-4}	1.05×10^{-4}	4.19×10^{-5}	2.97×10^{-4}	5.92×10^{-4}	2.36×10^{-5}
$S_{\text{gap}}\tau_{\text{gap}}$	0.005	0.005	0.003	0.0016	0.001	5.5×10^{-4}	2.5×10^{-4}	1.3×10^{-4}
$\bar{\tau}$	2.10×10^{-3}	1.38×10^{-4}	7.18×10^{-5}	3.63×10^{-5}	2.22×10^{-5}	1.80×10^{-5}	1.79×10^{-5}	3.26×10^{-6}
\overline{TL}	27	39	41	44	47	47	47	55

From the table it can be seen that the effect of the gap under the door is to significantly reduce the effective wall TL and on comparing the results in the above table with the table of problem 8.8, it can be seen that the required enclosure noise reduction will no longer be achieved in the 1000Hz octave band.

Problem 8.10

The required air flow may be calculated using equation 8.84 in the text, in which $\rho = 1.206\text{kg/m}^3$, $C_p = 1010\text{m}^2\text{s}^{-2}\text{C}^{-1}$, $\Delta T = 3^\circ\text{C}$ and $H = 0.05 \times 10^4\text{W}$. Thus:

$$V = \frac{0.05 \times 10^4}{3 \times 1010 \times 1.206} = 0.14 \text{ m}^3/\text{s}$$

The required Insertion Loss specifications for the silencer would be the same as the *TL* of the walls (not the noise reduction required of the enclosure as this excludes reverberant build-up in the enclosure) and this is found in problem 8.8.

Problem 8.11

The following steps would need to be taken.

1. Check local noise regulations for allowable levels in the residential area. Measure existing levels in dB(A) on the perimeter of the supermarket property at the closest location to the proposed compressor location over an extended period (about a week with a statistical noise analyser, or if this is impractical use a sound level meter).
2. Choose as the design criterion for the compressor noise in the community the smallest of the regulation level and the lowest measured existing level plus 5dB(A). If existing levels were determined using spot checks with a sound level meter, then the criterion may be more conservative, such as the existing level plus 2dB(A).
3. Use table 4.8 to convert the allowable dB(A) level in the community to an *NR* level and then use the corresponding *NR* curve (figure 4.8) to specify the allowable community noise levels in octave bands.
4. Calculate the noise reduction from the compressor site to the nearest community location due to atmospheric absorption, turbulence, ground effects and meteorological influences, or place a loudspeaker at the proposed compressor location and measure the noise reduction as a function of distance from it.
5. Use the sound power data for the compressor and the excess attenuation data together with equation 5.158 in the text to calculate the noise levels due to the compressor at the nearest community location in octave bands.
6. Thus calculate the required enclosure noise reduction and following that,

the corresponding required wall transmission loss.

7. Check compressor cooling requirements and if necessary design lined inlet and outlet ducts (with forced air ventilation) with the same Insertion Loss as the Transmission Loss of the enclosure walls.

Problem 8.12

- (a) Compressor is 80m from perimeter. The sound level at the receiver with no enclosure (assuming hard ground, zero reflection loss) may be calculated using equation 5.158 in the text with $DI_M = A_E = 0$, so that:

$$L_p = L_w - 10 \log_{10}(2\pi r^2) = 105 - 10 \log_{10}(2\pi \times 80^2) = 59 \text{ dB}$$

The required noise level = 38dB, so reduction required = 21dB at 500Hz.

Smallest panel of enclosure = $0.6\text{m} \times 0.6\text{m}$ (stud spacing), so 500Hz is well above the first panel resonance frequency (you can calculate this using equation 8.21 to be sure).

Critical frequency is:

$f_c = 0.55 \times 343^2 / (5400 \times h) = 12/h$ Hz, where h is in metres. So 500Hz is in the mass law range for all choices of panel.

From equation 8.79 in the text, the required panel $TL = NR + C$, and for an enclosure lined on the inside, $C = 7$ (see table 8.4, p376 in the text). Thus the required enclosure $TL = 21 + 7 = 28\text{dB}$ at 500Hz.

- (b) The wall TL may be calculated in the mass law range by using equation 8.36 in the text (with the assumption NOT made that $fm/\rho c > 1$), with a constant of 4 instead of 5.5 to account for octave band calculations. Thus:

$$\begin{aligned} TL &= 10 \log_{10} \left[1 + \left(\frac{\pi f m}{\rho c} \right)^2 \right] - 4 \\ &= 10 \log_{10} \left[1 + \left(\frac{\pi \times 500 \times 7800 \times h}{1.206 \times 343} \right)^2 \right] - 4 \quad \text{dB} \end{aligned}$$

When $h = 0.001\text{m}$, $TL = 25.4\text{dB}$ which is too low. As we are in mass law range a doubling of panel thickness will increase the TL by 6dB which is a bit high. Trying $h = 0.0016\text{m}$, gives $TL = 29.5\text{dB}$ which is OK, so choose a wall thickness of 1.6mm.

- (c) Should consider the effect of a door and the design of appropriate door seals, the need for cooling air, the introduction of the inlet air, the pipe penetration (to be isolated from the enclosure wall) for the compressed air and the need for vibration isolation of the enclosure from the compressor.

Problem 8.13

Equation 8.79 in the text gives $TL = NR + C$. From Table 8.3, $C = 5\text{dB}$ for an enclosure with surfaces lined with sound absorbing material. Thus,
 $TL = 15 + 5 = 20\text{dB}$

Problem 8.14

- (a) The performance of a machinery noise enclosure should not be expressed as a single number dB(A) rating because the dB(A) performance will be dependent on the spectrum shape of the noise generated by the enclosed source.
- (b) We may use equation 8.79 in the text to relate TL and noise reduction and thus define the minimum required TL . The TL due to the panel may be calculated using the procedure on p360 in the text.

$$d = 0.1\text{m}, m_1 = m_2 = 760 \times 0.013 = 9.88\text{kg/m}^2.$$

$$f_0 = 80\sqrt{2/(0.1 \times 9.88)} = 114\text{Hz}$$

$$f_{c1} = f_{c2} = \frac{0.55 \times 343^2}{1600 \times 0.013} = 3100\text{Hz}$$

$$TL_A = 20\log_{10}(9.88 \times 2) + 20\log_{10}(113.8) - 48 = 19.0\text{dB}$$

$$TL_{B1} = 19 + 20\log_{10}(3100/114) - 6 = 42\text{dB}$$

For line-line support:

$$\begin{aligned}
 TL_{B2} &= 20\log_{10}(9.88) + 10\log_{10}(0.6) + 30\log_{10}(3100) \\
 &\quad + 20\log_{10}(2) - 78 = 50 \text{ dB}
 \end{aligned}$$

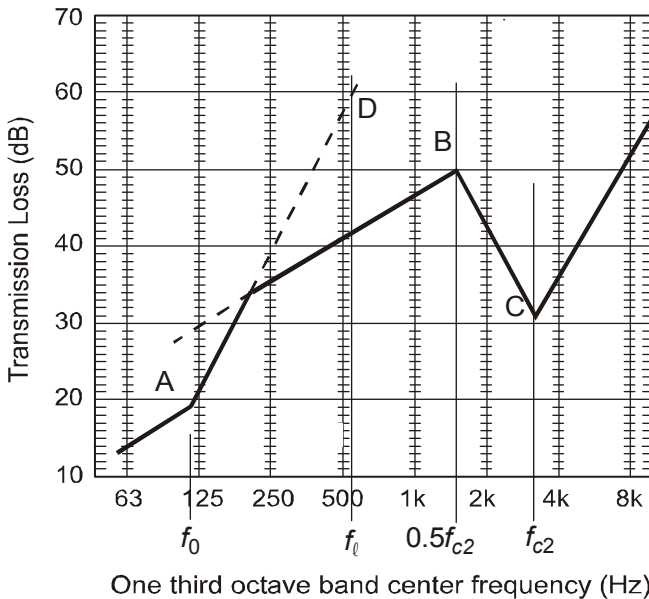
Assuming that there is sound absorbing material in the wall cavity, $TL_B = 50\text{dB}$.

$$\begin{aligned}
 TL_C &= 50.4 + 6 + 15\log_{10}(0.02) = 31 \text{ dB} \\
 f_\ell &= 55/0.1 = 550 \text{ Hz}
 \end{aligned}$$

The corresponding TL curve is plotted in the figure below and the octave band data are listed in the following table, where octave band results are obtained by using 1/3 octave values read from the graph and calculated using:

$$TL_{oct} = -10\log_{10} \left\{ (1/3) [10^{-TL_1/10} + 10^{-TL_2/10} + 10^{-TL_3/10}] \right\}$$

where TL_1 , TL_2 , and TL_3 , are the (1/3) octave band data read from the figure.



Octave Band Centre Frequency	63	125	250	500	1000	2000	4000	8000
TL (from plot)	14	21	35	41	47	43	36	52
C	13	11	9	7	5	4	3	3
NR	1	10	26	34	42	39	33	49
Required NR	10	15	20	25	30	35	40	13

It can be seen that the enclosure is deficient in the 63Hz, 125Hz, and 4000Hz octave bands, so it is not adequate.

- (c) • Use double, staggered stud wall.
 • Use point-line or point-point support by placing rubber grommets between the panel and stud at attachment points.
 • Use thicker panels.
 • Fix additional 13mm thick panels to existing panels with patches of silicone sealant.
 • Use panels of different thickness.

(d) From equation 8.84 in the text:

$$V = \frac{H}{\rho C_p \Delta T} = \frac{0.02 \times 50000}{1.206 \times 1010 \times 5} = 0.164 \text{ m}^3/\text{s}$$

(e) The sound power level is given by equation 8.70 in the text. Thus:

$$L_w = L_{p1} + TL + 10\log_{10} S_E - C = L_{p1} + NR + 10\log_{10} S_E$$

where $S_E = 2(5 \times 2.5 + 4 \times 2.5) + 4 \times 5 = 65\text{m}^2$ and $10\log_{10} S_E = 18\text{dB}$. The power level calculations may be summarised in the following table.

Octave Band Centre Frequency	63	125	250	500	1000	2000	4000	8000
L_{p1}	80	83	78	73	70	60	60	60
NR	10	15	20	25	30	35	40	20
L_w	108	116	116	116	118	113	118	98

- (f) Test surface 1m from machine of dimensions $2\text{m} \times 1\text{m} \times 1\text{m}$, so measurement surface is $4\text{m} \times 3\text{m} \times 2\text{m}$ (machine assumed to be resting on the ground), having an area of $S = 2(4 \times 2 + 3 \times 2) + 4 \times 3 = 40\text{m}^2$ (4 sides and 1 top). Machine surface area $S_m = 2(2 \times 1 + 1 \times 1) + 2 \times 1 = 8\text{m}^2$. The sound pressure is related to the sound power by equation 6.25 in the text with $\Delta_1 = 0$ because the measurements are made outdoors. The ratio, $S/S_m = 5$, so the near field correction $\Delta_2 = 0$ and the sound pressure level 1m from the machine surface is given by:

$$L_{pd} = L_w - 10\log_{10}S = L_w - 16\text{dB}$$

The results are tabulated in the following table:

Octave Band Centre Frequency	63	125	250	500	1000	2000	4000	8000
L_w	108	116	116	116	118	113	118	98
L_{pd}	92	100	100	100	102	97	102	82

- (g) With the enclosure in place, the mean square sound pressure is equal to the sum of the direct and reverberant field contributions. Sound pressure levels are converted to mean square sound pressures using equation 1.78 in the text and sound powers are converted to sound power levels and vice versa using equation 1.80 in the text. Thus the mean square sound pressure in the enclosure is:

$$\langle p^2 \rangle = \langle p_d^2 \rangle + \langle p_R^2 \rangle$$

The reverberant mean square pressure is:

$$\langle p_R^2 \rangle = \frac{4W\rho c}{R}$$

where R is the room constant and W is the source sound power. The enclosure constant C is given in terms of R by equation 8.82 in the text which can be rearranged (with the aid of equation 7.43) to give:

$$\frac{4}{R} = \frac{4}{3.33} \left(\frac{10^{C/10}}{0.3 \times S_E} - \frac{1}{S_E} \right)$$

The results are summarised in the following table, where for convenience, $\rho c = 400$.

Octave Band Centre Frequency	63	125	250	500	1000	2000	4000	8000
C	13	11	9	7	5	4	3	3
L_w	108	116	116	116	118	113	118	98
W	6.3×10^{-2}	0.40	0.40	0.40	0.63	0.20	0.63	6.3×10^{-3}
$4/R$	1.21	0.76	0.47	0.29	0.18	0.14	0.10	0.10
L_{pd}	92	100	100	100	102	97	102	82
$\langle p_d^2 \rangle$	0.63	4.0	4.0	4.0	6.3	2.5	6.3	0.063
$\langle p_R^2 \rangle$	30.5	122	75.2	46.4	45.4	11.2	25.2	0.25
$\langle p_d^2 \rangle + \langle p_R^2 \rangle$	31	126	79	50	52	14	32	0.31
L_{pT}	109	115	113	111	111	105	109	89
dB(A) correct.	-26	-16	-9	-3	0	1	1	-1
dB(A) sound pressure levels	92	100	100	100	102	97	103	88

- (h) In the 2000Hz band, $L_{p1} = 60\text{dB}$ and the sound pressure level at distance r from the enclosure is given by equation 8.72 in the text (with $D_\theta = 2$ due to the hard ground surface) as:

$$\begin{aligned} L_{p2} &= 60 + 10\log_{10}(65) + 10\log_{10}\left(\frac{2}{4 \times \pi \times 200^2}\right) - A_E \\ &= 24 - A_E \quad (\text{dB}) \end{aligned}$$

A_E is the excess attenuation given by equation 5.165 in the text.

A_g is included in D_θ above as the asphalt is hard resulting in essentially hemispherical spreading.

$A_a = 10.8/5 = 2\text{dB}$ (see table 5.3 on page 225 in the text).

$A_m = +5, -4\text{dB}$ (see table 5.10)

Thus the range of variability is 18 to 27dB.

Problem 8.15

Noise level inside enclosure = 101dB at 1000Hz.

Noise level on surface, 1m from enclosure = 91dB.

Noise level at 50m distance = 70dB.

- (a) Sound power level radiated by enclosure may be calculated using equation 6.25 in the text.

Test surface 1 m from enclosure of dimensions $3\text{m} \times 3\text{m} \times 3\text{m}$, so measurement surface is $5\text{m} \times 5\text{m} \times 4\text{m}$ (assuming that the enclosure is resting on the ground), having an area of

$S = 2(5 \times 4 \times 2) + 5 \times 5 = 105\text{m}^2$ (4 sides and 1 top). Machine surface area, $S_m = 2(3 \times 3 \times 2) + 3 \times 3 = 45\text{m}^2$. The sound pressure is related to the sound power by equation 6.25 in the text with $\Delta_1 = 0$ because the measurements are made outdoors. The ratio, $S/S_m = 105/45 = 2.3$, so the near field correction $\Delta_2 = 1$ and the sound power level of the enclosure is given by:

$$L_w = 91 + 10\log_{10}(105) - 1 = 110\text{dB}$$

- (b) We may use equations 5.158 and 5.161 with $r = 50\text{m}$ and $DI_M = A_E = 0$ so that:

$$L_p = 110 - 20\log_{10}(50) - 10\log_{10}(2\pi) = 68\text{dB}$$

- (c) As the measured noise level is 70dB, the excess attenuation due to atmospheric absorption and meteorological influences is $68 - 70 = -2\text{dB}$. The excess attenuation due to the ground effect is -3dB , so the total excess attenuation is -5dB .
- (d) The machine is probably not well vibration isolated from the enclosure, causing the enclosure wall to vibrate and radiate noise. Vibration could be transmitted by way of the floor or by direct connection of parts of the machine (or attached pipework) to parts of the enclosure. Also, pipework or other equipment not included in the enclosure but attached to the noisy machine could radiate noise which was not apparent prior to installation of the enclosure.

Problem 8.16

Adequate internal absorption is necessary to prevent the build-up of reverberant energy which will compromise the predicted acoustic performance.

Problem 8.17

- (a) The enclosure should be vibration isolated to prevent the walls from being excited to vibrate and thus radiate sound which in turn will compromise the enclosure performance.
- (b) One possible disadvantage associated with vibration isolation from the floor is increased difficulty in producing an adequate acoustic seal around the base of the enclosure.
- (c) Other factors which could degrade the enclosure performance are
- poor seals around doors and windows

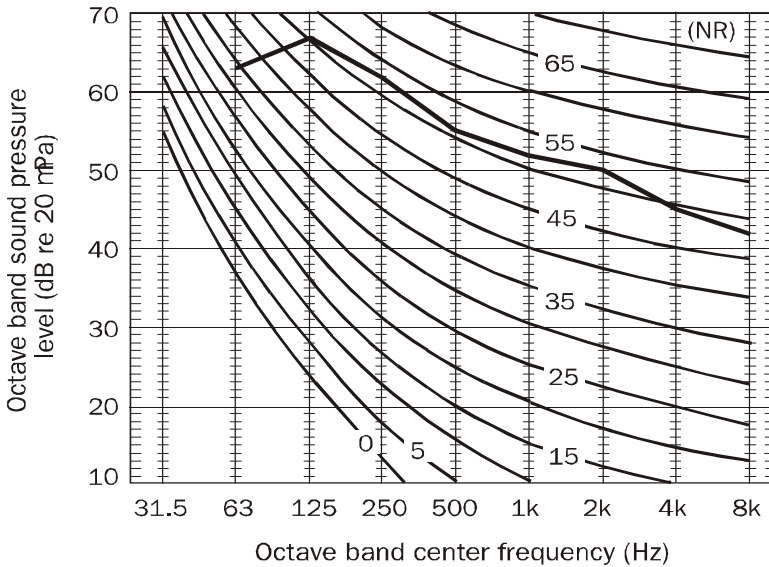
- inadequate TL performance of doors and windows
- inadequate internal absorption
- pipe penetrations in the enclosure walls not vibration isolated from the walls or poorly sealed acoustically.
- pipework or other equipment not included in the enclosure but attached to the noisy machine could radiate noise which was not apparent prior to installation of the enclosure.

Problem 8.18

- acoustic absorbing material left out of wall cavity or too rigid and touching both walls
- poor seals around windows and doors
- glass in double glazing too thin
- doors of insufficient acoustic performance
- floor vibration transmitted to enclosure walls because of inadequate vibration isolation
- wall stud spacing incorrect
- incorrect wall thickness or wall materials
- poor seal at base of enclosure
- poor bricklaying (if brick walls) leading to gaps in the mortar
- change from original noise sources
- tonal noise from the machine corresponding to a structural resonance or an acoustic resonance (wall cavity or enclosure space).
- pipework or other equipment not included in the enclosure but attached to the noisy machine could radiate noise which was not apparent prior to installation of the enclosure.

A test to determine whether the problem was airborne or structure-borne would be to turn the machines off and use loudspeakers in the enclosure to generate the same noise levels inside the enclosure. If the exterior noise levels are then the same with the loudspeakers operating as they were with the machine, then the problem is airborne flanking. If the noise external to the enclosure with the loudspeakers operating is lower, then the problem is likely to be structure-borne vibration or radiation from equipment attached to the noisy machine but not included in the enclosure.

Problem 8.19



- (a) The Noise Rating (NR) is obtained by plotting the un-weighted octave band data on a set of NR curves (see fig 4.7, p118 in text) as illustrated in the following figure. The NR value is 52.5.

A-weighted sound level is given by:

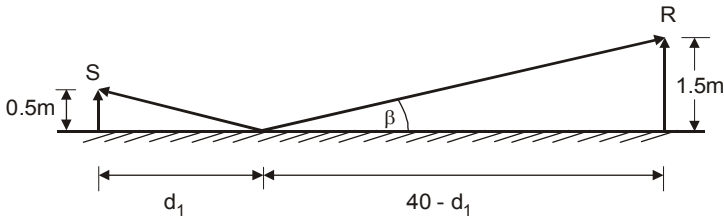
$$\begin{aligned}
 L_{pA} &= 10 \log_{10} \left(10^{(6.3 - 2.62)} + 10^{(6.7 - 1.61)} + 10^{(6.2 - 0.86)} \right. \\
 &\quad \left. + 10^{(5.5 - 0.32)} + 10^{5.2} + 10^{(5 + 0.12)} + 10^{(4.5 + 0.1)} + 10^{(4.2 - 0.11)} \right) \\
 &= 59.2 \text{ dB(A)}
 \end{aligned}$$

- (b) The acceptable noise level is obtained using table 4.10, p167 in the text and is $L_p = 40 + 20 - 10 = 50 \text{ dB(A)}$. The actual level is over 9dB(A) above the allowable level, a situation which is not acceptable and according to table 4.11, it will generate widespread complaints from the community. If the noise only occurs during the hours of 7am and 6pm, then the allowable level is $L_p = 40 + 20 = 60 \text{ dB(A)}$ and the existing level is thus acceptable.

- (c) The octave band levels essentially follow the shape of the NR curve (except at 63Hz, where the level is much lower), so the noise will sound neutral.
- (d) With no barrier, there are two propagation paths; the direct path and the ground reflected path.

With a barrier, there are 8 paths as listed below:

- over the top with no ground reflections
- over the top with a ground reflection on the source side
- over the top with a ground reflection on the receiver side
- over the top with ground reflections on both sides



- around each end with no ground reflections
 - around each end with a ground reflection
- (e) Attenuation of ground reflected wave (see figure)

Using similar triangles:

$$\frac{0.5}{d_1} = \frac{1.5}{40 - d_1}; \quad d_1 = 10 \text{ m}$$

$$\beta = \tan^{-1} \left(\frac{0.5}{10} \right) = 2.862^\circ$$

From table 5.2 on p209, $R_1 = 2 \times 10^{-5}$; $\rho = 1.206$. Thus at 500Hz:

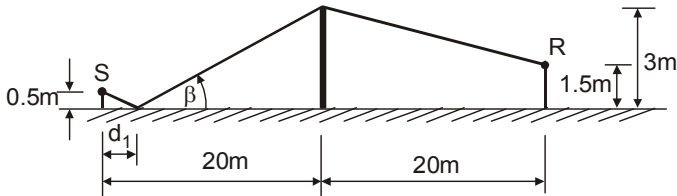
$$\frac{\rho f}{R_1} = \frac{1.206 \times 500}{2 \times 10^5} = 0.003$$

$$\beta \left(\frac{R_1}{\rho f} \right)^{1/2} = 52^\circ$$

From figure 5.20, the reflection loss on ground reflection is thus 3.9dB.

- (f) Attenuation due to barrier. First calculate the reflection loss for each path which involves a ground reflection.

• **Over the top, source side reflection**



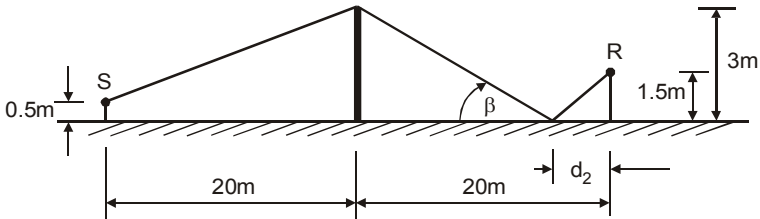
Using similar triangles:

$$\frac{d_1}{0.5} = \frac{20 - d_1}{3} ; \quad d_1 = 2.857 \text{ m}$$

$$\beta = \tan^{-1} \left(\frac{0.5}{2.857} \right) = 9.93^\circ ; \quad \beta \left(\frac{R_1}{\rho f} \right)^{1/2} = 181^\circ$$

and from figure 5.20, p231 in the text, $A_R = 7.9\text{dB}$

• **over the top, receiver side reflection**



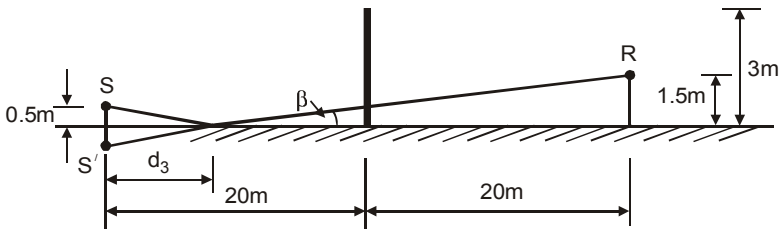
Using similar triangles:

$$\frac{d_2}{1.5} = \frac{20 - d_2}{3} ; \quad d_2 = 6.667$$

$$\beta = \tan^{-1}\left(\frac{1.5}{6.667}\right) = 12.7^\circ ; \quad \beta\left(\frac{R_1}{\rho f}\right)^{1/2} = 231^\circ$$

and from figure 5.20, p231 in the text, $A_R = 7.0\text{dB}$

● **around each end of the barrier**



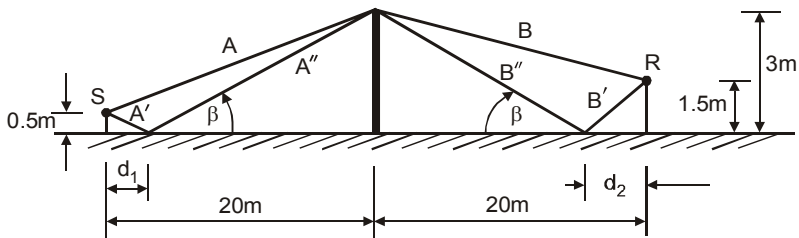
Using similar triangles:

$$\frac{d_3}{0.5} = \frac{40 - d_3}{1.5} ; \quad d_3 = 10\text{m}$$

which is the same as if there were no barrier, so $A_R = 3.9\text{dB}$

Now calculate Fresnel numbers for all paths over and around the barrier.

● **Over the top (see figure)**



The distance d , between source and receiver depends on the path which is being considered. For example, for waves reflected from the ground on the source side of the barrier only, the value of d (denoted d') below, is that from the image source to the receiver, etc.

$$A' = [0.5^2 + 2.857^2]^{1/2} = 2.9004\text{m}; \quad A'' = [3^2 + 17.143^2]^{1/2} = 17.404\text{m}$$

$$B' = [1.5^2 + 6.667^2]^{1/2} = 6.834\text{m}; \quad B'' = [3^2 + 13.333^2]^{1/2} = 13.663\text{m}$$

$$d' = [2^2 + 40^2]^{1/2} = 40.05\text{m}$$

$$A = [20^2 + 2.5^2]^{1/2} = 20.156\text{m}; \quad B = [20^2 + 1.5^2]^{1/2} = 20.056\text{m}$$

$$d = [40^2 + 1^2]^{1/2} = 40.01\text{m}$$

$$N_1 = (20.156 + 20.056 - 40.01) \times 2.92 = 0.58$$

$$N_2 = (2.900 + 17.404 + 20.056 - 40.05) \times 2.92 = 0.91$$

$$N_3 = (20.156 + 6.834 + 13.663 - 40.05) \times 2.92 = 1.76$$

$$N_4 = (2.900 + 17.404 + 6.834 + 13.663 - 40.01) \times 2.92 = 2.31$$

From figure 8.14, p389 in the text:

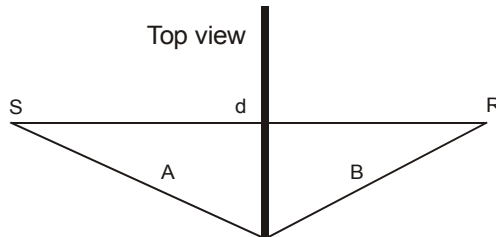
$$\Delta_{b1} = 11.5; \quad \Delta_{b2} = 12.8; \quad \Delta_{b3} = 15.8; \quad \Delta_{b4} = 17.0\text{dB}$$

It can be shown that in this case, the corrections of equation 8.88 are:

$$A_{b1} = 11.5 + 0.0 = 11.5; \quad A_{b2} = 12.8 + 0.1 = 12.9;$$

$$A_{b3} = 15.8 + 0.1 = 15.9; \quad A_{b4} = 17.0 + 0.2 = 17.2.$$

- **Around the ends** (see figure)



Direct waves, no reflection (one each side):

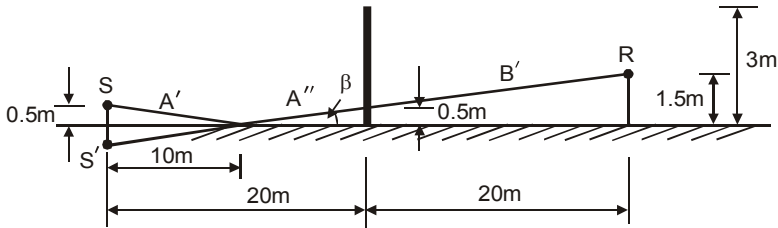
$$A = B = [20^2 + 5^2 + (1.5 - 0.5)^2/4]^{1/2} = 20.622 \text{ m}; \quad d = 40.01 \text{ m}.$$

From figure 8.14 in the text:

$$N_{5,6} = (20.622 \times 2 - 40.01) \times 2.92 = 3.6; \quad \Delta_{b5,6} = 18.7 \text{ dB}$$

$$\text{Thus } A_{b5,6} = 18.7 + 0.3 = 19.0 \text{ dB}$$

Reflected waves (one each side)



$$A' = A'' = [10^2 + 0.5^2 + 2.5^2]^{1/2} = 10.32 \text{ m}$$

$$B' = [20^2 + 1^2 + 5^2]^{1/2} = 20.64 \text{ m}; \quad d' = [40^2 + 2^2]^{1/2} = 40.05 \text{ m}$$

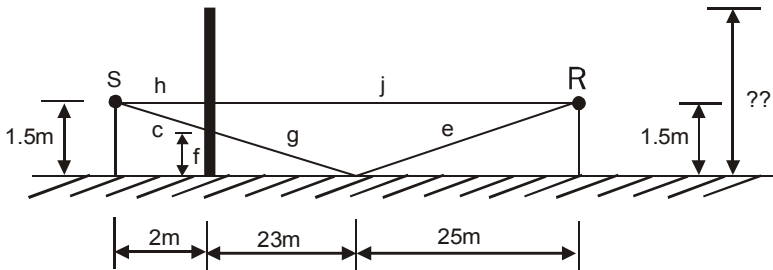
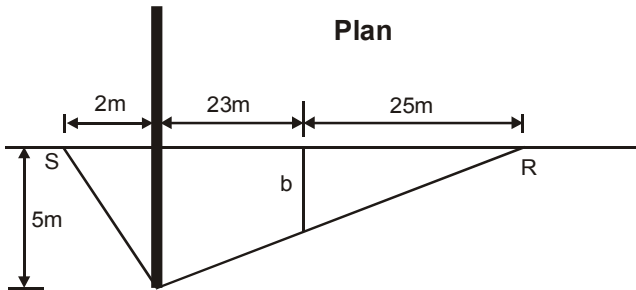
From figure 8.14 in the text:

$$N_{7,8} = (10.32 \times 2 + 20.64 - 40.05) \times 2.92 = 3.6; \quad \Delta_{b7,8} = 18.7 \text{ dB}$$

$$\text{Thus } A_{b7,8} = 18.7 + 0.3 = 19.0 \text{ dB}$$

NR due to barrier is then calculated using equation 1.97 in the text as:

$$\begin{aligned} NR &= 10 \log_{10} (10^{-0/10} + 10^{-3.9/10}) \\ &\quad - 10 \log_{10} (10^{-1.15} + 10^{-(1.29 + 0.79)} + 10^{-(1.59 + 0.70)} \\ &\quad + 10^{-(1.72 + 1.49)} + 2 \times 10^{-1.9} + 2 \times 10^{-(1.90 + 0.39)}) \\ &= 1.5 + 9.2 = 10.7 \text{ dB} \end{aligned}$$

Around edge (elevation)**Plan**

As the source and receiver are at the same height, the location of the ground reflection will be mid-way (in a horizontal direction only) between the source and receiver.

Path with no ground reflection

$$\text{Path } h = (2^2 + 5^2)^{1/2} = 5.385\text{m}$$

$$\text{Path } j = (48^2 + 5^2)^{1/2} = 48.260\text{m}$$

$$A + B - d = 5.385 + 48.260 - 50 = 3.64\text{m}$$

At 125Hz, $\lambda = 343/125 = 2.744\text{m}$ and $N = (2/\lambda) \times 3.64 = 2.65$. From figure 8.14 on p389 in the text, $A_b = 17.0\text{dB}$. The correction term given by equation 8.88 in the text (assuming an omnidirectional source) is $20\log_{10}(53.64/50) = 0.6\text{dB}$, so $A_b = 17.0 + 0.6 = 17.6\text{dB}$.

Path with ground reflection

From the figure, and using similar triangles, $b/5 = 25/48$; $b = 2.6\text{m}$

$$\text{Path } e = (25^2 + 1.5^2 + 2.604^2)^{1/2} = 25.180\text{m}$$

By similar triangles, $f/1.5 = 23/25$; $f = 1.38\text{m}$

$$\text{Path } g = (23^2 + 1.38^2 + 2.396^2)^{1/2} = 23.166 \text{ m}$$

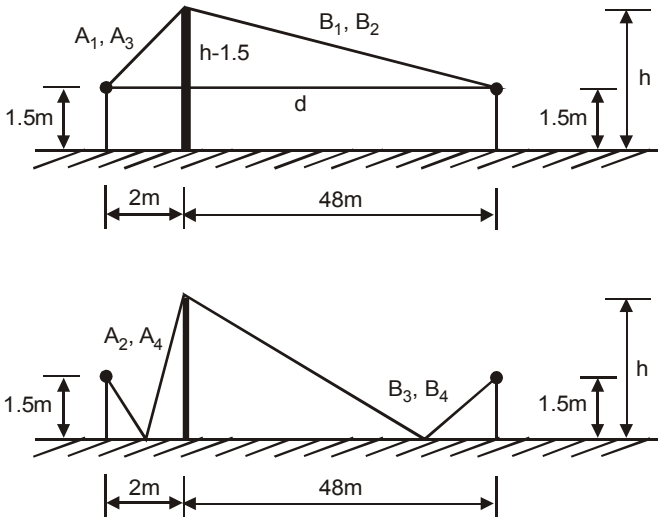
$$\text{Path } c = (0.12^2 + 2^2 + 5^2)^{1/2} = 5.387 \text{ m}$$

$$A + B - d = 5.387 + 23.166 + 25.180 - [50^2 + 3^2]^{1/2} = 3.64 \text{ m}$$

Thus, $N = (2/\lambda) \times 3.64 = 2.65$. from figure 8.14 on p389 in the text,

$\Delta_b = 17.0 \text{ dB}$. The correction term given by equation 8.88 in the text (assuming an omnidirectional source) is $20 \log_{10}(53.74/50.09) = 0.6 \text{ dB}$, so

$A_b = 17.0 + 0.6 = 17.6 \text{ dB}$. Losses due to ground reflection are zero.



The required barrier height can be found by trial and error. The results are summarised in the following table, where the subscript "1" refers to waves travelling over the barrier with no reflections, the subscript "2" implies a reflection on the source side, the subscript "3" implies a reflection on the receiver side and the subscript "4" implies a reflection on both sides. The distances "A" refer to the source side of the barrier (source to barrier top along the particular path specified by the associated subscript) and the distances "B" refer to distances on the receiver side. We will begin the trial with a barrier height of 3.0m. The correction term given by equation 8.88 in the text (assuming an omnidirectional source) is added to the barrier attenuation Δ_b to give A_b .

The barrier overall noise reduction is given by equation 1.97 in the text which can be expanded to give:

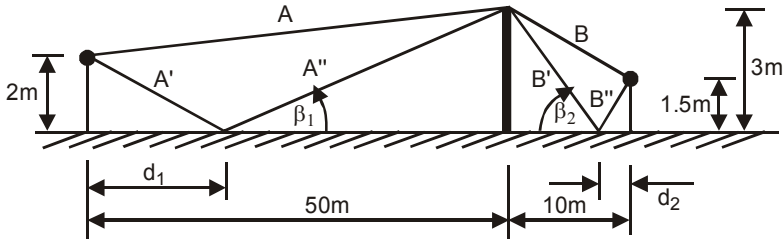
$$NR = 10 \log_{10}(10^0 + 10^0) - 10 \log_{10}(4 \times 10^{-1.76} + 10^{-A_{b1}/10} + 10^{-A_{b2}/10} + 10^{-A_{b3}/10} + 10^{-A_{b4}/10})$$

Barrier height	3.0m	3.5m	4.0m
A_1 (m)	2.50	2.828	3.202
A_2 (m)	4.92	5.385	5.852
A_3 (m)	2.50	2.828	3.202
A_4 (m)	4.92	5.385	5.852
B_1 (m)	48.02	48.042	48.065
B_2 (m)	48.02	48.042	48.065
B_3 (m)	48.21	48.260	48.314
B_4 (m)	48.21	48.260	48.314
N_1	0.38	0.63	0.923
N_2	2.07	2.43	2.79
N_3	0.45	0.73	1.04
N_4	2.28	2.66	3.036
Δ_{b1}	10.1	11.9	12.6
Δ_{b2}	16.1	17	17.3
Δ_{b3}	10.5	12.0	13.1
Δ_{b4}	17	17.1	18
A_{b1} (dB)	10.2	12.0	12.8
A_{b2} (dB)	16.6	17.5	17.9
A_{b3} (dB)	10.6	12.2	13.3
A_{b4} (dB)	17.5	17.6	18.7
NR (dB)	8.3	9.4	10.0

Thus the wall height should be about 4.0m.

Problem 8.21

The layout is illustrated in the following figure.



From similar triangles, $\frac{d_1}{50 - d_1} = \frac{2}{3}$; so $d_1 = 20.0\text{m}$

$$\frac{d_2}{10 - d_2} = \frac{1.5}{3}; \text{ so } d_2 = 3.333\text{m}$$

$$\lambda = 343/500 = 0.686\text{m}$$

$$A' = \sqrt{20^2 + 2^2} = 20.0998\text{m}$$

$$A'' = \sqrt{30^2 + 3^2} = 30.1496\text{m}$$

$$B' = \sqrt{6.667^2 + 3^2} = 7.3109\text{m}$$

$$B'' = \sqrt{3.333^2 + 1.5^2} = 3.6550\text{m}$$

$$A = \sqrt{50^2 + 1^2} = 50.0100\text{m}$$

$$B = \sqrt{10^2 + 1.5^2} = 10.1119\text{m}$$

There are 4 paths which will contribute to the sound level at the receiver so we need to calculate the Fresnel Number corresponding to each.

Path 1, A \Rightarrow B

$$N_1 = \frac{2}{0.686} [50.0100 + 10.1119 - (60^2 + 0.5^2)^{1/2}] = 0.35$$

Path 2, A' \Rightarrow A'' \Rightarrow B

$$N_2 = \frac{2}{0.686} [20.0998 + 30.1496 + 10.1119 - (60^2 + 3.5^2)^{1/2}] = 0.76$$

Path 3, A ⇒ B' ⇒ B''

$$N_3 = \frac{2}{0.686} [50.0100 + 7.3106 + 3.6553 - (60^2 + 3.5^2)^{1/2}] = 2.5$$

Path 4, A' ⇒ A'' ⇒ B' ⇒ B''

$$N_4 = \frac{2}{0.686} [20.0998 + 30.1496 + 7.3106 + 3.6553 - (60^2 + 0.5^2)^{1/2}] = 3.5$$

As the wall completely surrounds the factory, no sound is diffracted around its edges. The correction term given by equation 8.88 in the text is greatest for path 4 and is equal to 0.09dB which is negligible, so the correction will be ignored.

From figure 8.14, the attenuations corresponding to N_1 to N_4 are $\Delta_{N1} = 9.9$, $\Delta_{N2} = 12$, $\Delta_{N3} = 17$, $\Delta_{N4} = 18.5$. Adding 3dB to all ground reflections results in the following noise reductions corresponding to the 4 paths: Path 1, 9.9dB; Path 2, 15dB; Path 3, 20dB; Path 4, 24.5dB. From equation 1.97 in the text, the noise reduction due to the enclosure is:

$$\begin{aligned} NR &= 10 \log_{10} (10^{-0/10} + 10^{-3/10}) \\ &\quad - 10 \log_{10} (10^{-9.9/10} + 10^{-15/10} + 10^{-20/10} + 10^{-24.5/10}) \\ &= 1.76 + 8.31 = 10.1 \text{ dB} \end{aligned}$$

Problem 8.22

Referring to the following figures, we have for the reflection angles:

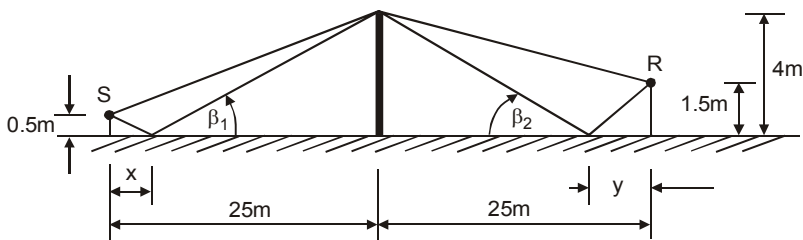
over top, source side,

$$\tan \beta_1 = \frac{4}{25 - x} = \frac{0.5}{x}; \quad x = 2.78 \text{ m}, \quad \beta_1 = 10.2^\circ$$

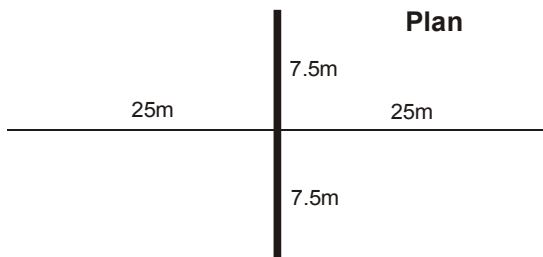
over top, receiver side,

$$\tan \beta_2 = \frac{4}{25 - y} = \frac{1.5}{y}; \quad y = 6.82 \text{ m}, \quad \beta_2 = 12.4^\circ$$

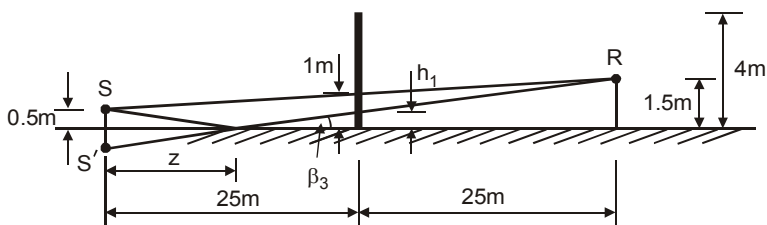
Over top (elevation)



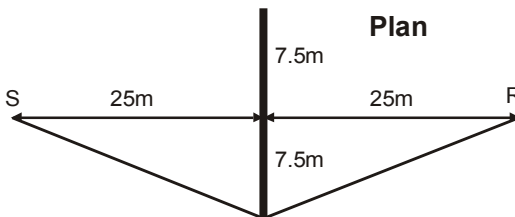
Plan



Around edge (elevation)



Plan



around edge, $\tan\beta_3 = \frac{1.5}{50 - z} = \frac{0.5}{z}$; $z = 12.5\text{ m}$, $\beta_3 = 2.3^\circ$

Flow resistivity of ground $= R_1 = 3 \times 10^7$ MKS rayls/m and $\rho = 1.206\text{ kg/m}^3$.
With no barrier, $\beta_4 = \beta_3 = 2.3^\circ$. The reflection loss, A_R is calculated using figure 5.20 in the text and the results are tabulated in the table below.

Octave band centre frequency (Hz)	$\frac{\rho f}{R_1}$	$\left(\frac{R_1}{\rho f} \right)^{1/2}$	A_{R1}	A_{R2}	A_{R3} and A_{R4}
63	2.5×10^{-6}	630	0.2	0.2	1.5
125	5.0×10^{-6}	447	0.3	0.3	2.0
250	1.0×10^{-5}	316	0.5	0.5	2.5
500	2.0×10^{-5}	223	0.7	0.7	3.5
1000	4.0×10^{-5}	158	1.0	1.0	5.0
2000	8.0×10^{-5}	112	1.5	1.5	6.0
4000	1.6×10^{-4}	79	2.3	2.3	7.0
8000	3.2×10^{-4}	56	3.0	3.0	6.5

Path 1 - over top of barrier with no ground reflections

$d = 50.010\text{ m}$, $A = 25.244$, $B = 25.125$ and $A + B - d = 0.359\text{ m}$

Path 2 - over top of barrier with ground reflection on the source side

$d = 50.040\text{ m}$, $B = 25.125$

$A = (2.78^2 + 0.5^2)^{1/2} + (4^2 + 22.22^2)^{1/2} = 25.402\text{ m}$ and

$A + B - d = 0.487\text{ m}$

Path 3 - over top of barrier with ground reflection on the receiver side

$d = 50.010\text{ m}$, $A = 25.244$

$B = (6.82^2 + 1.5^2)^{1/2} + (4^2 + 18.18^2)^{1/2} = 25.598\text{ m}$ and

$A + B - d = 0.802\text{ m}$

Path 4 - over top of barrier with ground reflection on both sides

$d = 50.010\text{ m}$, $A = 25.402$, $B = 25.598$ and $A + B - d = 0.990\text{ m}$

Paths 5&6 - around edges with no reflection

$B = A = (0.5^2 + 25^2 + 7.5^2)^{1/2} = 26.106\text{ m}$ and $A + B - d = 2.201\text{ m}$

Paths 7&8 - around edges with reflection in ground

Intersection height of diffracted wave with barrier edge

$$= 1.5 \times 12.5/37.5 = 0.5\text{m.}$$

$$A = 2(0.5^2 + 12.5^2 + (7.5/2)^2)^{1/2} = 26.120\text{m}$$

$$B = 2(1^2 + 25^2 + 7.5^2)^{1/2} = 26.120\text{m and}$$

$$A + B - d = 2.20\text{m}$$

Fresnel Number, $N = \frac{2f}{343}(A + B - d)$. Values of N for each path are

tabulated below.

Octave band centre frequency (Hz)	N_1	N_2	N_3	N_4	$N_{5\&6}$	$N_{7\&8}$
63	0.13	0.18	0.30	0.36	0.80	0.80
125	0.26	0.36	0.59	0.72	1.60	1.61
250	0.52	0.71	1.17	1.44	3.21	3.21
500	1.05	1.42	2.34	2.89	6.42	6.4
1000	2.09	2.84	4.68	5.77	12.8	12.8
2000	4.19	5.68	9.35	11.5	25.7	26
4000	8.37	11.4	18.7	23.1	51.3	51
8000	16.7	22.7	37.4	46.2	103	103

The noise reductions (NR or Δ_b) corresponding to the above Fresnel Numbers are calculated using figure 8.14, p389 in the text and are tabulated below. The correction term given by equation 8.88 in the text (assuming an omnidirectional source) will be less than 0.1dB overall and will be ignored here. The numbers in brackets indicate the sum of the ground reflection losses and barrier diffraction loss. All quantities are in dB.

Octave band centre frequency (Hz)	NR_1	NR_2	NR_3	NR_4	$NR_{5\&6}$	$NR_{7\&8}$
63	8.2	8.9 (9.1)	9.9 (10.1)	10.0 (10.4)	12.3	12.3 (13.8)
125	9.5	10.1 (10.4)	11.8 (12.1)	12.0 (12.6)	15.0	15.0 (17.0)
250	10.8	12.0 (12.5)	13.8 (14.3)	14.5 (15.5)	18.1	18.1 (20.6)
500	13.0	14.8 (15.5)	17.0 (17.7)	17.7 (19.1)	21.2	21.2 (24.7)
1000	16.1	17.9 (18.9)	19.8 (20.8)	20.7 (22.7)	24	24 (29)
2000	19.2	20.9 (22.4)	23.0 (24.5)	23.8 (26.8)	24	24 (30)
4000	22.3	23.9 (26.1)	24 (26.3)	24 (28.6)	24	24 (31)
8000	24	24 (27)	24 (27)	24 (30)	24	24 (30.5)

The barrier noise reduction is given by equation 1.97 in the text and the results of the calculations are summarised in the table below. The subscript "A" refers to the condition with no barrier and the subscript "B" refers to the condition with barrier. All quantities are in dB.

Octave band centre frequency (Hz)	$10 \log_{10} \sum 10^{-NR_{Ai}/10}$	$-10 \log_{10} \sum 10^{-NR_{Bi}/10}$	NR	SPL at receiver
63	2.3	1.8	4.1	65.9
125	2.1	3.7	5.8	69.2
250	1.9	6.0	7.9	64.1
500	1.6	8.9	10.5	49.5
1000	1.2	12.1	13.3	44.7
2000	1.0	14.8	15.8	40.2
4000	0.8	16.7	17.5	32.5
8000	0.9	17.3	18.2	33.8

The overall A-weighted level is calculated using the octave band levels as described on pages 101 and 102 in the text. The A-weighted level with the barrier is calculated using the numbers in the last column of the preceding table and is 58.2dB(A). The A-weighted level without the barrier is calculated using the octave band levels given in the problem and is 67dB(A). Thus the noise reduction due to the barrier is 9dB(A).

Problem 8.23

If the barrier were a building, the additional noise reduction may be calculated using equation 8.98, p394 of the text which may be rewritten as:

$$\Delta C = K \log_{10}(2\pi f b / 343)$$

The values of K are calculated using figure 8.17 in the text and trigonometry is used to calculate the angles, θ and φ . The results are summarised in the two following tables. Values for ΔC are in dB and the subscript on C refers to the path number.

	Path 1	Path 2	Path 3	Path 4	Paths 5&6	Paths 7&8
θ (degrees)	98	100	98	100	117	117
φ (degrees)	96	96	102	102	117	117
K	1.2	1.5	2.3	2.7	5.6	5.6

Octave band centre frequency (Hz)	ΔC_1	ΔC_2	ΔC_3	ΔC_4	$\Delta C_{5,6,7\&8}$
63	0.8	1.0	1.5	1.8	3.7
125	1.2	1.4	2.2	2.6	5.4
250	1.5	1.9	2.9	3.4	7.1
500	1.9	2.3	3.6	4.2	8.8
1000	2.2	2.8	4.3	5.0	10.4
2000	2.6	3.2	5.0	5.8	12.1
4000	3.0	3.7	5.7	6.7	13.8
8000	3.3	4.2	6.4	7.4	15.5

The noise reductions are calculated as before with the attenuation due to the barrier thickness added to each path. The results (in dB) are summarised in the table below.

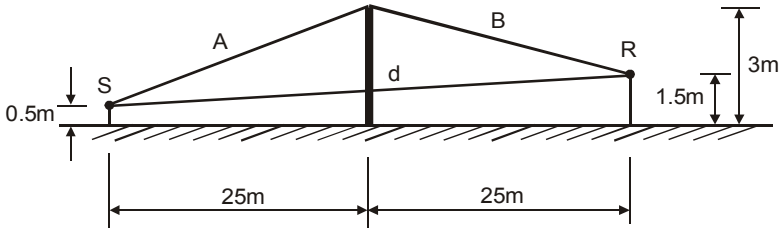
Octave band centre frequency (Hz)	NR_1	NR_2	NR_3	NR_4	$NR_{5\&6}$	$NR_{7\&8}$
63	9.0	10.1	11.6	12.2	16.0	17.5
125	10.7	11.8	14.3	15.2	19.4	22.4
250	12.3	14.4	17.2	18.9	25.2	27.8
500	14.9	17.8	21.5	23.3	30.0	33.5
1000	18.3	21.7	25.2	27.7	34.4	39.4
2000	21.8	25.6	29.5	32.6	36.1	42.1
4000	25.3	29.8	33.0	35.4	37.8	44.8
8000	27.3	31.2	33.4	37.4	39.5	46.0

The barrier noise reduction is given by equation 1.97 in the text and the results of the calculations are summarised in the table below. The subscript "A" refers to the condition with no barrier and the subscript "B" refers to the condition with barrier.

Octave band centre frequency (Hz)	$10 \log_{10} \sum 10^{-NR_{A_i}/10}$	$-10 \log_{10} \sum 10^{-NR_{B_i}/10}$	NR	SPL at receiver
63	2.3	3.6	5.9	64.1
125	2.1	6.0	8.1	66.9
250	1.9	8.6	10.5	61.5
500	1.6	12.0	13.6	46.4
1000	1.2	15.7	16.9	41.1
2000	1.0	19.3	20.3	35.7
4000	0.8	22.9	23.7	26.3
8000	0.9	24.5	25.4	26.6

The overall A-weighted level is calculated using the octave band levels as described on pages 101 and 102 in the text. The A-weighted level with the thick building as the barrier is calculated using the numbers in the last column of the preceding table and is 55.6(A) which is 2.6dB(A) less than for the thin barrier. Thus the effect of thickening the barrier is to increase the noise reduction by 2.6dB(A) to a total of approximately 11dB(A).

Problem 8.24



If the barrier is moved closer to the refrigeration unit, the noise reduction will increase because the Fresnel numbers will increase due to the increased path length from source to receiver over the top of the barrier and around the edges.

Problem 8.25

An elevation view of the situation is shown in the figure. As the barrier is indoors, equation 8.109 on p402 in the text is appropriate. Thus:

$$\begin{aligned}
 IL &= 10 \log_{10} \left(\frac{D_{\theta}}{4\pi r^2} + \frac{4}{S_0 \bar{\alpha}_0} \right) - 10 \log_{10} \left(\frac{D_{\theta} F}{4\pi r^2} + \frac{4K_1 K_2}{S(1 - K_1 K_2)} \right) \\
 &= \text{term1} - \text{term2}
 \end{aligned}$$

Source and receiver are each 25m from barrier. For an omnidirectional source on a hard floor $D_{\theta} = 2$; and $\frac{D_{\theta}}{4\pi r^2} = 6.366 \times 10^{-5}$.

$$r = d = [50^2 + 1^2]^{1/2} = 50.01 \text{ m}$$

$$A = (25^2 + 2.5^2)^{1/2} = 25.125 \text{ m} \quad B = (25^2 + 1.5^2)^{1/2} = 25.045 \text{ m}$$

$A + B - d = 0.160 \text{ m}$; $S = 2 \times 50 = 100 \text{ m}^2$ = area of gap between barrier and ceiling.

$$S_0 = 2(100 \times 5 + 50 \times 5 + 100 \times 50) = 11,500 \text{ m}^2; \quad \bar{\alpha}_0 = 0.08.$$

$$\text{Thus, } 4/S_0 \bar{\alpha}_0 = 4.348 \times 10^{-3}$$

$$S_1 = S_2 = S_0/2 + 3 \times 50 = 5,900\text{m}^2.$$

$$\bar{\alpha}_1 = \bar{\alpha}_2 = \frac{5750 \times 0.08 + 150 \times 0.15}{5900} = 0.082$$

Thus:

$$S_1 \bar{\alpha}_1 = S_2 \bar{\alpha}_2 = 482.5\text{m}^2 \quad \text{and} \quad K_1 = K_2 = \frac{100}{100 + 482.5} = 0.172$$

Referring to the first equation:

$$\text{term1} = 10 \log_{10} (6.366 \times 10^{-5} + 4.348 \times 10^{-3}) = -23.6\text{dB}$$

There is only one path over the top of the barrier, so $F = (3 + 10N)^{-1}$, where N is the Fresnel number for the path, calculated using equation 8.85 and figure 8.13 on page 388 in the text. Thus *term 2* can be written as:

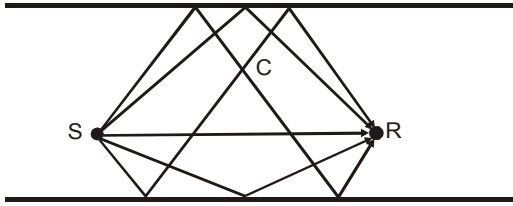
$$\begin{aligned} \text{term2} &= 10 \log_{10} \left(6.366 \times 10^{-5} F + \frac{4 \times 0.172^2}{100(1 - 0.172^2)} \right) \\ &= 10 \log_{10} (6.366 \times 10^{-5} F + 1.29 \times 10^{-3}) \end{aligned}$$

The calculations which are a function of frequency are summarised in the following table, where the barrier IL is given by $IL = \text{term 1} - \text{term 2}$.

Octave band centre frequency (Hz)	$N = \frac{2}{\lambda}(A + B - d)$	F	$\frac{D_0 F}{4\pi r^2}$	<i>term 2</i>	<i>IL</i>
63	0.059	0.28	1.78×10^{-5}	-29	5.4
125	0.117	0.240	1.53×10^{-5}	-29	5.4
250	0.233	0.188	1.12×10^{-5}	-29.1	5.5
500	0.466	0.131	8.34×10^{-6}	-29.1	5.5
1000	0.933	0.081	5.16×10^{-6}	-29.1	5.5
2000	1.865	0.046	2.93×10^{-6}	-29.1	5.5
4000	3.732	0.025	1.59×10^{-6}	-29.1	5.5
8000	7.46	0.013	8.20×10^{-7}	-29.1	5.5

Problem 8.26

- (a) The most dominant five paths from source to receiver are shown in the figure. We are to ignore any paths contributing less than 0.2dB to the



total. That is, we will ignore paths which contribute 10dB or more lower than the direct path. Of course, the correct way to do this problem is to use equation 7.110 in the text for a flat room with a diffusely reflecting floor (as there is furniture present) and specularly reflecting ceiling. However, here we are happy with an approximate solution. The direct path contribution is given by equations 5.158 and 5.160 in the text with DI_M and $A_E = 0$. Thus:

$$L_{pd} = L_w - 10 \log_{10}(4\pi \times 16) = L_w - 23 \text{ dB}$$

The second path (reflection from the floor) will be attenuated by:

$$\begin{aligned} & -10 \log_{10}(1 - \bar{\alpha}_f) + 10 \log_{10} \left(\frac{(1 + 1.2)^2 + 4^2}{4^2} \right) \\ & = -10 \log_{10}(1 - \bar{\alpha}_f) + 1.1 \end{aligned}$$

The third path (reflection from the ceiling) will be attenuated by:

$$\begin{aligned} & -10 \log_{10}(1 - \bar{\alpha}_c) + 10 \log_{10} \left(\frac{(2 + 1.8)^2 + 4^2}{4^2} \right) \\ & = -10 \log_{10}(1 - \bar{\alpha}_c) + 2.8 \end{aligned}$$

The fourth path (reflection from the floor then ceiling) will be attenuated by:

$$\begin{aligned}
& -10\log_{10}(1 - \bar{\alpha}_f)(1 - \bar{\alpha}_c) + 10\log_{10}\left(\frac{(1 + 3 + 1.8)^2 + 4^2}{4^2}\right) \\
& = -10\log_{10}(1 - \bar{\alpha}_f)(1 - \bar{\alpha}_c) + 4.9
\end{aligned}$$

The fifth path (reflection from the ceiling then floor) will be attenuated by:

$$\begin{aligned}
& -10\log_{10}(1 - \bar{\alpha}_f)(1 - \bar{\alpha}_c) + 10\log_{10}\left(\frac{(1.2 + 3 + 2)^2 + 4^2}{4^2}\right) \\
& = -10\log_{10}(1 - \bar{\alpha}_f)(1 - \bar{\alpha}_c) + 5.3
\end{aligned}$$

The contributions of each path to the final sound pressure level is obtained by subtracting the attenuations from the direct wave sound pressure level (arithmetically). The total sound pressure level is then obtained by logarithmically combining the contributions from each path (assuming incoherent combination).

The results are tabulated in the table below. Note that paths involving more reflections will result in a contribution of less than 0.2dB to the total and are thus not included. Perhaps you could prove this by trial and error.

frequency (Hz)	500	1000	2000
L_w	70	77	75
L_{pd}	47	54	52
L_{p2}	42.2	47.8	45.5
L_{p3}	38.2	38.2	29.2
L_{p4}	32.4	31	21.7
L_{p5}	31.9	30.5	21.2
L_{pt}	48.8	55.1	52.9

- (b) When the barrier is present, the only contributions to the sound level on the other side will be the wave reflected from the ceiling (L_{p3}) and the wave diffracted over the top, as all other paths are blocked by the barrier (do a sketch of the arrangement to prove this to yourself). For the wave diffracted over the top, we can write the following for the Fresnel number:

$$N = \frac{2 \times f}{c} \left[\sqrt{1.2^2 + 2^2} + \sqrt{1^2 + 2^2} - 4 \right] = 3.31 \times 10^{-3} f$$

The attenuation, Δ_b , is read from figure 8.14 in the text (point source) and corrected using equation 8.88 to give:

$$A_b = \Delta_b + 20 \log_{10} \left(\frac{2.282 + 2.282}{4} \right) = \Delta_b + 1.2$$

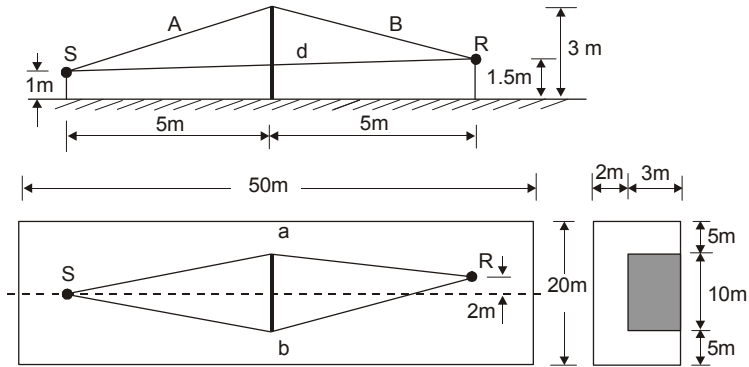
The contribution, L_{pb} of the wave diffracted over the top of the barrier to the sound field at the receiver is calculated using:

$$L_{pb} = L_{pd} - A_b$$

The results are summarised in the following table.

frequency (Hz)	500	1000	2000
L_{pd}	47	54	52
L_{p3}	39.9	33.9	24.4
N	1.65	3.3	6.6
A_b	16	19	22
L_{pb}	31	35	30
L_{pt}	39.0	39.9	33.3
reduction due to barrier	10	15	20

Assumptions:



- The barrier is sufficiently wide that the contribution at the receiver position due to refraction around the edges is negligible.
 - All waves at the receiver combine together incoherently.
 - The directivity of the source is uniform in all directions.
- (c) The noise reduction could be increased by moving the barrier sufficiently close to either the noise source or receiver that the wave reflected from the ceiling will also be blocked. In addition the barrier Fresnel number will be increased, further reducing the energy diffracted over the top of the barrier.

Problem 8.27

- (a) The situation is illustrated in the figure. As the barrier is indoors, equation 8.109 on p402 in the text is appropriate. Thus:

$$\begin{aligned}
 IL &= 10 \log_{10} \left(\frac{D_{\theta}}{4\pi r^2} + \frac{4}{S_0 \alpha_0} \right) - 10 \log_{10} \left(\frac{D_{\theta} F}{4\pi r^2} + \frac{4K_1 K_2}{S(1 - K_1 K_2)} \right) \\
 &= \text{term1} - \text{term2}
 \end{aligned}$$

First we calculate the Fresnel Numbers for diffraction over the top and around the edges using equation 8.85 and figure 8.13 on page 388 in the text.

Diffraction over the top

$$X_R = X_S = 5\text{m}; Z_R = 1.5\text{m}; Z_S = 1\text{m}; Y = 2\text{m}; h = 3\text{m}$$

$$Y_R = 2 \times 5 / (5 + 5) = 1$$

$$d = [(5 + 5)^2 + (1 + 1)^2 + (1.5 - 1)^2]^{1/2} = 10.210\text{m}$$

$$A = [5^2 + 1^2 + (3 - 1)^2]^{1/2} = 5.477\text{m}$$

$$B = [5^2 + 1^2 + (3 - 1.5)^2]^{1/2} = 5.315\text{m}$$

$$\text{Fresnel Number, } N = (2f/343)(A + B - d) = 3.393 \times 10^{-3}f$$

Diffraction around edge "a"

$$X_R = X_S = 5\text{m}; Z_R - Z_S = 2\text{m}; Y = 0.5\text{m}; h - Z_S = 5\text{m};$$

$$h - Z_R = 3\text{m}$$

$$Y_R = 0.5 \times 5 / (5 + 5) = 0.25$$

$$d = [(5 + 5)^2 + 4 \times 0.25^2 + 4]^{1/2} = 10.210\text{m}$$

$$A = [5^2 + 0.25^2 + 5^2]^{1/2} = 7.075\text{m}$$

$$B = [5^2 + 0.25^2 + 3^2]^{1/2} = 5.836\text{m}$$

$$\text{Fresnel Number, } N_a = (2f/343)(A + B - d) = 1.575 \times 10^{-2}f$$

Diffraction around edge "b"

$$X_R = X_S = 5\text{m}; Z_R - Z_S = 2\text{m}; Y = 0.5\text{m}; h - Z_S = 5\text{m};$$

$$h - Z_R = 7\text{m}$$

$$Y_R = 0.5 \times 5 / (5 + 5) = 0.25$$

$$d = [(5 + 5)^2 + 4 \times 0.25^2 + 4]^{1/2} = 10.210\text{m}$$

$$A = [5^2 + 0.25^2 + 5^2]^{1/2} = 7.075\text{m}$$

$$B = [5^2 + 0.25^2 + 7^2]^{1/2} = 8.606\text{m}$$

$$\text{Fresnel Number, } N_b = (2f/343)(A + B - d) = 3.190 \times 10^{-2}f$$

$$S_0 \bar{\alpha}_0 = 2(5 \times 20 + 5 \times 50 + 20 \times 50) \times 0.08 = 216.0\text{m}^2$$

$$S = 2 \times 5 \times 3 + 20 \times 2 = 70\text{m}^2$$

$$S_1 = S_2 = 30 + 1350 = 1380\text{m}^2$$

$$\bar{\alpha}_1 = \bar{\alpha}_2 = (216/2 + 30\bar{\alpha}_b)/(1350 + 30)$$

$$r = d = 10.210\text{m}; DI = 5, \text{ thus } D_\theta = 3.16 \text{ and}$$

$$D_\theta/4\pi r^2 = 2.51 \times 10^{-3}$$

$$\text{term1} = 10 \log_{10} \left(\frac{D_{\theta}}{4\pi r^2} + \frac{4}{S_0 \bar{\alpha}_0} \right) = -16.8 \text{ dB}$$

$$\text{and } F = \sum_{a,b,c} \frac{1}{3 + 10N_i}$$

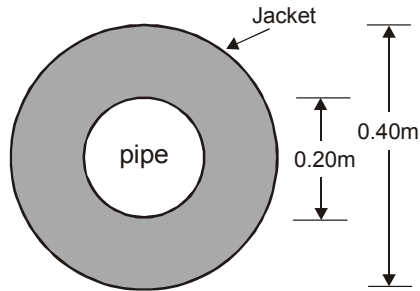
The quantities K_1 and K_2 are calculated using equation 8.111 in the text. They are frequency dependent quantities, but all other variables needed for their calculation have been evaluated above. We now proceed to evaluate the second term of equation 8.109 as a function of octave band centre frequency. Results for the second term as well as the overall Insertion Loss of the barrier are tabulated in the following table.

Octave band centre frequency (Hz)	63	125	250	500	1000	2000	4000
N (Top)	0.21	0.42	0.85	1.70	3.39	6.79	13.57
N_a	0.99	1.97	3.94	7.88	15.75	31.5	63.0
N_b	2.01	3.99	7.98	15.95	31.9	63.8	127.6
F	0.315	0.205	0.123	0.068	0.036	0.019	0.010
$\bar{\alpha}_b$	0.08	0.25	0.83	1.0	1.0	1.0	1.0
(table 7.1)							
$\bar{\alpha}_1, \bar{\alpha}_2$	0.08	0.08	0.10	0.10	0.10	0.10	0.10
K_1, K_2	0.388	0.377	0.345	0.337	0.337	0.337	0.337
$\frac{D_{\theta} F}{4\pi r^2} +$	1.09×10^{-2}	9.98×10^{-3}	8.02×10^{-3}	7.46×10^{-3}	7.39×10^{-3}	7.34×10^{-3}	7.32×10^{-3}
$\frac{4K_1 K_2}{S(1 - K_1 K_2)}$							
Insertion Loss (dB)	2.8	3.2	4.2	4.5	4.5	4.5	4.6

- (b) Moving the barrier closer to the source will increase the value of N which will reduce the value of F and thus the direct field contribution. The value of K_1 will be reduced and the term involving $K_1 K_2$ will be reduced slightly. Thus we can expect an increase in the barrier insertion loss. This effect is expected to be only 1 or 2dB but calculations as were done in part (a) are necessary to verify this.
- (c) Extending the barrier length will increase the barrier Insertion Loss because it will remove the contribution from waves diffracted around the barrier ends and will also reduce the reverberant field contribution because of a smaller gap area and also because of an increase in the effective room absorption. This effect should be slightly larger than the effect in part (b) above but would still be restricted to a few dB. The cost effectiveness of this action can only be assessed by completing the calculations as was done in part (a) of the problem and fully evaluating the benefit of the increased noise reduction vs the inconvenience of restricted passage.

Problem 8.28

This is a pipe lagging problem so we follow the procedure on pages 404 and 405 in the text with the errata corrected in equations 8.116, 8.117 and 8.119. The thickness of the jacket is $h = 6/2700 = 2.22\text{mm}$ and the diameter of the jacket, $d = 0.2 + 2 \times 0.1 + 0.0022 = 0.4022\text{m}$. The quantity, $1000(m/d)^{1/2} = 3862$. The longitudinal wavespeed is:



$$c_L = \sqrt{E/[\rho_m(1 - \nu^2)]} = \sqrt{71.6/[2700(1 - 0.34^2)]} = 5476 \text{ m/s}$$

The ring frequency is:

$$f_r = \frac{c_L}{\pi d} = \frac{5476}{\pi \times 0.4022} = 4,334 \text{ Hz}$$

The critical frequency is obtained using the equation on p337 as:

$$f_c = \frac{0.55 c^2}{c_L h} = \frac{0.55 \times 343^2}{5476 \times 0.00222} = 5,323 \text{ Hz}$$

For the Insertion Loss calculations, we use equation 8.112 on p404 for octave bands of 4000Hz and below and equation 8.115 for the 8000Hz octave band. The results of the calculations are summarised in the table below at octave band centre frequencies. Of course, if three 1/3 octave bands are averaged, then the result would be slightly different. In addition, equation 8.120 has also been used to generate an alternative set of Insertion Loss predictions. With equation 8.120, the Insertion Loss is given by:

$$IL = \frac{40}{1 + 0.12/0.2} \log_{10} \left(\frac{f\sqrt{6 \times 0.1}}{132} \right) = 25 \log_{10} (f \times 0.00587)$$

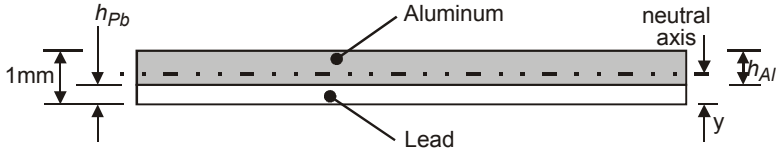
which is valid for frequencies defined by $f \geq 120/\sqrt{6 \times 0.1} = 155 \text{ Hz}$.

Octave band centre frequency (Hz)	ff_r or ff_c	C_r or C_c	X_r or X_c	Insertion Loss eqns. 8.112 - 8.119	Insertion Loss eqn.8.120
63	0.0145	0.1090	789.5	-18.8	-
125	0.0288	0.2163	1307	7.6	-
250	0.0577	0.4326	1731	17.9	4.2
500	0.115	0.8653	2120	25.5	11.7
1000	0.231	1.7305	2479	29.4	19.2
2000	0.461	3.4610	2717	19.4	26.7
4000	0.923	6.9220	1991	22.6	34.3
8000	1.503	15.849	4289	15.9	41.8

The Insertion Loss results are somewhat different between the two methods of calculation and as we shall see in the next problem, neither prediction scheme is particularly good. However, it is supposed that the true results lie somewhere between the two.

Problem 8.29

(a) The jacket cross section is shown schematically in the following figure.



Let x mm be the thickness of the aluminium part of the jacket. Then:

$$x \times 10^{-3} \times 2700 + (1 - x) \times 10^{-3} \times 11300 = 6$$

Thus $x = 0.616$ mm = thickness of Aluminium.

Thickness of lead = $1 - 0.616 = 0.384$ mm

Effective surface mass is given by:

$$m_{eff} = \rho_1 h_1 + \rho_2 h_2 = 6.00 \text{ kg/m}^2$$

The neutral axis location is given by:

$$y = \frac{(h_{Pb} + h_{Al}/2)E_{Al} + E_{Pb}h_{Pb}/2}{E_{Al} + E_{Pb}}$$

$$= \frac{(0.384 + 0.616/2) \times 71.6 + 16.5 \times 0.384/2}{71.6 + 16.5} = 0.598 \text{ mm}$$

The bending stiffness is given by:

$$B_{eff} = \frac{E_{Al}h_{Al}}{12} \frac{(h_{Al}^2 + (h_{Pb} + h_{Al}/2 - y)^2)}{1 - \nu_{Al}^2}$$

$$+ \frac{E_{Pb}h_{Pb}}{12} \frac{(h_{Pb}^2 + (y - h_{Pb}/2)^2)}{1 - \nu_{Pb}^2}$$

$$= \frac{71.6 \times 0.616}{12} \frac{(0.616^2 + 12 \times (0.384 + 0.308 - 0.598)^2)}{1 - 0.34^2}$$

$$+ \frac{16.5 \times 0.384}{12} \frac{(0.384^2 + 12 \times (0.598 - 0.192)^2)}{1 - 0.44^2}$$

$$= 2.017 + 1.392 = 3.41 \text{ kg m}^2 \text{ s}^{-2}$$

Assume that the effective Poisson's ratio is that of Aluminium and equal to 0.34. Thus the longitudinal wave speed is:

$$c_L = \frac{\sqrt{12}}{h} \sqrt{\frac{B_{eff}}{m_{eff}}} = \frac{\sqrt{12}}{0.001} \times \sqrt{\frac{3.409}{6}} = 2611 \text{ m/s}$$

The critical frequency is given by equation 8.3 in the text as:

$$f_c = \frac{c^2}{2\pi} \sqrt{\frac{m}{B}} = \frac{343^2}{2\pi} \sqrt{\frac{6}{3.409}} = 24.84 \text{ kHz}$$

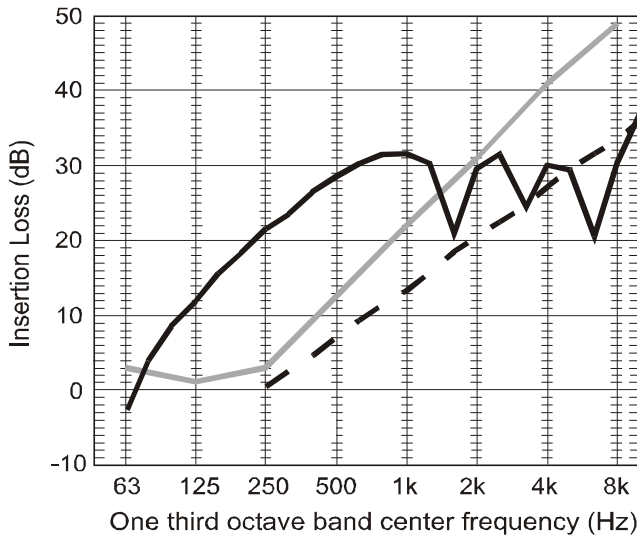
and the ring frequency is given on p404 in the text as:

$$f_r = \frac{c_L}{\pi d} = \frac{2611}{\pi \times 0.251} = 3311 \text{ Hz}$$

The jacket insertion loss may be calculated using either equation 8.120 or equations 8.112 to 8.119 in the text. We will use both methods here as a comparison. The quantities used in the equations are $m = 6 \text{ kg/m}^2$, $\ell = 0.05 \text{ m}$, $d = 0.15 + 2 \times 0.05 = 0.25 \text{ m}$ and $D = 0.15 \text{ m}$. The results of the calculations at the 1/3 octave band centre frequencies are summarised in the following table. Note that the calculations using equation 8.120 are only valid for frequencies defined by $f \geq 120/\sqrt{6 \times 0.05} = 220 \text{ Hz}$.

Octave band centre frequency (Hz)	X_r or X_m	C_r or C_c	Insertion Loss (dB)	
			Eqs. 8.112 - 8.119	Eq. 8.120
63	1279	0.1137	-2.4	-
80	1500	0.1444	4.1	-
100	1690	0.1805	8.5	-
125	1869	0.2256	12.1	-
160	2056	0.2888	15.7	-
200	2220	0.3610	18.6	
250	2380	0.4512	21.3	0.35
315	2543	0.5685	23.9	2.6
400	2708	0.7219	26.3	4.9
500	2859	0.9024	28.4	7.0
630	3010	1.1370	30.2	9.3
800	3157	1.4439	31.5	11.6
1000	3280	1.8048	31.7	13.7
1250	3378	2.2560	30.1	15.9
1600	3435	2.8877	21.1	18.3
2000	3403	3.6096	24.9	20.4
2500	3193	4.5120	31.4	22.6
3150	2258	5.6852	24.1	24.8
4000	2714	1.868	30.0	27.1
5000	3356	2.335	29.5	29.3
6300	4134	2.942	20.7	31.5
8000	5038	3.736	30.4	33.8
10000	5893	4.670	37.0	36.0

The data in the table are plotted in the following figure where the solid black line represents the theory embodied in equations 8.112-8.119 and the dashed black line represents the theory embodied in equation 8.120. In addition, some experimental data (solid grey line) from the text book (figure 8.18, second edition) are shown for comparison. It can be seen that neither theory provides a very good prediction of the measured data. Unfortunately there are no better theories available at present.

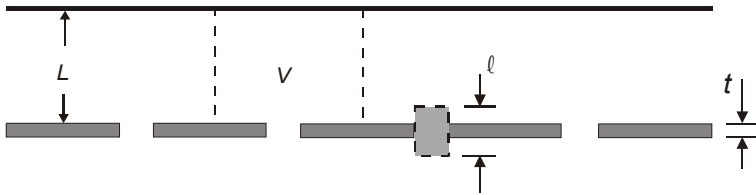


- (b) It is clear from the equations used for either calculation method that increasing the mass of the liner will increase the low frequency Insertion Loss. For the first prediction scheme, reducing c_L will also result in increased values for the low frequency Insertion Loss.
- (c) One advantage of porous acoustic foam over rockwool is that the porous foam will support the weight of the jacket indefinitely whereas rockwool will gradually compress and in a high vibration environment, it will turn to powder.

A disadvantage of foam is that it is not fireproof and if ignited it emits toxic gas. Another disadvantage is that the foam is much more expensive than rockwool.

Solutions to problems relating to muffling devices

Problem 9.1



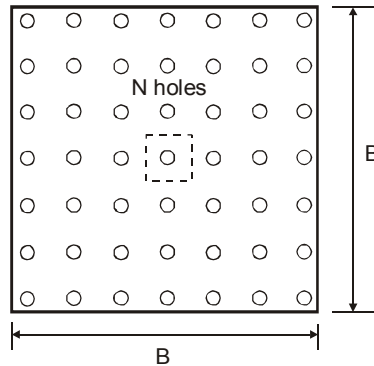
Referring to the figure, we can imagine that each hole in the perforated sheet represents a neck of a Helmholtz resonator with the volume associated with each neck being equal to the total volume behind the perforated sheet divided by the number of holes. Let L be the depth of the backing cavity and let us consider a section of sheet of dimensions $B \times B$ with a number of holes, N .

The total backing volume is then LB^2 and the volume associated with one hole is LB^2/N . The percent open area is given by:

$$P = \frac{100N}{B^2} \frac{\pi d^2}{4}.$$

Thus the effective resonator volume is:

$$V = \frac{100L}{P} \frac{\pi d^2}{4} = \frac{100LA}{P},$$



where A is the neck cross-sectional area.

From equation 9.38 in the text:

$$f_0 = \frac{c}{2\pi} \sqrt{\frac{A}{LV}} = \frac{c}{2\pi} \sqrt{\frac{P}{100L\ell}}$$

where $\ell = 2\ell_0 + t$ is the effective length of the neck and $2\ell_0$ is the total effective end correction for the hole. Comparing the above equation with equation 7.77 in the text gives the effective end correction of the perforate as:

$$2\ell_0 = 0.85d(1 - 0.22d/a)$$

The Helmholtz model is appropriate because the system is effectively a small mass of air vibrating against a stiffness represented by the backing volume.

Problem 9.2

- (a) Diameter, 0.4m, so higher order mode cut on frequency is $f = 0.586c/0.4 = 500$ Hz. Thus at 200 Hz only plane waves propagate.

$$p_i = A e^{j(\omega t - kx)} ; \quad p_r = B e^{j(\omega t + kx + \theta)}$$

$$p_T = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx + \theta)}$$

$$u_T = \frac{1}{\rho c} (A e^{j(\omega t - kx)} - B e^{j(\omega t + kx + \theta)})$$

$$\begin{aligned} Z = \frac{p_T}{u_T} &= \rho c \left[\frac{A e^{-jkx} + B e^{j(kx + \theta)}}{A e^{-jkx} - B e^{j(kx + \theta)}} \right] = \rho c \left[\frac{A + B e^{j(2kx + \theta)}}{A - B e^{j(2kx + \theta)}} \right] \\ &= \rho c \left[\frac{A/B + e^{j(2kx + \theta)}}{A/B - e^{j(2kx + \theta)}} \right] \end{aligned}$$

Minimum pressure occurs when $\theta = -2kx + \pi$; $x = 1.8$ m

At 200 Hz, $k = 2\pi f/c = 2\pi \times 200/343 = 3.664$;

$\theta = -2 \times 3.664 \times 1.8 + \pi = -10.048^\circ$.

Adding 4π , gives $\theta = 2.518^\circ = 144.2^\circ$

$$\frac{A}{B} = \frac{10^{8/20} + 1}{10^{8/20} - 1} = \frac{3.5119}{1.5119} = 2.323$$

At $x = 2$, $2kx = 14.656$ and $2kx + \theta = 4.608^\circ = 264$ degrees. Thus:

$$Z = 413 \left[\frac{2.323 + \cos(4.608) + j\sin(4.608)}{2.323 - \cos(4.608) - j\sin(4.608)} \right] = 413 \left[\frac{2.219 + j(-0.995)}{2.427 - j(-0.995)} \right]$$

Multiplying numerator and denominator by complex conjugate of the denominator gives:

$$\begin{aligned} Z &= 413 \left[\frac{2.219 - j0.995}{2.427 + j0.995} \right] = 413 \left[\frac{(2.219 - j0.995) \times (2.427 - j0.995)}{(2.427 + j0.995) \times (2.427 - j0.995)} \right] \\ &= 413 \left[\frac{4.395 - j4.623}{6.88} \right] \end{aligned}$$

Thus, $Z = 264 - j278$

- (b) $\text{Re}\{Z_A\} = \text{Re}\{Z/A_{\text{orifice}}\}$ The hole impedance is in parallel with the rigid plate containing it. If the rigid plate impedance is effectively infinity, the combined specific acoustic impedance is that of the hole and this is what is measured by the standing wave. The acoustic impedance of the hole is the specific acoustic impedance divided by the area of the hole only. If cross flow dominates resistance, $M = \text{Re}\{Z_A\} A_{\text{orifice}}/\rho c$
Thus,

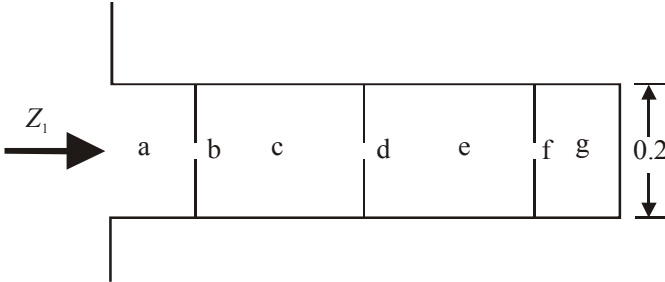
$$M = \text{Re}\{Z\}/\rho c = \frac{263.8}{413} = 0.64$$

- (c) End correction = no flow correction $\times (1 - M)^2$. Hole radius = 0.05 m
Thus total end correction is:

$$\ell_0 = \left[0.61 \times 0.05 + \frac{8 \times 0.05}{3\pi} (1 - (1.25 \times 0.25)) \right] \times (1 - 0.639)^2 = 0.0078 \text{ m}$$

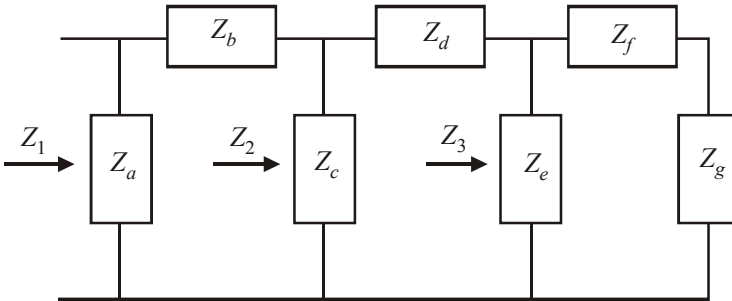
Problem 9.3

A quarter wave tuning stub works by changing the wall impedance of a duct on which it is mounted so that downstream propagating waves are reflected back upstream. The viscous losses at the entrance to the stub also account for some of the energy loss, because of the large particle motions near the edges of the entrance. The stub can also act by changing the radiation impedance (and thus the amount of power radiated) of the source producing the noise.

Problem 9.4

- (a) The impedance looking into the stub must be zero as we are ignoring the resistive component.

The equivalent electrical circuit is shown in the figure below:



$$\text{Impedance looking in at location 3} = Z_3 = \frac{Z_e(Z_f + Z_g)}{Z_e + Z_f + Z_g}$$

$$\text{Impedance looking in at location 2} = Z_2 = \frac{Z_c(Z_d + Z_3)}{Z_c + Z_d + Z_3}$$

$$\text{Impedance looking in at location 1} = Z_1 = \frac{Z_a(Z_b + Z_2)}{Z_a + Z_b + Z_2}$$

If we neglect resistive impedance, then $Z_1 = 0$ and

$Z_b = -Z_2 = -\frac{Z_c(Z_d + Z_3)}{Z_c + Z_d + Z_3}$. If we now substitute the expression for Z_3

into the preceding equation, we obtain:

$$Z_b \left(Z_c + Z_d + \frac{Z_e(Z_f + Z_g)}{Z_e + Z_f + Z_g} \right) + Z_c \left(Z_d + \frac{Z_e(Z_f + Z_g)}{Z_e + Z_f + Z_g} \right) = 0 \quad (1)$$

We now need to substitute in physical dimensions in place of the impedances. $Z_b = Z_d = Z_f = j\frac{\rho\omega\ell}{A}$ and the effective length of the hole is given by:

$$\ell = 2\ell_0 = \frac{8d}{3\pi} \left(1 - 1.25 \frac{d}{0.2} \right).$$

The density, $\rho = 1.206$, $A = \pi d^2/4$ and $\omega = 200\pi$.

The volumes between the orifices are given by:

$$V_e = V_c = \frac{\pi D^2 L}{4 \cdot 3} = 0.01047L \text{ and}$$

$$V_a = V_g = 0.5V_e = 0.005236L.$$

Required length, $L = 0.5 \times \lambda/4 = c/8f = 343/800 = 0.429\text{m}$.

Using equation 9.35, we obtain:

$$Z_c = Z_e = -j \frac{1.206 \times 343^2}{0.01047 \times 0.4288 \times 2 \times \pi \times 100} = -j5.03 \times 10^4.$$

$$Z_a = Z_g = 2Z_c = -j1.01 \times 10^5.$$

Equation (1) can now be solved by computer for d . The result is $d = 44\text{mm}$.

- (b) The device with baffles would have a much larger resistive impedance than the one without baffles because of all the cross-sectional changes at which there will be viscous losses. The larger resistive impedance will lower the quality factor and thus the peak attenuation, although the bandwidth of significant attenuation will increase.

Problem 9.5

- (a) To calculate the resistive impedance, equation 9.29 on p417 in the text may be used. First we will evaluate the variables used in the equation.

$$\rho c = 343 \times 1.206 = 414, A = \pi \times 0.044^2/4 = 0.00152 \text{ m}^2,$$

$$k = 2\pi \times 100/343 = 1.832,$$

$$t = h = \sqrt{2 \times 1.8 \times 10^{-5} / (1.206 \times 2\pi \times 100)} = 2.180 \times 10^{-4},$$

$w = 0$, $\varepsilon = 0$ and $M = 0$. Substituting these values into equation 9.29, we obtain:

$$R_s = \frac{414}{0.00152} \left[0.288 \times 1.832 \times 2.180 \times 10^{-4} \log_{10} \left(\frac{4 \times 0.00152}{\pi \times (2.180 \times 10^{-4})^2} \right) \right]$$

$$= 146 \text{ MKS Rayls}$$

- (b) At the design frequency (100Hz), the Insertion Loss is found by substituting R_s for Z_s in equation 9.46 in the text to give:

$$IL = 20 \log_{10} \left(1 + \frac{Z_d}{R_s} \right)$$

If the side branch is mounted an odd number of quarter wavelengths from the end of the duct, the reactive part of the impedance Z_d will theoretically be infinite (based on equation 9.14) and the side branch Insertion Loss will also be infinite as indicated by the above equation. However, in practice, the quarter wave tube is of finite cross section and Z_d cannot therefore be too large. This results in the Insertion Loss being finite and usually limited to 25 to 30dB.

Problem 9.6

- (a) Closed end side branches have less viscous losses than open ended tubes because of one less cross-sectional change and corresponding edge. Thus the quality factor and therefore the peak attenuation for open ended tubes will be smaller, although the bandwidth of significant attenuation will increase.
- (b) Equation 9.46 in the text may be used here. The downstream duct impedance for an infinite duct is simply $\rho c/A$. Thus the ratio Z_d/Z_s in equation 9.46 is given by:

$$\frac{Z_d}{Z_s} = \frac{1}{j(\omega - 10^4/\omega - j\omega \times 10^{-4})}$$

When the Insertion Loss is at its peak, the side branch reactive impedance, Z_s is zero. Thus:

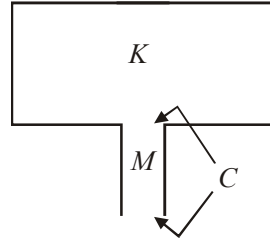
$$\omega^2 = 10^4 \quad \text{and} \quad \omega = 100 \text{ rad/s}$$

The Insertion Loss is then:

$$IL = 20 \log_{10} \left| 1 + \frac{Z_d}{Z_s} \right| = 20 \log_{10} \left| 1 + \frac{10^4}{\omega} \right| = 40 \text{ dB}$$

Problem 9.7

The equation is simply a differential equation of the vibration of a mass acting against a spring. The quantity, M , represents the vibrating mass of air in the resonator neck, the quantity, C , represents the viscous damping losses at the entrance and exit of the neck caused by motion of the air particles relative to the edges of the neck and the quantity, K , represents the stiffness of the volume of air in the cavity of the resonator as shown in the figure.



Problem 9.8

- (a) Helmholtz resonator - cylindrical cavity, 100mm radius, 200mm high.
Neck 40mm long with a radius of 30mm.

$$\text{Volume of cylinder, } V = \pi \times (0.1)^2 \times 0.2 = 6.28 \times 10^{-3} \text{ m}^3$$

$$\text{Area of neck, } A = \pi \times (0.03)^2 = 2.83 \times 10^{-3} \text{ m}^2.$$

$$\text{Length of neck, } \ell = 0.04\text{m} + \text{end corrections.}$$

$$\text{end correction} = 0.61 \times 0.03 + \frac{8 \times 0.03}{3\pi} (1 - 1.25 \times 0.3) = 0.0342 \text{ m}$$

$$\text{so } \ell = 0.0742 \text{ m.}$$

From equation 9.38 in the text:

$$\omega_0 = c\sqrt{A/\ell V} = 343 \left(\frac{2.83 \times 10^{-3}}{0.0742 \times 6.28 \times 10^{-3}} \right)^{1/2} = 844.6 \text{ rad/sec}$$

Thus, $f_0 = 844.6/2\pi = 134\text{Hz}$.

- (b) The quality factor may be calculated using equation 9.40 in the text. Thus:

$$Q = \frac{\rho c}{R_s} \sqrt{\ell/AV} = \frac{413.66}{1000} \left(\frac{0.0742}{2.83 \times 6.28 \times 10^{-6}} \right)^{1/2} = 26.7$$

- (c) If we define effectiveness as the frequency range within 3dB of the maximum, then the bandwidth of effectiveness is obtained using equation 7.22 in the text as $\Delta f = f/Q = 135/26.7 = 5.0\text{Hz}$. Thus we can expect the device to be effective between 132 and 137Hz.

- (d) Incident plane wave of 80dB L_p and 135Hz.

$$\text{Power dissipated} = \frac{\langle p^2 \rangle}{|Z_s|} = \frac{\langle p^2 \rangle}{R_s} \text{ at resonance.}$$

The mean square sound pressure is:

$$\langle p^2 \rangle = 4 \times 10^{-10} \times 10^8 = 0.04 \text{Pa}^2$$

$$\text{Thus the power dissipated} = \frac{0.04}{1000} = 40 \mu\text{Watts}$$

- (e) Power in a plane wave $= IA = \frac{\langle p^2 \rangle A}{\rho c} = 40 \times 10^{-6}$

$$\text{Thus area of plane wave, } A = 413.6 \times 40 \times 10^{-6} / 0.04 = 0.4136 \text{m}^2$$

- (f) The sabine absorption of the resonator, $S\bar{\alpha} = 0.414 \text{m}^2$

Cross-sectional area of resonator $= \pi \times 0.1^2 = 0.031 \text{m}^2$. Thus the cross-sectional area of the resonator volume is 13 times smaller than its Sabine absorption area. Thus to make the wall look anechoic, we would need $1/0.414 = 2.4$ resonators per square metre.

- (g) From equation 1.4 in the text, an ambient temperature variation from -5°C to 45°C corresponds to a speed of sound change from

$\sqrt{1.4 \times 8.314 \times 268 / 0.029}$ to $\sqrt{1.4 \times 8.314 \times 318 / 0.029}$, which is a range from 328 to 357m/s. From equation 9.38, it can be seen that the resonance frequency of the resonator will vary from 808 to 880 rad/sec which corresponds to a range from 129 to 140Hz which is outside the 3dB range discussed in part (c). The problem could be overcome by

using two additional types of resonator with resonance frequencies of 131.5 and 137.5Hz respectively at 20°C.

Problem 9.9

- (a) The total acoustic pressure anywhere along the tube is the sum of the incident and end reflected pressures and may be written in terms of two complex constants, A_i and A_r , representing the complex amplitudes of the incident and reflected (from $x = 0$) waves respectively. Thus:

$$p_t = p_i + p_r = A_i e^{j(\omega t + kx)} + A_r e^{j(\omega t - kx)}$$

Using equations 1.6, and 1.7, the acoustic particle velocity may be written as:

$$u_t = u_i + u_r = \frac{1}{\rho c} (A_i e^{j(\omega t + kx)} - A_r e^{j(\omega t - kx)})$$

At $x = 0$, $\frac{p_t}{S u_t} = Z_L$. Thus:

$$\frac{\rho c}{S} \frac{A_i + A_r}{A_i - A_r} = Z_L = Z_c \frac{A_i + A_r}{A_i - A_r}$$

At $x = L$:

$$\frac{p_t}{S u_t} = Z_i = Z_c \frac{A_i e^{jkL} + A_r e^{-jkL}}{A_i e^{jkL} - A_r e^{-jkL}}$$

Expanding the exponents gives:

$$\begin{aligned}
 Z_i &= Z_c \frac{A_i \cos(kL) + jA_i \sin(kL) + A_r \cos(kL) - jA_r \sin(kL)}{A_i \cos(kL) + jA_i \sin(kL) - A_r \cos(kL) + jA_r \sin(kL)} \\
 &= Z_c \frac{A_i + jA_i \tan(kL) + A_r - jA_r \tan(kL)}{A_i + jA_i \tan(kL) - A_r + jA_r \tan(kL)} \\
 &= Z_c \frac{A_i + A_r + j \tan(kL)(A_i - A_r)}{A_i - A_r + j \tan(kL)(A_i + A_r)}
 \end{aligned}$$

Rearranging gives:

$$Z_i = \frac{\frac{A_i + A_r}{A_i - A_r} + j \tan(kL)}{1 + \frac{A_i + A_r}{A_i - A_r} j \tan(kL)} = Z_c \frac{\frac{Z_L}{Z_c} + j \tan(kL)}{1 + \frac{Z_L}{Z_c} j \tan(kL)}$$

Rearranging gives:

$$Z_i = Z_c \left[\frac{Z_L + jZ_c \tan(kL)}{Z_c + jZ_L \tan(kL)} \right]$$

$$\text{(b) (i) } Z_c = \frac{\rho c}{S} = \frac{414}{0.2 \times 0.2} = 10,400 \text{ MKS Rayls}$$

$$\text{(ii) At } x = 0, p_r/p_i = 0.5; \text{ thus } A_r/A_i = 0.5 \text{ and}$$

$$\frac{A_i + A_r}{A_i - A_r} = \frac{1 + 0.5}{1 - 0.5} = 3$$

Thus, $Z_L = 3Z_c = 31,000$ MKS Rayls

$$\begin{aligned}
 \text{(iii) } Z_i &= 10350 \frac{31050 + j10350 \tan(200\pi \times 10.29/343)}{10350 + j31050 \tan(200\pi \times 10.29/343)} \\
 &= 10350 \frac{31050}{10350} = 31000 \text{ MKS Rayls}
 \end{aligned}$$

Problem 9.10

(a) The wavenumber is:

$$\kappa_n^2 = (\omega/c)^2 - [(n_x\pi/L_x)^2 + (n_y\pi/L_y)^2]$$

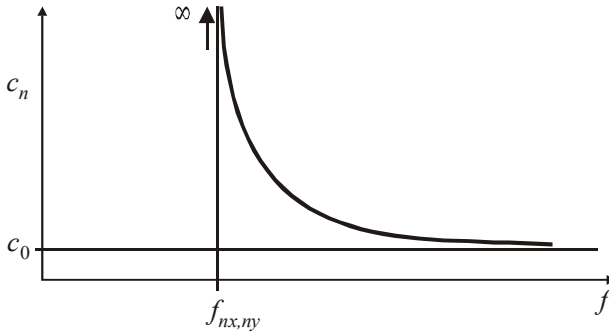
(i) Cut-on when $\kappa^2 = 0$. That is when:

$$(\omega/c)^2 = [(n_x\pi/L_x)^2 + (n_y\pi/L_y)^2]$$

(ii) Phase speed is defined as $c_n = \omega/\kappa_n$. Thus:

$$c_n = \left[\frac{\omega^2}{\left(\left(\frac{\omega}{c} \right)^2 - \left(\frac{n_x\pi}{L_x} \right)^2 - \left(\frac{n_y\pi}{L_y} \right)^2 \right)} \right]^{1/2}$$

This equation is shown sketched in the figure below, where $\omega = 2\pi f$.



(b) Drop in L_p over a distance of $L_y/4$ for 2,2 mode.

Cut-on frequency is given by:

$$\begin{aligned} \omega_{2,2} &= 343 \left[\left(\frac{4\pi}{L_y} \right)^2 + \left(\frac{2\pi}{L_y} \right)^2 \right]^{1/2} \\ &= \frac{343}{L_y} [16\pi^2 + 4\pi^2]^{1/2} = \frac{4819}{L_y} \end{aligned}$$

Excitation frequency $= 0.75\omega_{2,2} = 3614/L_y$ rad/sec. The wavenumber is then:

$$\kappa_n^2 = \left[\frac{3614}{343L_y} \right]^2 - \left[\frac{16\pi^2 + 4\pi^2}{L_y^2} \right]$$

Thus, $\kappa_{2,2} = \pm j \frac{9.293}{L_y}$ at the excitation frequency. The acoustic pressure

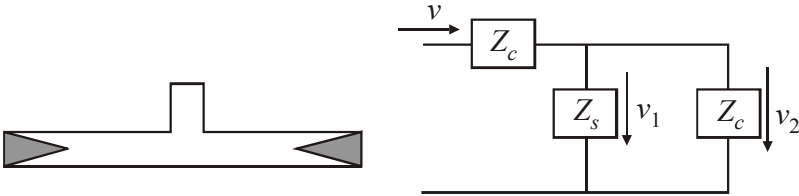
is related to the distance along the duct as $p(x) \propto e^{-j\kappa_n x}$. Thus

$$\frac{p(x_1)}{p(x_2)} = \frac{e^{-j\kappa_{2,2}x_1}}{e^{-j\kappa_{2,2}x_2}}. \text{ Setting } x_1 = 0 \text{ and } x_2 = L_y/4, \text{ we can write the following}$$

for the reduction in sound pressure level as the wave travels from x_1 to x_2 .

$$\Delta L_p = 20 \log_{10} \left[\frac{1}{e^{-j\kappa_{2,2}L_y/4}} \right] = 20 \log_{10} \left[\frac{1}{e^{-9.294/4}} \right] = 20.2 \text{ dB}$$

Problem 9.11



(a) The transmitted power is given by:

$$W_t = 0.5 \operatorname{Re} \{ p_2 v_2^* \} = 0.5 \operatorname{Re} \{ Z_c v_2 v_2^* \} = 0.5 \operatorname{Re} \{ Z_c \} |v_2|^2$$

A similar expression can be derived for the incident power. Thus the transmission coefficient is given by:

$$\tau = \frac{W_t}{W_i} = \frac{0.5 \operatorname{Re} \{ Z_c \} |v_2|^2}{0.5 \operatorname{Re} \{ Z_c \} |v|^2} = \left| \frac{v_2}{v} \right|^2$$

The characteristic impedance is defined as $Z_c = \rho c/A$. Thus:

$$R + jX = \frac{Z_s}{Z_c}$$

The circuit equations are $v = v_1 + v_2$ and $v_1 Z_s = v_2 Z_c$ from which:

$$v = v_2 \left(1 + \frac{Z_c}{Z_s} \right) \quad \text{and} \quad \left| \frac{v_2}{v} \right|^2 = \frac{1}{\left| 1 + \frac{1}{R_s + jX_s} \right|^2}$$

The transmission coefficient can then be written as:

$$\begin{aligned} \tau &= \left| \frac{v_2}{v} \right|^2 = \frac{1}{\left| 1 + \frac{1}{R_s + jX_s} \right|^2} = \frac{|R_s + jX_s|^2}{|R_s + jX_s + 1|^2} \\ &= \frac{R_s^2 + X_s^2}{(R_s + 1)^2 + X_s^2} \end{aligned}$$

- (b) The power reflection coefficient, $|R_p|^2$, is related to the transmission coefficient, τ , by $|R_p|^2 = 1 - \tau$. Thus:

$$|R_p|^2 = 1 - \frac{R_s^2 + X_s^2}{(R_s + 1)^2 + X_s^2} = \frac{(R_s + 1)^2 + X_s^2 - R_s^2 - X_s^2}{(R_s + 1)^2 + X_s^2}$$

Rearranging gives:

$$|R_p|^2 = \frac{2R_s + 1}{(R_s + 1)^2 + X_s^2}$$

Problem 9.12

- (a) The Insertion Loss is calculated using equation 9.54 as it is a constant volume velocity source. Assuming that the damping term of Equation (7.33) is negligible, the resonance frequency of the muffler is defined by:

$$IL = 10 \log_{10} \left(1 - \left(\frac{2\pi \times 20}{\omega_0} \right)^2 \right)^2 = 10.$$

Thus $\omega_0 = 61.6 \text{ rad/sec}$. To be a little conservative, use $\omega_0 = 60 \text{ rad/sec}$. The required chamber volume is then found using equation 9.38 in the

text. The cross-sectional area of the inlet pipe is $A = \pi \times 0.15^2/4 = 0.01767 \text{ m}^2$. Thus the required chamber volume is:

$$V = \frac{343^2 \times 0.01767}{0.3 \times 65^2} = 1.92 \text{ m}^3$$

- (b) The attenuating device could be made smaller by using a low pass filter as described on pp.433-438 in the text.

Problem 9.13

Low pass acoustic filter (see figure 9.11 in text). Head loss ≈ 4 velocity heads due to tube inlets and exits, so total pressure drop, $\Delta p = 2\rho U^2$. We now must calculate the flow speed, U . Assume that the choke tube diameter is the same as the inlet and exit pipes and equal to 0.1m. Flow rate at STP = $250,000 \text{ m}^3/\text{day} = 2.894 \text{ m}^3/\text{sec}$. The mass flow rate can be calculated from the Universal Gas Law. Thus:

$$P\dot{V} = \frac{\dot{m}}{M}RT$$

and:

$$\dot{m} = \frac{MP\dot{V}}{RT} = \frac{0.029 \times 101.4 \times 10^3 \times 2.894}{8.314 \times 288} = 3.55 \text{ kg/s}$$

At operating conditions, $T = 623^\circ \text{K}$ and $P = 12 \times 10^6 \text{ Pa}$. Thus:

$$\dot{V} = \frac{\dot{m}RT}{MP} = \frac{3.55 \times 8.314 \times 623}{0.029 \times 12 \times 10^6} = 0.0529 \text{ m}^3/\text{s}$$

The velocity is thus:

$$U = \frac{0.0529 \times 4}{\pi \times 0.1^2} = 6.735 \text{ m/s}$$

The gas density is:

$$\rho = \frac{\dot{m}}{\dot{V}} = \frac{3.54}{0.0529} = 66.9 \text{ kg/m}^3$$

Thus the pressure drop, Δp is:

$$\Delta p = 2 \times 67.2 \times 6.735^2 = 6.1 \text{ kPa}$$

which is much less than 0.5% of 12MPa, so it is OK.

Speed of sound in the gas is:

$$c_g = \sqrt{\gamma RT/M} = (1.3 \times 8.314 \times 623/0.029)^{1/2} = 482 \text{ m/s}$$

Following the design procedure on pages 437 and 438 of the text, we have

1. $f_0 = 0.6 \times 10 = 6 \text{ Hz}$
2. Try $V_1 = V_2 = 1 \text{ m}^3$
4. Choke tube diameter = 0.1m and length = 1.8m (assuming the chambers are cylinders 1m long and 0.64m diameter).
7. Resonance frequency is:

$$\begin{aligned} f_0 &= \frac{c_g}{2\pi} \left[\frac{A_c}{\ell_c} \left(\frac{1}{V_1} + \frac{1}{V_2} \right) \right]^{1/2} \\ &= \frac{482}{2\pi} \left[\frac{\pi \times 0.1^2}{4 \times 1.8} (1 + 1) \right]^{1/2} = 7.2 \text{ Hz} \end{aligned}$$

which is close enough to 6Hz for now.

$$\text{Choke tube x-sectional area} = \pi \times 0.01/4 = 0.007854 \text{ m}^2$$

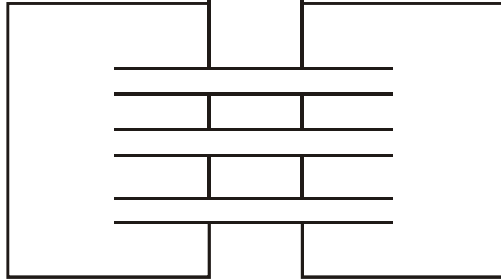
Equation 9.71 in the text may be used to calculate the Insertion Loss for the muffler. Substituting the values into this equation gives $IL = 30 \text{ dB}$ which is too much. Thus try changing the volumes to 0.75 m^3 and the choke tube length to 1.6m. This gives an Insertion Loss of 17.6dB which is too small. Try changing the volumes to 0.9 m^3 and the choke tube length to 1.7m. This gives an Insertion Loss of 25.9dB which is too large. Try changing the choke tube length to 1.4m. This gives an Insertion Loss of 20.8dB which is OK.

Thus, the final design is for 2 volumes, each of 0.9 m^3 with a 0.1m diameter choke tube, 1.4m long connecting them.

In practice, conservatism would usually dictate sticking with the 30dB design if it is practical.

Problem 9.14

As the tubes are short, they may be treated as lumped elements and the design procedure outlined on pages 437 and 438 in the text may be used. The impedance of the three tubes is three times that for a single tube.

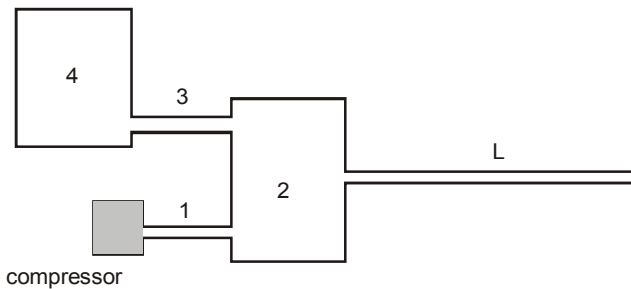


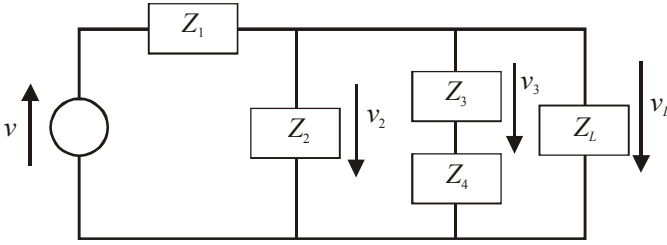
The design frequency is 40Hz. Thus the required resonance is $0.65 \times 40 = 26\text{Hz}$. To simplify matters, use the largest allowable chamber volumes and smallest allowable choke tube diameter. Using equation 9.80, we obtain:

$$26 = \frac{343}{2 \times \pi} \left(\frac{\pi \times 0.05^2}{3 \times 4 \times \ell_c} (0.03^{-1} + 0.03^{-1}) \right)^{1/2}$$

Thus $26\ell_c^{1/2} = 11.403$ and $\ell_c = 192\text{mm}$.

The filter thus consists of 2 volumes, each 0.03m^3 , connected by three 0.05m diameter tubes which are approximately 0.2m long.

Problem 9.15



- (a) The equivalent acoustical circuit is shown above.
- (b) The Insertion Loss of the muffler is given by equation 9.64 in the text. The circuit equations are:

$$\begin{aligned}
 v &= v_2 + v_3 + v_L \\
 v_2 Z_2 &= v_3 (Z_3 + Z_4) = v_L Z_L \\
 v_3 &= v_L \frac{Z_L}{Z_3 + Z_4} \\
 v_2 &= v_L \frac{Z_L}{Z_2} \\
 v &= v_L \left(1 + \frac{Z_L}{Z_3 + Z_4} + \frac{Z_L}{Z_2} \right)
 \end{aligned}$$

Thus the Insertion Loss is given by:

$$IL = 20 \log_{10} \left| 1 + \frac{Z_L}{Z_3 + Z_4} + \frac{Z_L}{Z_2} \right|$$

- (c) For no reflections from the pipe exit:

$$Z_L = \frac{\rho c}{A} = \frac{413.6 \times 4}{\pi \times 0.02^2} = 1.317 \times 10^6$$

The volume impedances are:

$$Z_2 = Z_4 = -j \frac{\rho c^2}{V \omega} = -j \frac{1.206 \times 343^2}{0.2 \times 2\pi \times 10} = -j 1.129 \times 10^4$$

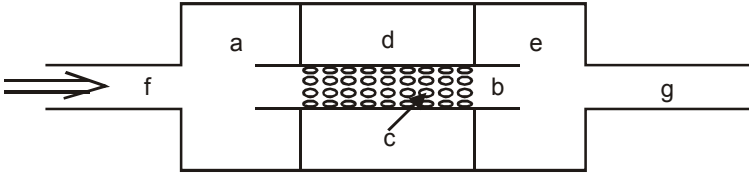
and the tube impedances are:

$$Z_1 = Z_3 = j \frac{\rho c}{A} \tan\left(\frac{2\pi L}{\lambda}\right) = j \frac{413.6 \times 4}{\pi \times 0.02^2} \tan\left(\frac{6\pi}{343}\right) = j7.24 \times 10^4$$

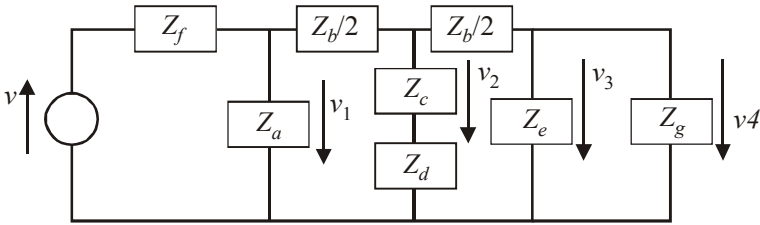
The Insertion Loss is then:

$$\begin{aligned} IL &= 20 \log_{10} \left| 1 + \frac{1.317 \times 10^6}{j(7.24 \times 10^4 - 1.129 \times 10^4)} + \frac{1.317 \times 10^6}{-j1.129 \times 10^4} \right| \\ &= 20 \log_{10} |1 - j21.54 + j116.7| \\ &\approx 39.6 \text{ dB} \end{aligned}$$

Problem 9.16



(a) Equivalent circuit diagram.



(b) System equations:

$$v = v_1 + v_2 + v_3 + v_4$$

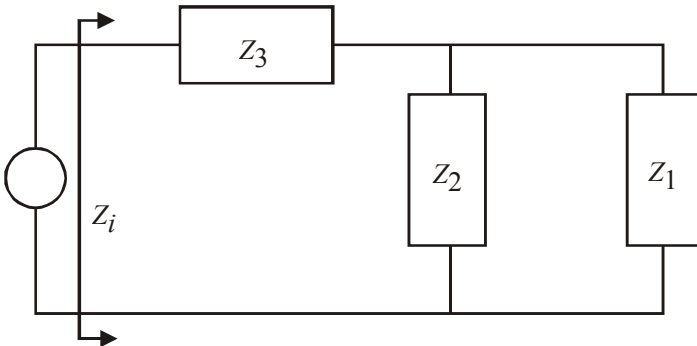
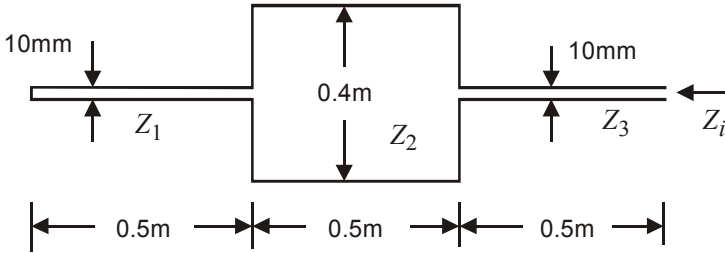
$$v_1 Z_a = (v_2 + v_3 + v_4)(Z_b/2) + (v_3 + v_4)(Z_b/2) + v_4 Z_g$$

$$v_2(Z_c + Z_d) = (v_3 + v_4)(Z_b/2) + v_4 Z_g$$

$$v_3 Z_e = v_4 Z_g$$

- (c) Inductive (with resistive part) impedances are Z_b , Z_c , Z_f and Z_g .
Capacitive impedances are Z_a , Z_d and Z_e .

Problem 9.17



- (a) Impedance looking into the tube is:

$$\begin{aligned}
 Z_i &= Z_3 + \frac{1}{(1/Z_2) + (1/Z_1)} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} \\
 &= \frac{Z_3 Z_1 + Z_3 Z_2 + Z_1 Z_2}{Z_1 + Z_2}
 \end{aligned}$$

- (b)

$$Z_3 = \frac{4j\rho c}{\pi d_3^2} \tan(kL_3); \quad Z_2 = -\frac{4j\rho c^2}{\pi d_2^2 L_2 \omega}$$

and:

$$Z_1 = -\frac{4j\rho c}{\pi d_1^2} \cot(kL_1)$$

Thus:

$$Z_i = \frac{\left[-\frac{(4j\rho c)^2}{\pi^2 d_1^2 d_3^2} \cot(kL_1) \tan(kL_3) - \frac{(4j\rho c)^2 c}{\pi^2 d_2^2 d_3^2 L_2 \omega} \tan(kL_3) \right. \\ \left. + \frac{(4j\rho c)^2 c}{\pi^2 d_1^2 d_2^2 L_2 \omega} \cot(kL_1) \right]}{-\frac{4j\rho c}{\pi} \left[\frac{\cot(kL_1)}{d_1^2} + \frac{c}{d_2^2 L_2 \omega} \right]}$$

- (c) Resonance occurs when inductive impedance = capacitive impedance, or $Z_i = 0$.

Eliminating $(4j\rho c/\pi)^2$ from previous expression for Z_i and setting the result = 0, we obtain:

$$-\frac{\cot(kL_1) \tan(kL_3)}{d_1^2 d_3^2} - \frac{c \tan(kL_3)}{d_2^2 d_3^2 L_2 \omega} + \frac{c \cot(kL_1)}{d_1^2 d_2^2 L_2 \omega} = 0$$

Substituting for $k = \omega/343$, $L_1 = L_2 = L_3 = 0.5$, $d_1 = d_3 = 0.01$ and $d_2 = 0.4$, we obtain:

$$-\omega \times 8 \times 10^{-2} \cot(\omega/686) \tan(\omega/686) \\ - 343 \times 10^{-4} \tan(\omega/686) + 343 \times 10^{-4} \cot(\omega/686) = 0$$

Simplifying gives:

$$-\omega + 0.429(-\tan(\omega/686) + \cot(\omega/686)) = 0$$

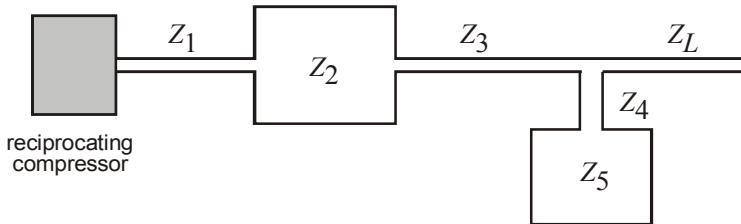
Solving by trial and error:

ω	Value of expression	ω	Value of expression
1	293	17	0.3
10	19.4	17.3	-0.28
20	-5.3	17.15	0.017
18	-1.6	17.1586	-0.000045

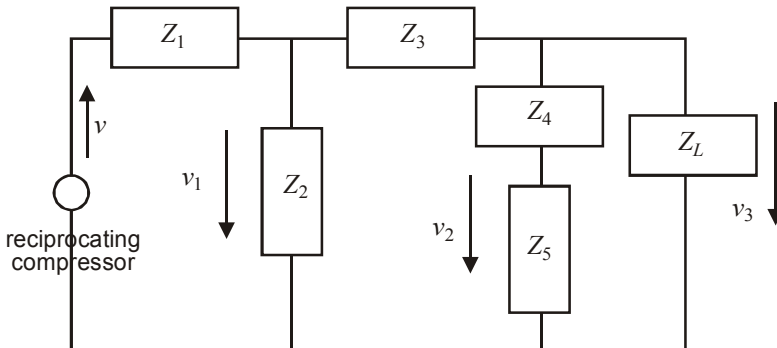
Thus the resonance frequency = $17.1586/2\pi = 2.7\text{Hz}$.

- (d) At resonance, there will be a pressure maximum at the closed end and a maximum particle velocity at the open end.

Problem 9.18



(a)



- (b) Constant volume velocity source. Thus $v = v_1 + v_2 + v_3$. The Insertion Loss is given by:

$$IL = 20 \log_{10} \left| \frac{v}{v_3} \right|$$

Examining pressure drops, we may write:

$$Z_2 v_1 = Z_3(v_2 + v_3) + v_2(Z_4 + Z_5)$$

Thus:

$$v_1 = \frac{Z_3 v_2}{Z_2} + \frac{Z_3 v_3}{Z_2} + \frac{Z_4 v_2}{Z_2} + \frac{Z_5 v_2}{Z_2}$$

Also:

$$v_2(Z_4 + Z_5) = v_3 Z_L; \quad \text{thus} \quad v_2 = \frac{v_3 Z_L}{Z_4 + Z_5}$$

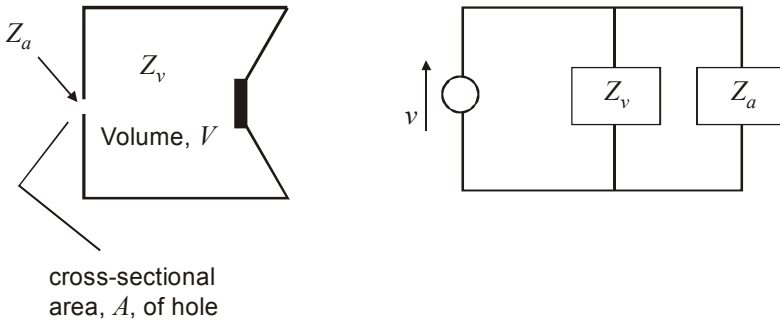
Substituting for v_2 in the expression for v_1 gives:

$$v_1 = v_3 \left(\frac{Z_3 Z_L}{Z_2(Z_4 + Z_5)} + \frac{Z_3}{Z_2} + \frac{Z_4 Z_L}{Z_2(Z_4 + Z_5)} + \frac{Z_5 Z_L}{Z_2(Z_4 + Z_5)} \right)$$

The Insertion Loss is then:

$$\begin{aligned} IL &= 20 \log_{10} \left| \frac{(v_1 + v_2 + v_3)}{v_3} \right| \\ &= 20 \log_{10} \left| 1 + \frac{Z_L}{Z_4 + Z_5} + \frac{Z_3 Z_L}{Z_2(Z_4 + Z_5)} + \frac{Z_3}{Z_2} \right. \\ &\quad \left. + \frac{Z_4 Z_L}{Z_2(Z_4 + Z_5)} + \frac{Z_5 Z_L}{Z_2(Z_4 + Z_5)} \right| \end{aligned}$$

- (c) The result of (b) is no longer valid if the dimensions of the chambers represented by impedances Z_2 and Z_5 exceed one quarter of a wavelength of sound.

Problem 9.19

- (a) Load seen by speaker (ignoring external air load), Z_s given by:

$$\frac{1}{Z_s} = \frac{1}{Z_v} + \frac{1}{Z_a}$$

where:

$$Z_v = -j \frac{\rho c^2}{V \omega} \quad \text{and} \quad Z_a = j \frac{\rho \omega \ell}{A}$$

and ℓ is the effective length of the orifice. From equation 9.16, the end correction for the side of the hole in free space is $8a/3\pi$. If we assume that the enclosure is cylindrical of diameter D , then the end correction for the side of the hole in the enclosure is $\ell_0 = \frac{8a}{3\pi}(1 - 2.5a/D)$ and the total

effective length of the hole is then $\ell = \frac{8a}{3\pi}(2 - 2.5a/D)$. If the cylinder

diameter is equal to its length, then $D = (4V/\pi)^{1/3}$ and

$$\ell = \frac{8a}{3\pi} \left(2 - 2.5a \left(\frac{\pi}{4V} \right)^{1/3} \right).$$

Thus:

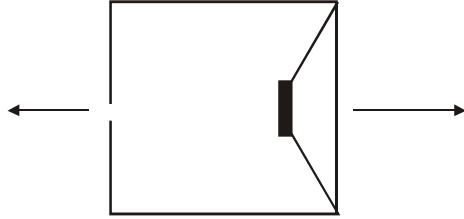
$$\begin{aligned} \frac{1}{Z_s} &= j \frac{V \omega}{\rho c^2} - j \frac{\pi a^2}{\rho \omega \ell} \\ &= j \frac{V \omega}{\rho c^2} - j \frac{3\pi^2 a}{8\rho \omega \left(2 - 2.5a \left(\frac{\pi}{4V} \right)^{1/3} \right)} \end{aligned}$$

Thus:

$$Z_s = -j \frac{16\rho\omega c^2(1 - 1.153aV^{-1/3})}{16V\omega^2(1 - 1.153aV^{-1/3}) - 3\pi^2ac^2}$$

- (b) At low frequencies, the second term in the denominator will be larger than the first and so the phase of the impedance will be $+j$ which means that the acoustic pressure leads the acoustic particle velocity by 90° . As the particle displacement also leads the velocity by 90° , the particle displacement at the orifice and the acoustic pressure will be in phase. As the box is small compared to a wavelength, the acoustic pressure will be in phase with the displacement of the cone; thus the out flow from the orifice will be in the same direction as the cone motion.

At higher frequencies, the second term in the denominator will become smaller than the first and the phase of the impedance will be $-j$, a 180° shift from the lower frequency case. Thus in this case the out flow from the orifice will be in the opposite direction to the cone motion and will thus reinforce the out flow at the cone as shown in the figure.



The crossover frequency is thus when the two terms in the denominator are equal, which is when:

$$16V\omega^2(1 - 1.153aV^{-1/3}) = 3\pi^2ac^2$$

or:

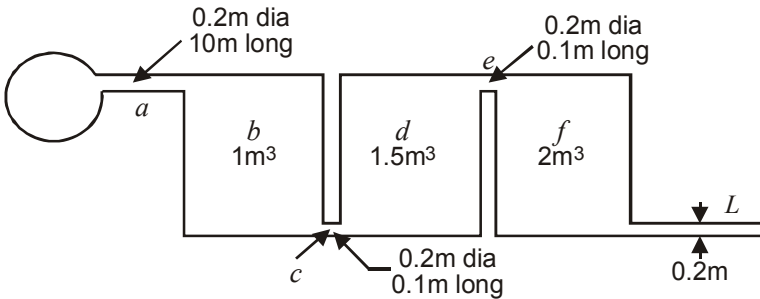
$$\omega_0 = \sqrt{\frac{3\pi^2ac^2}{16V(1 - 1.153aV^{-1/3})}}$$

- (c) We need to solve the above equation for V , given that $\omega_0 = 2\pi \times 100$. Rearranging the above equation and substituting values for variables, we obtain:

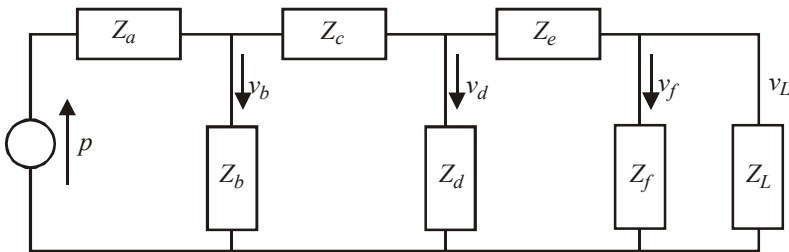
$$\begin{aligned}
 V &= \frac{3\pi^2 ac^2}{16\omega_0^2(1 - 1.153aV^{-1/3})} \\
 &= \frac{3\pi^2 \times (0.01/\pi)^{1/2} \times 343^2}{16 \times 4 \times \pi^2 \times 10^4(1 - 1.153 \times (0.01/\pi)^{1/2}V^{-1/3})} \\
 &= \frac{0.0311}{1 - 0.0651V^{-1/3}}
 \end{aligned}$$

Solving by trial and error gives $V = 0.039\text{m}^3$.

Problem 9.20



Equivalent acoustical circuit



Referring to the figures above and using equation 9.55 in the text for a constant pressure source, we may write the following Insertion Loss and acoustical circuit equations:

$$IL = 20 \log_{10} \left| \frac{P}{v_L Z_L} \right| \quad (1)$$

$$P = (v_b + v_d + v_f + v_L) Z_a + v_b Z_b \quad (2)$$

$$v_b Z_b = v_d Z_d + Z_c (v_d + v_f + v_L) \quad (3)$$

$$v_d Z_d = v_f Z_f + Z_e (v_f + v_L) \quad (4)$$

$$v_f Z_f = v_L Z_L \quad (5)$$

Using eq. (4), we can write:

$$v_d = v_f \left(\frac{Z_f}{Z_d} + \frac{Z_e}{Z_d} \right) + \frac{v_L Z_e}{Z_d} \quad (6)$$

Using eqs. (5) and (6), we can write:

$$v_d = v_L \left(\frac{Z_L}{Z_d} + \frac{Z_e Z_L}{Z_d Z_f} + \frac{Z_e}{Z_d} \right) \quad (7)$$

Using eq. (3):

$$v_b = v_d \left(\frac{Z_d}{Z_b} + \frac{Z_c}{Z_b} \right) + v_f \frac{Z_c}{Z_b} + v_L \frac{Z_c}{Z_b} \quad (8)$$

Using eqs. (5), (7) and (8):

$$v_b = v_L \left[\left(\frac{Z_d}{Z_b} + \frac{Z_c}{Z_b} \right) \left(\frac{Z_L}{Z_d} + \frac{Z_e Z_L}{Z_d Z_f} + \frac{Z_e}{Z_d} \right) + \frac{Z_c Z_L}{Z_b Z_f} + \frac{Z_c}{Z_b} \right] \quad (9)$$

Using eqs. (2), (5), (7) and (9):

$$\begin{aligned} \frac{P}{Z_L v_L} &= \frac{Z_a + Z_b}{Z_L} \left[\left(\frac{Z_d}{Z_b} + \frac{Z_c}{Z_b} \right) \left(\frac{Z_L}{Z_d} + \frac{Z_e Z_L}{Z_d Z_f} + \frac{Z_e}{Z_d} \right) + \frac{Z_c Z_L}{Z_b Z_f} + \frac{Z_c}{Z_b} \right] \\ &\quad + Z_a \left[\frac{Z_L}{Z_d} + \frac{Z_e Z_L}{Z_d Z_f} + \frac{Z_e}{Z_d} + \frac{Z_L}{Z_f} + 1 \right] \end{aligned}$$

The Insertion Loss is then:

$$IL = 20 \log_{10} \left| \frac{Z_a + Z_b}{Z_L} \left[\left(\frac{Z_d}{Z_b} + \frac{Z_c}{Z_b} \right) \left(\frac{Z_L}{Z_d} + \frac{Z_e Z_L}{Z_d Z_f} + \frac{Z_e}{Z_d} \right) + \frac{Z_c Z_L}{Z_b Z_f} + \frac{Z_c}{Z_b} \right] + Z_a \left[\frac{Z_L}{Z_d} + \frac{Z_e Z_L}{Z_d Z_f} + \frac{Z_e}{Z_d} + \frac{Z_L}{Z_f} + 1 \right] \right|$$

where the impedances are defined as:

$$Z_a = \frac{j\rho c}{A_a} \tan(k \ell_a) + R_a; \quad Z_b = -j \frac{\rho c^2}{V_b \omega}; \quad Z_d = -j \frac{\rho c^2}{V_d \omega}$$

$$Z_c = j \frac{\rho c}{A_c} \tan(k \ell_c) + R_c; \quad Z_e = j \frac{\rho c}{A_e} \tan(k \ell_e) + R_e; \quad Z_L = \frac{\rho c}{A_L}$$

From equation 9.29, p.417:

$$R_a = \frac{\rho c}{A_a} \left[\frac{\omega}{c} \frac{t D_a w_a}{2 A_a} \left(1 + (\gamma - 1) \sqrt{\frac{5}{3\gamma}} \right) + 0.288 \frac{\omega}{c} d \log_{10} \left(\frac{4 A_a}{\pi h_a^2} \right) + \varepsilon \frac{\omega^2}{c^2} \frac{A_a}{2\pi} + M \right]$$

R_c and R_e are defined similarly to R_a , except that subscripts c and e are substituted for subscript a respectively.

$$\rho c = 1.206 \times 343 = 414; \quad A_e = A_c = A_a = \pi \times 0.2^2/4 = 0.00314 \text{ m}^2.$$

$$\text{Pipe end corrections, } \ell_0 = (8 \times 0.1)(1 - 1.25 \times 0.2/1)/(3 \times \pi) = 0.064 \text{ m}$$

$$t = \sqrt{2\mu/(\rho\omega)} = 0.00550/\sqrt{\omega}; \quad (\mu = 1.8 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1})$$

$h_c = h_e =$ largest of t or 0.05 ; $h_a =$ largest of t or tube inlet radius.

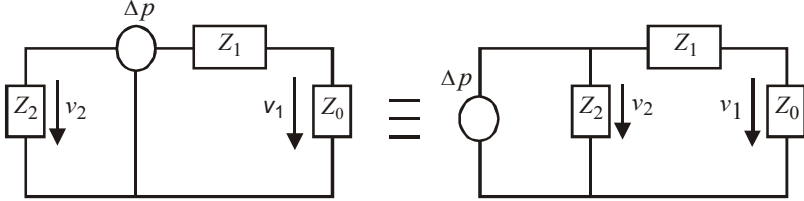
$$w_a = 10 \text{ m}, \quad w_e = w_c = 0.1 \text{ m}, \quad \gamma = 1.4, \quad V_b = 1, \quad V_d = 1.5, \quad V_f = 2 \text{ m}^3$$

Assume that M is small enough to neglect.

$$\varepsilon_a = \varepsilon_c = \varepsilon_e = 0$$

Problem 9.21

- (a) Fan and plenum equivalent circuit diagram is shown in the figure below.



Power flow through Z_0 is the radiated sound power and is equal to:

$$\frac{1}{2} \operatorname{Re} \{ \Delta p_{Z_0} v_1^* \} = \frac{1}{2} \operatorname{Re} \{ Z_0 v_1 v_1^* \} = \frac{|v_1|^2}{2} \operatorname{Re} \{ Z_0 \}$$

v_1 is an rms quantity and Δp_{Z_0} is the pressure drop across Z_0 . Equating circuit pressure drops gives:

$$v_1(Z_1 + Z_0) = v_2 Z_2 = \Delta p$$

$$v_1 = \frac{\Delta p}{Z_1 + Z_0}$$

$$W_0 = \frac{|v_1|^2}{2} \operatorname{Re} \{ Z_0 \} = \frac{|\Delta p|^2}{2} \frac{\operatorname{Re} \{ Z_0 \}}{|Z_1 + Z_0|^2}$$

At low frequencies, $\operatorname{Re} \{ Z_0 \} = 0$, and so $W_0 = 0$

- (b) For Z_2 , use the analysis leading up to equation 9.35 in the text. For Z_1 use the analysis leading to equation 9.14 for the imaginary part and equation 9.29 for the real part.

The expressions are valid over the frequency range from zero up to where the neck diameter or plenum dimensions approach 0.2 of a wavelength.

- (c) The resistive component consists of viscous friction losses due to air particles vibrating back and forth against the edge of the inlet.
- (d) Varying the fan position along the duct will have the effect of adding an inductive impedance between the fan and plenum and will also change impedance, Z_1 . Thus the radiated sound power will vary.

Problem 9.22

The speed of flow, $U_0 = 0.1\text{m/s}$. The speed of sound in the gas is:

$$c_g = \sqrt{\frac{\gamma RT}{M}} = \left(\frac{1.3 \times 8.314 \times (273 + 900)}{0.03} \right)^{1/2} = 650\text{m/s}$$

Following the design procedure on p.432 in the text, the desired resonance frequency is given by:

$$25 = 10 \log_{10} \left(1 - (\omega/\omega_0)^2 \right)^2 ;$$

$$\text{Thus, } \omega_0 = \frac{\omega}{\sqrt{18.78}} = \frac{2\pi \times 50}{\sqrt{18.78}} = 72.5\text{rad/s}$$

Using equation 9.60 in the text, the design equation is:

$$\left[\frac{1}{1.3} - \frac{18 \times 0.1}{314.2 V} - \frac{2 \times 0.01}{A^2 \times 650^2} \right] \frac{72.5^2 A^{1.5} V}{10\sqrt{\pi} \times 8 \times 10^{-3} \times 0.01} > 1$$

Rewriting gives:

$$\left[0.77 - \frac{5.73 \times 10^{-3}}{V} - \frac{4.734 \times 10^{-8}}{A^2} \right] 3.707 \times 10^6 A^{1.5} V > 1$$

To begin, set each term in brackets = (0.77/3). Thus:

$$A_{\min} = \left(\frac{3 \times 4.734 \times 10^{-8}}{0.77} \right)^{1/2} = 4.3 \times 10^{-4}\text{m}^2$$

$$V_{\min} = \frac{3 \times 5.73 \times 10^{-3}}{0.77} = 0.022\text{m}^3$$

However, this will not satisfy the minimum requirement for $A^{1.5}V$. Thus increase A and V by a factor of 2. Thus try $V = 0.04\text{m}^3$ and $A = 9.6 \times 10^{-4}\text{m}^2$ ($d = 35\text{mm}$).

Design equation becomes:

$$[0.769 - 0.143 - 0.051] 4.41 = 2.53, \text{ which is a bit large}$$

Try reducing d to 30mm; $A = 7.06 \times 10^{-4}$ and the design equation is

$$[0.769 - 0.143 - 0.095]2.78 = 1.48, \text{ which is OK}$$

The required tail pipe length, ℓ , is:

$$\ell = \frac{7.06 \times 10^{-4}}{0.04} \left(\frac{650}{72.5} \right)^2 = 1.42 \text{ m}$$

Design summary (to achieve 25dB at 50Hz)

Volume = 40litres

Tail pipe diameter = 30mm

tail pipe length = 1.42m

Could use a smaller tail pipe length and larger tail pipe diameter if a larger volume were chosen.

Problem 9.23

Use maximum allowable

OD = 0.6m.

Inside diameter

= 0.3m = $2h$

Liner thickness,

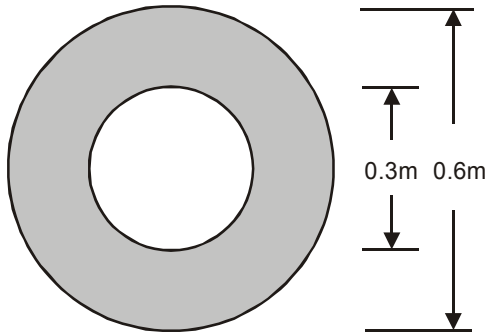
$\ell = 0.15\text{m}$

Thus $\ell/h = 1.0$

$M = -17/343 = -0.05$

At 500Hz,

$$\frac{2h}{\lambda} = \frac{0.3 \times 500}{343} = 0.437$$



From Figure 9.15 in the text, for $M = 0$, $\ell/h = 1.0$ and $2h/\lambda = 0.437$, the figure with the highest attenuation is the top right figure corresponding to $R_1\ell/\rho c = 2$. For a square duct lined on 4 sides (equivalent to a circular duct), the attenuation would be 6.3dB per length of lined duct equal to the duct radius.

From Figure 9.17, top right figure, the attenuation for $M = -0.1$, is 7.1 dB per length of duct equal to the duct radius. As $M = -0.05$, use an attenuation rate of $0.5(6.3 + 7.1) = 6.7\text{dB}$ per length of duct equal to the duct radius.

From figure 9.22 in the text, (expansion ratio = $(0.6/0.3)^2 = 4$), the required liner attenuation for an overall attenuation of 15dB is 10.8dB.

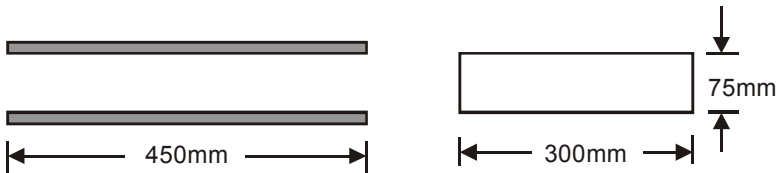
Thus the required length of liner = $0.15 \times 10.8/6.7 = 0.24\text{m}$

Additional noise reduction if direction of sound is distributed equally in all directions at the duct inlet (for example, if the duct were venting a machine enclosure) can be found from figure 9.21, p.459 in the text, where:

$$\frac{\sqrt{S}}{\lambda} = \left(\pi \times \frac{0.3^2}{4} \right)^{1/2} \times \frac{500}{343} = 0.39$$

From the figure, the additional attenuation is 7dB.

Problem 9.24



The total attenuation is made up of

- entrance losses (figure 9.21, p.459)
- exit losses (table 9.5, p.464, numbers in brackets)
- liner attenuation (figure 9.15, p.449)

Mach no., $M = 0.0$. Area of open duct = $75 \times 300 \times 10^{-6} = 0.0225$, $\sigma = 0$. To maximise the attenuation choose the curve corresponding to $\ell/h = 4$ and $R_1 \ell / \rho c = 4$ in figure 9.15 which corresponds to $M = 0.0$. For the large dimension, $2h = 0.3$ and duct length is equal to $3h$, and for the small dimension, $2h = 0.075$ and the duct length is equal to $12h$. The following table may now be generated.

Octave band centre frequency (Hz)	$2h_1/\lambda$		attenuation (dB) - lining only		
	short	long	short sides	long sides	all sides
500	0.437	0.109	7.5	19.1	26.6
1000	0.875	0.219	4.6	24.2	30.2
2000	1.749	0.437	2.4	29.5	31.9
4000	3.499	0.875	0.9	24.0	24.9

The total loss may now be calculated with the aid of the following table.

Octave band centre frequency (Hz)	\sqrt{S}/λ ($S=0.3 \times 0.075$)	Entrance loss (dB)	Exit loss (dB) $D = \sqrt{\frac{4A}{\pi}} = 0.169$	lining loss (dB)	total loss (dB)
500	0.219	3.1	4.2	26.6	33.9
1000	0.437	7.3	1.6	30.2	39.1
2000	0.875	9.6	0.6	31.9	42.1
4000	1.749	10.0	0	24.9	34.9

- (a) It is clearly possible to achieve 30dB or more in each of the octave bands, 1 2 & 4kHz. In fact a thinner liner would most likely be adequate.
- (b) The best attenuation possible at 500Hz is 34dB.

Problem 9.25

Dissipative muffler - 3 attenuations: inlet, outlet and lined section (no expansion loss as it is mounted on an enclosure). Try beginning with the smallest allowed cross section of duct. Thus, $S = 0.25 \text{ m}^2$ and $\sqrt{S}/\lambda = f \times \sqrt{S}/c = 1.458 \times 10^{-3} f$ and $2h/\lambda = 0.5f/c = 1.458 \times 10^{-3} f$

frequency (Hz)	$\frac{\sqrt{S}}{\lambda} = \frac{2h}{\lambda}$	inlet loss	outlet loss	total atten. needed	Liner atten needed
125	0.18	2	5.5	9	1.5
1000	1.46	10	0	15	5
2000	2.91	10	0	15	5

If we use the largest outer cross section allowed, the ratio of liner thickness to half airway width is 1.0. Using curve 3 in Figure 9.16, assuming a flow speed of $M=0.1$, and using $R_1 \ell / \rho c = 8$, we obtain the following attenuations for 0.25 m length of duct lined on all 4 sides.

125 Hz 1.2 dB
 1000Hz 3.4 dB
 2000 Hz 1.0 dB

It is clear that the critical frequency is 2000 Hz and to satisfy the requirement of 5 dB at this frequency, we need a length of duct equal to $5 \times 0.25/1.0 =$

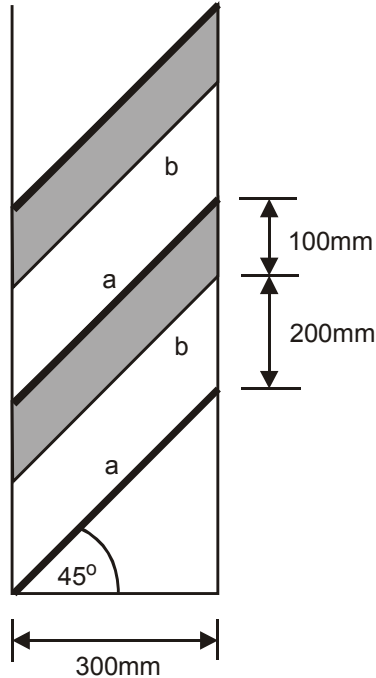
1.25 m. Note that many other solutions would be acceptable as well.

Problem 9.26

- (a) From the figure to the right, using similar triangles, it can be seen that the ratio, ℓ/h is $100/200 = 0.5$ (as one side only of the airway is lined). It may also be assumed that the flow speed is small enough to ignore.

- (b) For the ratio $\ell/h = 0.5$, an acceptable value of $R_1 \ell / \rho c$ is 2. Thus the required flow resistance of the liner is $R_1 = 2 \times 413.6 / 0.0707 = 11,700$ MKS Rayls.

- (c) $h = 2\ell = 200/\sqrt{2} = 141.4\text{mm}$. Duct cross-sectional area $= (0.2/\sqrt{2}) \times 0.4 = 0.0566\text{m}^2$. The effective duct length, $L = 300\sqrt{2} = 424\text{mm} = 3h$. Wavelength, $\lambda = 343/f$. The exit loss is obtained from Table 9.5 in the text and the inlet loss is from figure 9.21 in the text (assuming diffuse field input). The lined duct loss is from figure 9.15 in the text (curve 2, top right figure). The results are summarised in the following table.

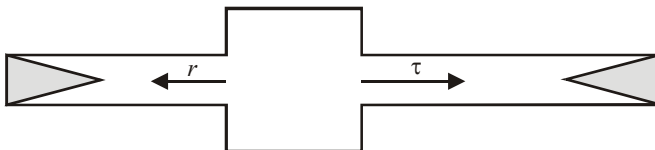


Octave band centre frequency (Hz)	$2h/\lambda$	lined duct loss (dB) length, $3h$	\sqrt{S}/λ	Inlet Loss (dB)	Exit Loss (dB)	Total Atten. (dB)
63	0.052	0.1	0.044	0	12.3	14
125	0.103	0.6	0.087	0	8.2	9
250	0.206	2.1	0.17	2	4.2	9
500	0.412	6.6	0.35	6.1	1.2	14
1000	0.825	8.2	0.69	8.8	0.1	18
2000	1.65	3.0	1.39	10	0	13
4000	3.30	0.9	2.78	10	0	11
8000	6.60	0.3	5.55	10	0	10

Problem 9.27

- (a) Insertion Loss is the difference in sound level at the end of the duct with and without the silencer in place. Transmission Loss is the difference in sound pressure level measured at the inlet and outlet of the silencer.

(b)



Power transmitted down duct with no muffler = 1.

Power transmitted with muffler = τ .

Thus $IL = -10\log_{10}\tau$

Power incident on muffler = 1.

Power transmitted by muffler = τ

Thus Transmission Loss, $TL = -10\log_{10}\tau = IL$

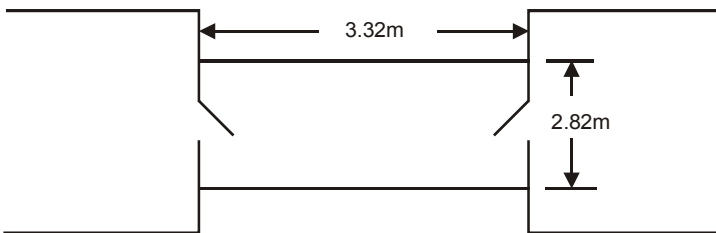
- (c) Dissipative attenuators absorb energy and contain surfaces lined with sound absorbing material. They are cost effective for high frequency noise.

Reactive mufflers change the radiation impedance "seen" by the sound

source (tonal noise or system resonances) and reflect energy back to the source. They can also dissipate energy through viscous losses at entrances and exits of small air passage ways. Reactive mufflers are cost effective for low frequency broadband noise. They can also be tuned to attenuate tonal noise at one or more frequencies.

Active mufflers act in a similar way to reactive mufflers but the impedance change, sound reflection or absorption is provided by a sound source such as a loudspeaker. These mufflers are cost effective for low frequency tonal noise problems and in many cases they are preferred to reactive mufflers because of their relatively small size and installation convenience. They are also preferred in dirty environments where reactive mufflers can become clogged.

Problem 9.28



- (a) Room 3.32m long, 2.82m wide and 2.95m high. Doors at each end are 2.06m high, 0.79m wide. Can treat it like a plenum chamber.

$\bar{\alpha} = 0.1$ and the sound power attenuation or transmission loss is:

$$TL = -10 \log_{10} \left[\frac{A}{R} + \frac{A \cos \theta}{\pi r^2} \right]$$

$$A = 2.06 \times 0.79 = 1.627 \text{m}^2, r = 3.32 \text{m}$$

$$R = \frac{S \bar{\alpha}}{(1 - \bar{\alpha})}$$

$$= 2(3.32 \times 2.82 + 3.32 \times 2.95 + 2.82 \times 2.95) \times 0.1 / 0.9$$

$$= 6.106 \text{m}^2$$

Thus:

$$TL = -10 \log_{10} \left[\frac{1.627}{6.106} + \frac{1.627}{\pi \times 3.32^2} \right] = 5.0 \text{ dB}$$

(b) Increasing $\bar{\alpha}$ to 0.5 gives:

$$R = 6.106 \times \frac{0.9}{0.1} \times \frac{0.5}{0.5} = 54.95 \text{ m}^2$$

Thus,

$$TL = -10 \log_{10} \left[\frac{1.627}{54.95} + \frac{1.627}{\pi \times 3.32^2} \right] = 11.2 \text{ dB}$$

That is, a 6dB improvement.

(c) If the direct line of sight were prevented, then the direct field term contribution will be zero. Thus for case (a) above:

$$TL = -10 \log_{10} \left[\frac{1.627}{6.106} \right] = 5.7 \text{ dB}$$

and for case (b) above,

$$TL = -10 \log_{10} \left[\frac{1.627}{54.95} \right] = 15.3 \text{ dB}$$

That is, there is little difference in the first case where the reverberant field contribution dominates.

(d) Sound power leaving the doorway of the first room is given by the sound intensity directed towards the door multiplied by the door opening area. Using equations 7.33 and 1.75 in the text, we obtain:

$$W = \frac{\langle p^2 \rangle}{4\rho c} A = \frac{4 \times 10^{-10} \times 10^{8.5} \times 1.627}{4 \times 413.6} = 1.244 \times 10^{-4} \text{ W}$$

The sound power level is:

$$L_w = 10 \log_{10} W + 120 = 81.0 \text{ dB re } 10^{-12} \text{ W}$$

Thus the sound power incident on the second doorway is:

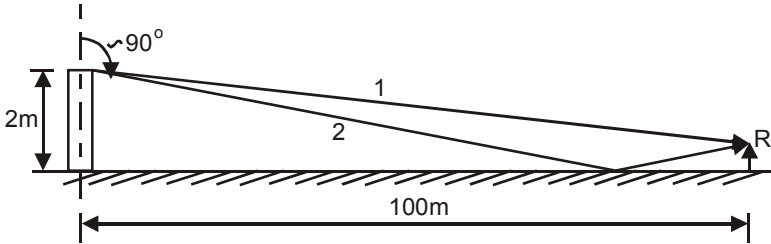
$$81.0 - 5.0 = 76.0 \text{ dB.}$$

Assuming no reflection from the doorway, this corresponds to a sound pressure level in the doorway given by equation 6.2 in the text as (where

the correction for $\rho c \neq 400$ has been included):

$$\begin{aligned} L_p &= L_w - 10 \log_{10} A + 0.15 \\ &= 76.0 - 10 \log_{10}(1.627) + 0.15 = 74.0 \text{ dB} \end{aligned}$$

Problem 9.29



At a distance of 50m, the angular orientation from the stack axis of the line joining the stack to the observer is approximately 90° . The directivity index may be obtained using figure 9.27 in the text. The Strouhal number is:

$$N_s = fd/c = 500 \times 1/343 = 1.458$$

Thus from figure 9.27 in the text, $DI_M = -9.6\text{dB}$.

The sound pressure level at the receiver is related to the sound power radiated by the stack using equation 5.158 in the text which may be written as:

$$L_p = L_w - K + DI_M - A_E \quad (\text{dB re } 20\mu\text{Pa})$$

$$L_w = 135\text{dB}$$

$$K = 10 \log_{10}(2\pi r^2) = 10 \log_{10}(2\pi \times 100^2) = 48\text{dB} \quad (\text{equation 5.161 in text})$$

$$A_E = A_a + A_g + A_m + A_b + A_f \quad (\text{equation 5.165 in text})$$

$$A_a = 0.2 - 0.3\text{dB} \quad (\text{table 5.3, p. 225 in text})$$

$$A_g = -3\text{dB} \quad (\text{attenuation of ground reflected wave} = \text{attenuation of direct wave})$$

$$A_m = (+3, -1)\text{dB} \quad (\text{table 5.10, p.243 in text})$$

$$A_b = A_f = 0$$

Thus:

$$L_p = 135 - 48 - 9.6 - 0.2 + 3 - (+3, -1) = 77 - 81 \text{ dB re } 20\mu\text{Pa}$$

If meteorological influences are ignored, $L_p = 80\text{dB re } 20\mu\text{Pa}$.

Solutions to problems in vibration isolation

Problem 10.1

- (a) The difference in levels between the plant room and apartment for the two cases of the plant operating and the test source operation are obtained by subtracting appropriate rows in the table given in the problem and are given in the table below

Octave band centre frequency (Hz)	63	125	250	500
Equipment operating	25	36	38	40
Test source operating	34	37	38	39

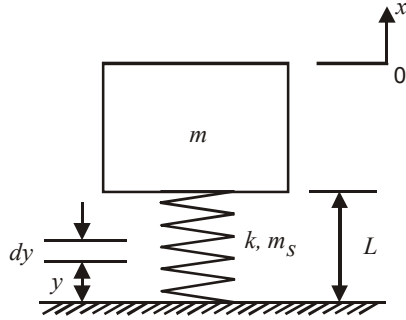
Inspection of the above table indicates that in the 63Hz band, the problem is dominated by structure-borne noise whereas at higher frequencies, air-borne noise dominates. Thus improved vibration isolation will only help the low frequency noise problem and after allowing for the A-weighting correction of table 3.1, it can be seen that reducing the 63Hz problem will not significantly reduce the A-weighted noise level in the apartment.

Problem 10.2

The single degree of freedom model gives inaccurate estimations of vibration transmission in the audio frequency range because it treats the spring and supported mass as lumped elements and does not include the effects of wave transmission along the spring. The wave transmission effects are negligible at sub-audio frequencies but are often important mechanisms of vibration transmission in the audio frequency range.

Problem 10.3

- (a) Assuming that the displacement of the end of the spring attached to the mass is described by $x(t)$. An intermediate point on the spring, a distance of y from the fixed base will have a displacement of $(y/L)x(t)$. The total kinetic energy of the spring is:



$$\begin{aligned}
 T_s &= \int_0^L \frac{1}{2} \rho S \left(\frac{y}{L} \dot{x}^2 \right) dy = \frac{1}{2} \frac{\rho S}{L^2} \dot{x}^2 \frac{L^3}{3} \\
 &= \frac{1}{2} (\rho S L) \frac{\dot{x}^2}{3} \\
 &= \frac{1}{2} m_s \dot{x}^2
 \end{aligned}$$

The total kinetic energy of the spring and mass is:

$$T_{tot} = T_s + T_m = \frac{1}{2} \left(\frac{m}{3} \right) \dot{x}^2 + \frac{1}{2} m \dot{x}^2 = \frac{1}{2} \left(m + \frac{m_s}{3} \right) \dot{x}^2$$

If $x(t) = A \sin(\omega t)$, then $\dot{x}(t) = \omega A \sin(\omega t)$ and:

$$T_{\max} = \frac{1}{2} \left(m + \frac{m_s}{3} \right) (\omega A)^2$$

The maximum potential energy stored in the spring is:

$$V = \frac{1}{2} k x_{\max}^2 = \frac{1}{2} k A^2$$

where k is the spring stiffness.

Equating T_{\max} with V_{\max} gives:

$$\frac{1}{2} \left(m + \frac{m_s}{3} \right) (\omega A)^2 = \frac{1}{2} k A^2$$

Thus:

$$\omega_n = \sqrt{\frac{k}{m + \frac{m_s}{3}}}$$

and the resonance frequency f_0 is thus:

$$f_0 = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m + m_s/3}}$$

(b) Effective Young's modulus, E , is given by:

$$E = \frac{\text{force/area}}{\text{spring extension/spring length}} = \frac{kx/S}{x/L} = \frac{kL}{S}$$

Longitudinal wave speed is thus given by:

$$c_L = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{kL}{S\rho}} = f_s \lambda_s = 4f_s L$$

Thus the surge frequency is given by:

$$f_s = \frac{1}{4} \sqrt{\frac{k}{S\rho L}} = 0.25 \sqrt{\frac{k}{m_s}}$$

(c) The wave motion in the spring satisfies the one dimensional wave equation given by:

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c_L^2} \frac{\partial^2 \xi}{\partial t^2}$$

where ξ is the spring longitudinal displacement as a function of axial location x .

The solution for this equation is the same as for the acoustic case. That is:

$$\xi = A e^{j(\omega t - (\omega/c_L)x)} + B e^{j(\omega t + (\omega/c_L)x + \theta)}$$

One boundary condition is that at $x = 0$, $\xi = 0$. Substituting this into the solution to the wave equation gives $Be^{j0} = -A$. Thus the wave equation solution becomes:

$$\xi = A \left(e^{j(\omega t - (\omega/c_L)x)} - e^{j(\omega t + (\omega/c_L)x)} \right)$$

The second boundary condition is that the inertia force of the mass at $x = L$ is equal to the spring force. That is:

$$m \frac{\partial^2 \xi}{\partial x^2} = -kL \frac{\partial \xi}{\partial x}$$

where k is the spring stiffness.

Substituting the wave equation solution into this gives:

$$\begin{aligned} & -m\omega^2 \left(e^{j(\omega t - (\omega/c_L)L)} - e^{j(\omega t + (\omega/c_L)L)} \right) \\ &= jkL \frac{\omega}{c_L} \left(e^{j(\omega t - (\omega/c_L)L)} + e^{j(\omega t + (\omega/c_L)L)} \right) \end{aligned}$$

which can be rewritten as:

$$\begin{aligned} & m\omega^2 \left(e^{j(\omega/c_L)L} - e^{-j(\omega/c_L)L} \right) \\ &= jkL \frac{\omega}{c_L} \left(e^{-j(\omega/c_L)L} + e^{j(\omega/c_L)L} \right) \end{aligned}$$

As shown in part (b), $kL = c_L^2 \rho S$. Using this relation and rearranging the above equation gives:

$$\frac{\omega L}{c_L} \tan \left(\frac{\omega L}{c_L} \right) = \frac{\rho SL}{m} = \frac{m_s}{m} = \frac{1}{N}$$

The surge frequency is $2\pi \times \omega$, where ω is the solution of the above transcendental equation. As stated in the problem, the upper frequency bound will be $0.9 \times$ this value.

Problem 10.4

As shown in the figure, let the frame stiffness be represented by k_2 and the isolator stiffness by k_1 . The effective stiffness, k_e is then given by:

$$k_e = \frac{k_1 k_2}{k_1 + k_2}$$

Thus the resonance frequency is given by:

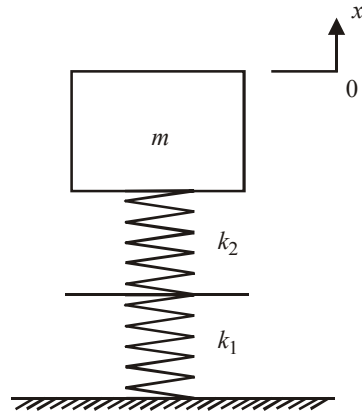
$$\omega_1 = \sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}$$

With a rigid frame, the resonance frequency is given by:

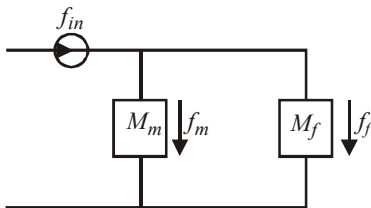
$$\omega_2 = \sqrt{\frac{k_1}{m}}$$

The ratio of the two (which is plotted in figure 10.7) is:

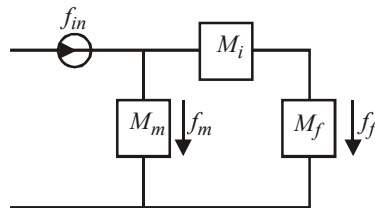
$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{k_2}{k_1 + k_2}} = \sqrt{\frac{1}{1 + k_1/k_2}}$$

**Problem 10.5**

- (a) The equivalent mobility electrical circuits are shown in the two figures below.



without isolator



with isolator

(b) For the circuit without the isolator:

$$f_m M_m = f_f M_f$$

and:

$$f_{in} = f_m + f_f = f_f \left(1 + \frac{M_f}{M_m} \right) = f_f \left(\frac{M_m + M_f}{M_m} \right)$$

The force transmissibility is the ratio of f_f/f_{in} , which is:

$$T_1 = \frac{f_f}{f_{in}} = \frac{M_m}{M_m + M_f}$$

For the circuit with the isolator:

$$f_m = f_f \frac{M_i + M_f}{M_m}$$

and:

$$f_{in} = f_m + f_f = f_f \left(1 + \frac{M_i + M_f}{M_m} \right) = f_f \left(\frac{M_m + M_i + M_f}{M_m} \right)$$

The force transmissibility is the ratio of f_f/f_{in} , which is:

$$T_2 = \frac{f_f}{f_{in}} = \frac{M_m}{M_m + M_f + M_i}$$

The force transmissibility with the isolator compared to that without the isolator is then T_2/T_1 and is:

$$T_F = \frac{T_2}{T_1} = \frac{M_m + M_f}{M_m + M_f + M_i}$$

which is the same as equation 10.31.

Problem 10.6

$2b = 0.7$ m; machine width = 0.9m

$2e = 1.0$ m; machine depth = 1.2m

$2h = 0.2$ m

$a = 0.5 - h = 0.4$ m; machine height = $2 \times (0.4 - h) = 0.6$ m

vertical dimension of mass, $2d = 2(a - h) = 0.6$ m

$k = 4000$ N/m

Radius of gyration about vertical y-axis (eq. 10.18 in text):

$$\delta_y = \sqrt{[(0.9/2)^2 + (1.2/2)^2]/3} = 0.4330 \text{ m}$$

Radius of gyration about horizontal x-axis (eq. 10.18 in text):

$$\delta_x = \sqrt{[(0.6/2)^2 + (1.2/2)^2]/3} = 0.3873 \text{ m}$$

Radius of gyration about horizontal z-axis (eq. 10.18 in text):

$$\delta_z = \sqrt{[(0.6/2)^2 + (0.9/2)^2]/3} = 0.3122 \text{ m}$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{4000 \times 4}{50}} = 2.847 \text{ Hz}$$

Now the rocking modes will be calculated, first in the x-y plane (about the z-axis) and then in the z-y plane (about the x-axis):

$$W_x = (\delta_z/b) \sqrt{k_x/k_y} = \frac{0.3122}{0.35} \sqrt{1/4} = 0.446$$

$$M_x = a/\delta_z = 0.4/0.3122 = 1.281$$

From figure 10.5 in the text, $\Omega_a = 0.38$ and $\Omega_b = 1.19$. Thus:

$$f_a = 0.38 \times 2.847 \times 0.35/0.3122 = 1.21 \text{ Hz} \quad \text{and}$$

$$f_b = 1.19 \times 2.847 \times 0.35/0.3122 = 3.80 \text{ Hz}$$

$$W_z = (\delta_x/e) \sqrt{k_z/k_y} = \frac{0.3873}{0.5} \sqrt{1/4} = 0.3873$$

$$M_z = a/\delta_x = 0.4/0.3873 = 1.033$$

From figure 10.5 in the text, $\Omega_a = 0.33$ and $\Omega_b = 1.09$. Thus:

$$f_c = 0.33 \times 2.847 \times 0.5 / 0.3873 = 1.21 \text{ Hz and}$$

$$f_d = 1.09 \times 2.847 \times 0.5 / 0.3873 = 4.01 \text{ Hz}$$

The resonance frequency of the rotational mode is calculated using equation 10.17 in the text and is:

$$f_y = \frac{1}{\pi} \sqrt{\frac{0.35^2 \times 4000 + 0.5^2 \times 4000}{50 \times 0.433^2}} = 4.01 \text{ Hz}$$

Problem 10.7

From equation 10.31, the increase in force transmission is:

$$T_F = \frac{M_m + M_f}{M_m + M_f + M_i}$$

From the question, $M_f = 0.2M_i = 2M_m$. Thus:

$$T_F = \frac{0.1 + 0.2}{0.1 + 0.2 + 1} = 0.231$$

which corresponds to a reduction in force transmission by a factor of 4.3.

Problem 10.8

Referring to figure 10.11 in the text and the discussion on pages 496, the optimal absorber is characterised by:

$$\frac{k_2}{k_1} = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$2\pi f_2 = \sqrt{\frac{k_2}{m_2}}$$

$$\zeta^2 = \frac{3(m_2/m_1)}{8(1 + m_2/m_1)^3}$$

The excitation frequency is $3000/60 = 50\text{Hz}$ and this should be equal to the

resonance frequency of the absorber mass and spring system.

Thus $(100\pi)^2 = k_2/m_2$ or $k_2 = 9.87 \times 10^4 \times m_2$

Using the first equation and substituting the above for k_2 , and the given values for k_1 and m_1 , we obtain:

$$\frac{9.87 \times 10^4 \times m_2}{10^7} = \frac{1000 m_2}{(1000 + m_2)^2}$$

which results in a negative mass m_2 . The text indicates that the damping mass should be as large as possible; thus it seems impractical to try to satisfy the second of the above three design equations. Thus let the design mass $m_2 = 20\%$ of $m_1 = 200\text{kg}$. Thus the required stiffness is given by:

$$k_2 = (100\pi)^2 \times m_2 = 19.7\text{MN/m}$$

The required damping may be obtained from equation 10.47 as:

$$\zeta = \sqrt{\frac{3 \times 200/1000}{8(1 + 200/1000)^3}} = 0.21$$

Problem 10.9

Damping a vibrating surface will only reduce the resonant response. Thus damping will only result in a reduction in sound radiation if the resonant modes are contribution most to the radiated sound field. This is generally the case when the surface is excited mechanically. However, if the surface is excited by an acoustic wave on the side opposite that which is causing the radiation problem, then it is likely that the sound radiation will be dominated by vibration modes which are being forced to vibrate at frequencies well above their resonances. In this case damping the surface will not reduce the sound radiation directly but may reduce it a little because adding mass to the vibrating structure will decrease its mobility for excitation by the incoming sound field. See pages 504-506 in the text.

Problem 10.10

(a) 100dB below 1 volt corresponds to a voltage of $1 \times 10^{-100/20}$ Volts =

10 μ Volts.

Accelerometer mass, m (grams), is approximately equal to sensitivity in mV/g. Smallest detectable acceleration (in g) is the smallest detected voltage in milli-volts divided by m . Thus smallest detectable acceleration is $10^{-2}/m$ "g" which in metres/sec is 0.01 divided by the accelerometer weight in grams.

- (b) The required relation can be determined by considering the mass loading effect of the accelerometer. To obtain results within 3dB of the correct level, the accelerometer mass must satisfy the requirement that $m < 3.7 \times 10^{-4}(\rho c_L h^2/f)$.

For steel, $\rho = 7800$ and $c_L = 5150/0.954$. Thus the condition $mf_u < 15,580 h^2$ must be satisfied.

- (c) From part (a), if the smallest acceleration to be detected is 0.01g, then the lightest accelerometer which can be used is 1 gram. Substituting $m = 1$ and $h = 1$ in the relation of (b) above gives $f_u = 15.58\text{kHz}$.

Problem 10.11

- (a) For a large plate vibrating as a piston, sound will be radiated as plane waves with no near field. Thus at any point the pressure and acoustic particle velocity are related by $p = \rho cu$. As the acoustic particle velocity adjacent to the plate is equal to the plate velocity, the velocity of the plate is given by:

$$u = \frac{p}{\rho c} = \frac{2 \times 10^{-5} \times 10^{80/20}}{413.6} = 4.836 \times 10^{-4} \text{ m/s}$$

The r.m.s. acceleration is thus given by:

$$\dot{u}_{rms} = 2\pi f u = 2\pi \times 1000 \times 4.836 \times 10^{-4} = 3.04 \text{ m/s}^2$$

- (b) The r.m.s. displacement is given by:

$$d_{rms} = \frac{u}{2\pi f} = \frac{4.836 \times 10^{-4}}{2\pi \times 1000} = 0.078 \mu\text{m}$$

- (c) At high frequencies accelerations are generally large compared to displacements, whereas the opposite is true at very low frequencies. An accelerometer is thus the best means of measuring the acceleration at 1kHz, provided that the accelerometer did not significantly mass load the plate (see equation 10.53 in the text). For thin plates at high frequencies, a measurement of the sound pressure close to the plate may be the best way of determining the plate response.

Problem 10.12

- (a) Adding damping will reduce the sound radiated by a vibrating surface if the surface vibration modes which are excited are resonant as is usually the case if the structure or surface is excited mechanically (but not acoustically). A full discussion of this concept may be found on pages 504-506 of the text.
- (b) Adding stiffness to a vibrating surface will only decrease its sound radiation or increase its transmission loss at frequencies below the first resonance frequency of the surface. If the surface is excited at resonance by a tonal excitation source, then adding stiffness will increase its resonance frequencies and if the carefully done will result in a reduction in radiated sound.
- (c) Adding mass to a vibrating surface will reduce the sound radiation and increase the panel transmission loss at frequencies above the first resonance frequency of the surface and below the surface critical frequency.

Problem 10.13

Vibration velocities measured in octave bands on a diesel engine are listed in the following table.

Octave band centre frequency (Hz)	63	125	250	500	1k
rms vibration velocity (mm/s)	5	10	5	2	0.5
rms acceleration estimate ($v/2\pi f$) (m/s^2)	1.98	7.85	7.85	6.28	3.14
rms displacement estimate ($v/2\pi f$) (μm)	12.6	12.7	3.18	0.64	0.08

- (a) Overall rms velocity = $(5^2 + 10^2 + 5^2 + 2^2 + 0.5^2)^{1/2} = 12.4 \text{ mm/s}$
 (b) Overall velocity in dB re $10^{-6} \text{ mm/s} = 20 \log_{10}(12.4/10^{-6}) = 142\text{dB re } 10^{-6} \text{ mm/s}$
 (c) Estimate of the overall acceleration level in dB re $10^{-6} \text{ m/s}^2 = 20 \log_{10}(13.29/10^{-6}) = 142\text{dB re } 10^{-6} \text{ m/s}^2$
 (d) Estimate of the overall displacement level in dB re $10^{-6} \mu\text{m} = 20 \log_{10}(18.22/10^{-6}) = 145\text{dB re } 10^{-6} \mu\text{m}$

Problem 10.14

- (a) I would mount the accelerometer so that its axis was normal to the beam surface and thus parallel to the beam displacement.
- (b) Longitudinal waves would cause the accelerometer to vibrate normal to its axis and the cross-axis sensitivity of the accelerometer will result in a signal due to the longitudinal wave. For a properly selected and aligned accelerometer the effect can be very small but for any other accelerometer the effect could be significant depending on the relative levels of the bending and longitudinal displacements and the actual cross axis sensitivity of the accelerometer.

Problem 10.15

- (a) Using the relation, $f_0 = \frac{1}{2\pi} \sqrt{\frac{g}{d}}$, we obtain $f_0 = 11.1\text{Hz}$

- (b) We may use equation (10.14) in the text. The value of X is $3000/(60 \times 11.1) = 4.49$. Thus:

$$T_F = \sqrt{\frac{1 + (2 \times 0.05 \times 4.49)^2}{(1 - 4.49^2)^2 + (2 \times 0.05 \times 4.49)^2}} = 0.057$$

Thus the reduction in transmitted vertical force is $-20 \log_{10}(0.057) = 25\text{dB}$

- (c) We may use equation (10.31) in the text. First we must calculate the isolator mobility. The overall spring stiffness is found by setting equations (10.2) and (10.3) in the text, equal. The result is $k = 1000 \times 9.81 / 0.002 = 4.905\text{MN/m}$. The isolator mobility is then calculated using $M_i = \frac{j\omega}{k_i}$ to give:

$$M_i = \frac{j2\pi \times 50}{4.905e+6} = j6.405 \times 10^{-5} \text{m/s/N}$$

The mobility of the supported mass is calculated using $M_m = 1/j2\pi fm = -j3.183 \times 10^{-6} \text{m/s/N}$.

Using equation (10.31), we obtain:

$$T_F = \left| \frac{-3.183 \times 10^{-6} - 2 \times 10^{-5}}{-3.183 \times 10^{-6} - 2 \times 10^{-5} + 6.405 \times 10^{-5}} \right| = 0.567$$

Thus the reduction in transmitted vertical force is now $-20 \log_{10}(0.567) = 5\text{dB}$; thus there is an increase of 20dB .

Problem 10.16

The critical damping ratio is given by:

$$\frac{4\pi\zeta}{\sqrt{1 - \zeta^2}} = 0.5$$

Thus:

$$\zeta = \sqrt{\frac{0.25}{16\pi^2 + 0.25}} = 0.04$$

and $\eta = 0.02$.

This loss factor is about 10 to 20 times greater than would be expected from a sheet of steel, so one might conclude that the product would be effective. One application would be for lining of parts bins.

Solutions to problems in active noise control

Problem 11.1

Acoustic mechanisms associated with active noise control include:

1. Suppression of the primary source by changing its input impedance with a control source
2. Reflection of energy as a result of causing an impedance mismatch at the control source
3. Absorption of energy by the control source
4. Local cancellation at the expense of increased levels elsewhere

Applications:

- (a) Feasible, reference sensor would be tacho signal, control source should be downstream of primary source and remote from turbulence generating parts of the duct system, and error sensor should be downstream of control source (out of source near field). Mechanism involved is suppression of primary source by changing its radiation impedance.
- (b) Not feasible if the source of noise is the grille. This is because it would be difficult to obtain a causal reference signal. If the source of noise is upstream of the grille, then active control may be feasible. A reference signal could be obtained from a microphone (with a turbulence filter) placed upstream. The control source would be placed at least 1.2m downstream of the reference sensor (depending on the controller time delay) and the error sensor would be placed 0.5 to 1m downstream of the control source, and it may even work better if placed on the room side of the grille. Mechanism would be reflection and absorption of primary energy.
- (c) Not feasible if global control is needed. Possible to establish zones of local noise reduction.

- (d) Not feasible due to difficulty in obtaining a causal reference signal.
- (e) Feasible, reference sensor would be tacho signal on rotating shaft related to noise producing machine, control sources should be in factory corner if room is small, otherwise they should be near the locations where noise reduction is needed. Mechanism involved is suppression of primary source by changing its radiation impedance for small room and local cancellation for large room.
- (f) Feasible, reference sensor would be tacho signal from aircraft engine, control sources would be placed in cabin ceiling or in seat headrests, and error sensors should be as close as possible to the passengers. Mechanism involved is suppression of primary source by changing its radiation impedance, although local cancellation may dominate in some cases
- (g) Not generally feasible due to complexity of radiated sound field.
- (h) Feasible. Feedback system is needed. Control sources could be actuators on the fuselage skin or loudspeakers in the passenger headrests. Microphones would be best located in passenger head rests.
- (i) Feasible. Reference signal would be derived from electricity mains signal, control sources could be shakers on the tank or loudspeakers surrounding the transformer and very close to it. Error sensors would need to surround the transformer and be located further away than the control sources. Mechanism is suppression of primary noise by changing the radiation impedance of the transformer tank.

Errata in the 3rd edition of Engineering Noise Control

p xi, Change “Noise Reduction Index (NRI)” to “Noise Reduction Coefficient (NRC)”

p xv, change “FWHA” to “FHWA”

p xviii In line 19, change “Noise Reduction Index” to “Noise Reduction Coefficient”

p16, In line 3, change the equation to $(1/hf)\sqrt{E/\rho} > 2$

p16, line 10, change $D_p = 1.346E$ to $D_p = 1.099E$

p16, Change Eq. (1.3) to

$$D_C = \frac{D_F}{1 + \frac{D_F}{E_W} \left(\frac{2R}{t} + \frac{\rho_w}{\rho} v^2 \right)}$$

p18, In Eq. (1.5), change "332" to "331"

p27, Change Eq. 1.40a to $\varphi = \frac{f(k(ct \pm r))}{r}$

p29, 3 lines above Eq. (1.50), change "1.36" to "1.41"

p34, Change the reference just above Eq. (1.69) to "Fahy, 1995"

p35, First line under Eq. (1.67), change "1.65" to "1.64"

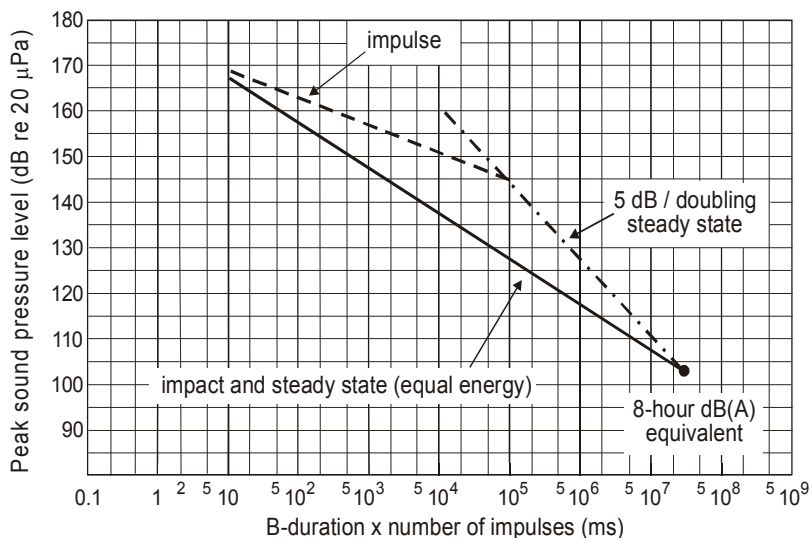
p41, 4 lines above Section 1.10., replace "pet" with "per"

p45, 2 lines under Eq. (1.89) and in Eq. (1.90), remove the subscript, "t" from p_t .

- p51, Table 1.3, line 3, replace " U " with " u "
- p51, Table 1.3, line 5, replace " Z_d " with " Z_A "
- p51, Heading 1.12.2, replace " Z " with " Z_s "
- p72, line immediately below the figure, add "is the" after the word, "ordinate"
- p76, Line 13, change "sound" to "sounds"
- p87, 2 lines above Example 2.1, the text should read, "Figure 2.10(b) is an alternative representation of Figure 2.10(a)"
- p111, line 4, change "1252" to "61252".
- p134, 3rd line, replace H with H'
- p142, The number "3" and "0.3" should be replaced by "3.01" and "0.301" respectively in Equations (4.37) to (4.41) inclusive
- p143, Replace equation 4.43 and the 2 lines preceding it with:
The daily noise dose (DND), or "noise exposure", is defined as equal to 8 hours divided by the allowed exposure time, T_a with L_B set equal to 90. That is:

$$DND = 2^{(L'_{Aeq,8h} - 90)/L}$$

- p143, Replace the sentence following equation (4.42) with: "If the number of hours of exposure is different to 8, then to find the actual allowed exposure time to the given noise environment, the "8" in Equation (4.42) is replaced by the actual number of hours of exposure."
- p144, 3rd equation down should be:
- $$T_a = 8 \times 2^{-(91.2 - 90.0)/3} = 8/2^{0.39} = 6.1 \text{ hours}$$
- p147, Replace Figure 4.6 with the more accurate figure below.



p147, 4 lines under Figure 4.6, change "1414" to "1474".

p149, 5th and 6th lines from the top, change "645" to "60645" in four places.

p150, 13 lines from the bottom, change Figure 4.6 to Figure 4.7.

p153, First line after the headings in Table 4.6, change "0.06" to "0.6".

p157, Fig 4.9 caption, add "MAF" = minimum audible field.

p160, On y-axis, change label from "dB re 20 mPa" to "dB re 20 μ Pa"

p165, First paragraph in section 4.9, replace "1995" with "1995, 1999".

p176, Line above Eq. (5.6), change " r " to " $r = a$ "

p176, In Eq (5.6), change " r " to " a "

p177, In Eq. (5.7), change " r " to " a "

p179, 2 lines under figure 5.2, replace "(x,y)" with "O" and label the observer as O in Figure 5.2

p192, 2 lines above Eq. (5.71), add "each of which has a radius of a_i " immediately after "sources"

p192, 2 lines above Eq. (5.72), change " a " to " a_i "

p192, Line above Eq. (5.72), change " ka " to " ka_i "

p192, In Eq. (5.72), change " a " to " a_i " in 5 places

p192 Eq. 5.71 and below, change Q to \bar{Q} in 4 places

p192 last line add "amplitude" immediately after "velocity"

p193 Eq 5.73 and below change Q to \bar{Q} in 2 places

p225, In Table 5.3 caption, change "Sutherland et al., 1974" to "Sutherland and Bass, 1979"

p226, 13 lines above Eq. (5.171), change "2613" to "9613".

p226, Paragraph beginning "Note that ISO" only applies to overall A-Weighted calculations and should be deleted here. The paragraph following this one should also be deleted as the meteorological effects should not be taken into account in two separate places - either they should be included in the barrier calculations or calculated separately but not both.

p229, Interchange the 63 Hz and 2000 Hz labels on the curves in Fig. 5.19.

p232, Eq. 5.181, change "-0.09" to "-0.9"

p236, In Eq. (5.188) change "10.3" to "10.0"

p241, Table 5.9, $-3.0 < v < +0.5$ should be replaced with $-3.0 < v < -0.5$

p244, ISO 9613-2 procedures for calculating ground effects and shielding effects are based on an assumption of downwind propagation from the sound source to the receiver. Thus the only correction term (Equation (5.193)) that is offered by ISO for meteorological effects is a term to reduce the A-weighted calculated sound pressure level for long time averages of several months to a year. Thus section 5.11.12.4. should be deleted and replaced with the paragraph above.

p251, In Figure 6.1, in the centre on the right hand side replace $\gamma = 1/\kappa$ with $\gamma = \kappa$

- p253, 2 lines above section 6.6, change "1989" to "1995".
- p259, The equation numbered "6.12" should be numbered "6.11"
- p264, The equation numbered "6.25" should be numbered "6.24"
- p264, 2 lines below Eq. 6.20, replace S_1 with $1/S_1$
- p264, 3 lines below Eq. 6.20, replace S_2 with $1/S_2$
- p267, The first equation should be numbered "6.26"
- p267, In Fig 6.3, there are two curves labelled "4". The lower curve should be labelled "5"
- p292, 3 lines above Eq. 7.52, change $\langle p_k^2(t) \rangle$ to $\langle p_k^2(0) \rangle$ and add "at time $t=0$ " after "mode k "
- p292, 2 lines above Eq. 7.52, change $\langle p_k^2(t) \rangle$ to $\langle p_k^2(0) \rangle$
- p292, In Eq. 7.52, change $\langle p_k^2(t) \rangle$ to $\langle p_k^2(0) \rangle$
- p293, 3 lines above Eq. 7.55, change p_k to $p_k(0)$
- p293 6 lines from the bottom, there should be a minus sign before log
- p294, 5 lines from the bottom, change (2000) to (2001)
- p294, Eq. (7.59), replace $\frac{0.16V}{S}$ with $\frac{0.16V}{S^2}$
- p295, Eq. (7.64), multiply each of the three terms in brackets by -1
- p295, 2 lines beneath Eq. (7.62), add "energy" before "reflection"
- p295, 2 lines above Equation (7.64), change "2001" to "2000"
- p296, lines 2 and 3, change " S_x , S_x and S_x " to , " S_x , S_y and S_z "
- p301, In each of the top two lines of the table, add "(m²)" after $S\bar{\alpha}$
- p303, Section 7.7.2, change "NRI" to "NRC" in three places and change "Noise Reduction Index" to "Noise Reduction Coefficient" in two places. Also change Eq. 7.76 to:

$$NRC = \frac{(\bar{\alpha}_{250} + \bar{\alpha}_{500} + \bar{\alpha}_{1000} + \bar{\alpha}_{2000})}{4} \quad (7.76)$$

p303, 2 lines from bottom, change "20 mm" to "20 μ m"

p304, Caption of Figure 7.6, line 1, change "porous surface" to "rigidly backed porous material" and in the last line, change "L" to ℓ

p310, Immediately following Equation (7.88), add the following: "Note that for square, clamped-edge panels, the fundamental resonance frequency is 1.83 times that calculated using Equation (8.21). For panels with aspect ratios of 1.5, 2, 3, 6, 8 and 10 the factors are 1.89, 1.99, 2.11, 2.23, 2.25 and 2.26 respectively."

p310, Equation 7.85 should be: $\zeta_c = \left(\frac{f}{f_c} \right)^{1/2}$

p311, End of second full paragraph, change "Elbert" to "Elfert"

p329, Eq. (7.122), replace T_{60u} with $\frac{1}{T_{60u}}$

p330, 10th line, change "2000" to "2001"

p339, 12th line from the bottom, change "1973" to "1988"

p343, 5 lines above the figure, change "ASTM E90-66T" to "ASTM E413-87"

p347, replace the line immediately above section 8.2.4 and the last word in the line above that with "contour value at 2000 Hz is increased by 1 dB." and add "Note that IIC , R_w and STC values are all reported as integers."

p352, 3 lines under Equation (8.36), change "below" to "above".

p353, change x-axis label to $f(\text{Hz})$ (log scale)"

p354, 2nd and 3rd lines from the bottom, replace "8.37" with "8.38"

p355, 2nd line after Eq. 8.44, replace $f_{c2}/2$ with $f_{c1}/2$

p355, 3rd line, replace "8.37" with "8.38"

p359, In Eq. 8.50, replace $10 \log_{10} m_1$ with $20 \log_{10} m_1$

p360, change x-axis label to "frequency (Hz) (log scale)"

p360, on the x-axis of the figure, change " $0.5 f_{c2}$ " to " $0.5 f_{c1}$ "

p360, first line of item (b) in the caption, change to "Line-point support (f_{c2} is the critical frequency of the point supported panel)"

p360, Under "Point B", item (a), replace " $30 \log_{10} f_{c2}$ " with " $20 \log_{10} f_{c1} + 10 \log_{10} f_{c2}$ "

p360, Under "Point B", items (b) and (c), replace " $40 \log_{10} f_{c2}$ " with " $20 \log_{10} f_{c1} + 20 \log_{10} f_{c2}$ "

p360, Eq (a) under "Point C", add the term, " $20 \log_{10} (f_{c2} / f_{c1})$ " to the RHS of the equation

p360, last Eqn., change f_1 to f_i

p361, replace Eq. 8.55 with:

$$D = \begin{cases} \frac{2}{h} & \text{if } f < 0.9 \times f_{c1} \\ \frac{\pi f_{c1}}{8 f \eta_1 \eta_2} \sqrt{\frac{f_{c2}}{f}} & \text{if } f > 0.9 \times f_{c1} \end{cases}$$

$$h = \left[1 - \left(\frac{f}{f_{c1}} \right)^2 \right]^2 \left[1 - \left(\frac{f}{f_{c2}} \right)^2 \right]^2$$

p363, 6 lines from the bottom of the page, change the equation to:

$$20 + 20 \log_{10}(2500/100) - 6 = 42.0 \text{ dB}$$

p363, 4 lines from the bottom of the page, change "77" to "78" and "61" to "60" in 2 places

p363, last line, change "61" to "60" and "52" to "51"

p365, Section 8.2.6.2, 5 lines down, replace the sentence beginning with "Alternatively" with the following: "This mechanism can be considered to approximately double the loss factor of the base panels. Alternatively, the panels could be connected together with a layer of visco-elastic material to give a loss factor of about 0.2."

p365, Section 8.2.6.2, 9 lines down, after the words "(0.3 to 0.6 m)", add the words, "or connected with a layer of visco-elastic material or even nailed together".

p371, In the 500 Hz column, 7th number from the bottom, replace S1" with "51"

p379, 2 lines above "Example 8.4", change "Example 8.7" to "Example 8.8"

p380, replace the example table with the following table.

	Octave band centre frequency (Hz)							
	63	125	250	500	1000	2000	4000	8000
TL from Table 8.2	30	36	37	40	46	54	57	59
$\bar{\alpha}_w$ from Table 7.1	0.013	0.013	0.015	0.02	0.03	0.04	0.05	0.06
$\bar{\alpha}_f$ from Table 7.1	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.03
$S_i \alpha_i$ (m)	0.463	0.463	0.525	0.68	1.05	1.36	1.67	2.04
$S_E/S_i \bar{\alpha}_i$	67	67	59	45.6	29.5	22.8	18.6	15.2
$10 \log_{10}(S_E/S_i \bar{\alpha}_i)$	18	18	18	17	15	14	13	12
NR (dB)	12	18	19	23	31	40	44	47

p381, 3rd line down, Equation (8.75) should be (8.65), 6 lines down Equation (8.76) should be (8.66) and 8 lines down, Equation (8.6), should be (8.65).

p381, 4th Eq. in section 3, "30.5/30" should be "30.5/31"

p391, At the end of the paragraph above the figure, add the following sentences. "When paths involving the ground reflected wave on the source side are considered, the straight line distance, d , used in Equation (8.85) is the distance between the image source and the receiver. The same reasoning applies to paths involving ground reflections on the receiver side."

p394, 3 lines following Eq. 8.98, replace "barrier" with "barrier".

p395, replace the four equations for A_b with the following in the same order

$$A_b = 15.8 + 20 \log_{10}[5.8/4.5] = 18.0 \text{ dB}; A_R = 1.3 \text{ dB}; A_b + A_R = 19.3 \text{ dB}$$

$$A_b + A_R = 19.3 \text{ dB}$$

$$A_b = 19.8 + 20 \log_{10}[7.2/4] = 24.9 \text{ dB}; A_R = 2.6 \text{ dB}; A_b + A_R = 27.5 \text{ dB}$$

p395, 6 lines from the bottom, replace "4.6" with "4.7"

p395, Solution, item 1, last line, change "5.18" to "5.20".

$$A_b = 19.5 + 20 \log_{10}[7.5/4.5] = 23.9 \text{ dB}; A_R = 5 \text{ dB}; A_b + A_R = 28.9 \text{ dB}$$

p396, replace the two equations for A_b with the following in the same order.

$$A_b = 12.0 + 20 \log_{10}[4.5/4] = 13.0 \text{ dB}$$

p396, Item 3, lines 2 and 3, change the numbers to 19.3 dB, 19.3 dB, 27.5

$$A_b = 19.8 + 20 \log_{10}[7.2/4] = 24.9 \text{ dB}$$

dB, 28.9 dB, 28.9 dB, 13 dB, 24.9 dB and 24.9 dB

p396, Item 3, line 4, change "5.18" to "5.20".

p396, Item 3, line 4, change "10 dB" to "12 dB"

p399, Figure 8.19, replace r with R

p399, Replace Eq. (8.100) with:

$$\ell'_s = R \theta \cos \alpha$$

$$h'_s = H_b - R \theta \sin \alpha$$

$$\alpha = \frac{1}{2}(\pi - \theta) - \beta$$

$$\beta = \cos^{-1}(H_b/A)$$

$$\theta = \pm \cos^{-1}[1 - (A^2/2R^2)], \quad |R| > A/2$$

p400, 1st paragraph, change "Figure 8.12" to "Figure 8.14"

p401, Eq. (8.107) should be:

$$N = \pm \frac{2}{\lambda} \left\{ \left[\left(X_S^2 + (h_b - Z_S)^2 \right)^{1/2} + \left(X_R^2 + (h_b - Z_R)^2 \right)^{1/2} + b \right]^2 + Y^2 \right\}^{1/2} - d \}$$

p404, Figure 8.21 is missing (see following figure)

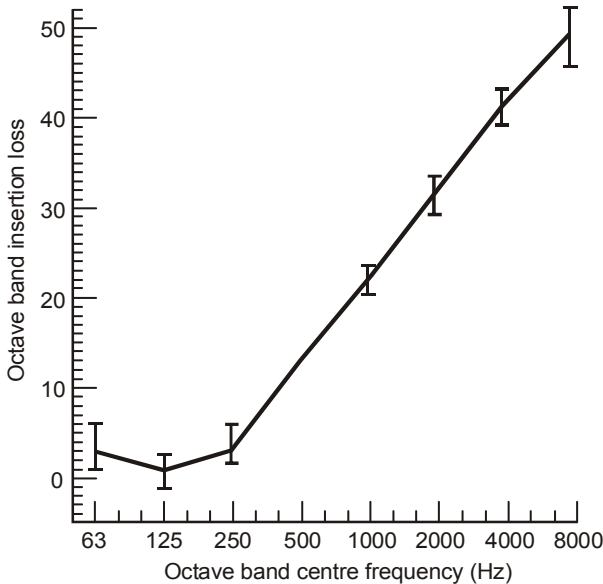


Figure 8.21 Typical pipe lagging insertion loss for 50 mm glass-fibre, density 70-90 kg/m³, covered with a lead / aluminium jacket of surface density, 6 kg/m². The I symbols represent variations in measured values for three pipe diameters (75 mm, 150 mm and 360 mm).

p405, Replace Equations (8.116), (8.117) and (8.119) with the following:

$$X_c = [41.6(m/h)^{1/2} \xi_c (1 - 1/\xi_c)^{-1/4}] - [258h/\ell \xi_c] \quad (8.116)$$

$$C_c = 0.232 \xi_c \ell / h \quad (8.117)$$

$$X_m = [226(m/h)^{1/2} \xi_c (1 - \xi_c^2)] - [258h/(\ell \xi_c)] \quad (8.119)$$

p415, lines 6 and 7 under Eq 9.16, replace with, "the end correction. In this case, $\xi = 0$. For a"

p417, replace the text between Eqs. (9.25) and (9.26) with:

"An alternative expression for the effective length, which may give slightly better results than Equation (9.25), for grazing flow across the holes, and which only applies for flow speeds such that $u_t/(\omega d) > 0.03$, is (Dickey and Selamet, 2001)"

p429, Move Equation (9.52) up one line.

p432, Item 5, line 1, Replace "Equation (8.48)" with "Equation (9.52)"

p439, line following Equation (9.81), replace μ with f_m

p444, In Table 9.2, "19" should be "-19"

p453, 454, Replace the legend in the figures with

curve no	$\frac{\sigma}{\rho h}$
1	0.01
2	0.1
3	0.5
4	1

p459, Figure 9.21, x-axis label, change "S" to "A" and in the caption add "open" immediately before "duct".

p461, In the equation in the centre of the page, change "6" to "5"

p461, 4 lines below the equation in the middle of the page, change "5.5" to "7"

p461, 8 lines below the equation in the middle of the page, change "12.5" to "13"

p462, line 1, change "1.2" to "1.0"

p462, Figure 9.23 caption, last line, change "1992" to "1987"

p464, Replace Table 9.5 with the following:

Duct diameter (mm)	Octave band centre frequency (Hz)					
	63	125	250	500	1000	2000
150	18(20)	13(14)	8(9)	4(5)	1(2)	0(1)
200	16(18)	11(12)	6(7)	2(3)	1(1)	0(0)
250	14(16)	9(11)	5(6)	2(2)	1(1)	0(0)
300	13(14)	8(9)	4(5)	1(2)	0(1)	0(0)
400	10(12)	6(7)	2(3)	1(1)	0(0)	0(0)
510	9(10)	5(6)	2(2)	1(1)	0(0)	0(0)
610	8(9)	4(5)	1(2)	0(1)	0(0)	0(0)
710	7(8)	3(4)	1(1)	0(0)	0(0)	0(0)
810	6(7)	2(3)	1(1)	0(0)	0(0)	0(0)
910	5(6)	2(3)	1(1)	0(0)	0(0)	0(0)
1220	4(5)	1(2)	0(1)	0(0)	0(0)	0(0)
1830	2(3)	1(1)	0(0)	0(0)	0(0)	0(0)

p470, Figure 9.27, caption, and Eq. (9.115), replace " D " with " d "

p471, Eq. (9.116) and (9.117) and 2 lines below Fig. 9.28, replace " D " with " d "

p476, 3rd and 6th line of the first paragraph, change "1979" to "1978"

p478, line above Equation (10.14), change "1979" to "1978"

p479, Figure 10.2, replace the lowest y-axis label (currently 0) with 0.02

p483, In Equation (10.18) and 2 lines above it, replace " e " with " q " to avoid confusion with the distance, e , between spring supports.

p484, In Figure 10.6, the force should be shown as acting on mass m_2 , not

mass m_1 .

- p485 In Eqs. (10.25a,b), the left hand side should be squared.
- p487, line above Equation (10.31), change "1986" to "1988"
- p495, Equation (10.42), remove the symbol " d " from the right hand side.
- P496, Equation (10.48), replace " d " with $|F|/k_1$
- p496, Equation (10.47), the numerator on the RHS should be $3(m_2/m_1)^3$
- p498, 8 lines from the top of the page, change "1979" to "1978"
- p513, Table 11.2, 3rd line in 2000 Hz column should be "25"
- p513, Table 11.2, the 8000 Hz column should be replaced with 13, 15, 18, 27, 35, 35, 26, 32, 32, 34, 42 and 44 respectively and the BFI column for the two tubeaxial entries should be " 7 "
- p513, Remove the paragraph containing Equation (11.2) and remove "(11.2)" in the second to bottom line.
- p514, last Equation, label (11.2)
- p515, Example 11.1 table, replace "30" with "36"
- p517, Equation (11.10), change to:

$$L_w = 72 + 13.5 \log_{10} kW \quad (\text{dB re } 10^{-12} \text{ W})$$
- p526, last line, change "8.8" to "8.3"
- p528, replace the values in the table with the following.

0	72	77	80	81	80	76	69	63
60	74	79	82	83	82	78	71	65
120	61	66	69	70	69	65	58	52
180	55	60	63	64	63	59	52	46

- p535, 4 lines above Eq.(11.33), and 2 lines after Eq. (11.34), change "534" to "60534".
- p536, 4 lines from the bottom, change "534" to "60534".
- p541, Following Eq. 11.64, insert the statement, "If the second term in brackets of Equation (11.64) exceeds 0.3, it is set equal to 0.3".
- p542, line 3, change "534" to "60534".
- p542, Immediately before Equation (11.67), add the following: "Note that the final spectrum levels must all be adjusted by adding or subtracting a constant decibel number so that when A-weighted and added together, the result is identical to the A-weighted overall levels form Equations (11.65) and (11.66)."
- p543, 1 line and 4 lines above Eq. (11.70), change "534" to "60534".
- p544, Equation 11.73, second term on the right should have the " \log_{10} " removed and "17.27" replaced with "17.37", so it reads "-17.37(.....)"
- p544, Replace the last paragraph with, "The octave band external sound pressure levels may be calculated using Equations (11.73) and (11.76) with octave band sound power levels used in Equation (11.76) instead of overall sound power levels."
- p552, The constant in Equation (11.89) should be "55", not "53".
- p558, Replace the paragraph following Table 11.29 with the following:
"The road surface or condition correction is taken as zero for either sealed roads at speeds above 75 km/hr or gravel roads. For speeds below 75 km/hr on impervious sealed roads, the correction is -1 dB. For pervious road surfaces, the correction is -3.5 dB. For concrete roads with deep random grooves greater than 5 mm in width, the correction is, $C_{cond} = 4 - 0.03P$ where P is the percentage of heavy vehicles."
- p559, Replace the nine lines following Eq. 11.102 with the following:
"Low barriers such as twin beam metal crash barriers can have less

effect than soft ground. So if these are used with any proportion, P_d , of soft ground, their effect should be calculated by looking at the lower noise level (or the most negative correction) resulting from the following two calculations:

- Soft ground correction ($0 < P_d < 1.0$), excluding the barrier correction; and
- hard-ground correction ($P_d = 0$) plus the barrier correction."

p560, Remove the sentence beginning 12 lines from the bottom of the page, "Note that the two values for β must add up to 180° "

p561, In the heading and first line, change "FWHA" to "FHWA"

p561, 6 lines from the bottom, add "Menge, et al.," before "1998".

p562, 4 lines under Equation (11.108), add "Menge, et al.," before "1998".

p563, 5th line in first paragraph, and 3 lines under Equation (11.109), replace "1995" with "U.K. DOT, 1995a".

p563, 3 lines under Equation (11.111), replace "1995" with "U.K. DOT, 1995a,b".

p563, p564, Replace the last two lines of page 563 and the top three lines of page 564 with the following:

"Note that different vehicle types must be considered as separate trains. For any specific train type consisting of N identical units, the quantity SEL_{ref} is calculated by adding $10\log_{10}N$ to SEL_v . In addition the track correction, C_2 from Table 11.32 must also be added so that:

$$SEL_{ref} = SEL_v + 10 \log_{10} N + C_2$$

p564, The second entry of "Freight vehicles, tread braked, 2 axles" should actually be "Freight vehicles, disc braked, 4 axles"

p565, Lines 1 and 3, change SEL to SEL_{ref} .

- p565, table 11.32, add C_2 , after "Correction" in the column 2 label.
- p567, In Equation (11.121), remove the minus sign
- p568, Add equation numbers, 11.122 and 11.123 to the equations at the top of the page.
- p580, 10 lines above Equation (12.1), change "1985" to "1986".
- p609, line 2 in the table for fresh water, change "988" to "998".
- p609, line in the table for iron, Young's Modulus = 206, density = 7,600, $\sqrt{E/\rho} = 4910$, $\eta = 0.0005$ and $\nu = 0.27$.
- p609, line in the table for Nylon, move the "6.6" next to "nylon" and Young's Modulus = 2, density = 1,140, $\sqrt{E/\rho} = 1,320$.
- p609, line in table for lead, loss factor = 0.015
- p609, line in table for concrete, loss factor = 0.005 - 0.02
- p610, the last column of numbers is the density and the 2nd last column is Young's modulus.
- p617, In figure captions, change "C.6" to "C.5" and "C.5" to "C.6".
- p621, Change number of Eq. 1.36 to C.24.
- p622, In Equation (C.29), replace Z_N with $Z_N/\rho c$
- p623, In Equation (C.30), replace θ with β in three places.
- p645, Missing references.
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- p661, In the Zinoviev reference, replace "In print" with " **269**, 535-548."
- p663, after last line, add, "ANSI S3.6 – 1997. Specification for Audiometers."
- p667, line 1, replace "E90-99" with "E90-02".
- p715, Change "Noise Reduction Index" to "Noise Reduction Coefficient".